

TURBULENCE, HEAT and MASS TRANSFER 1

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Preface

Turbulence plays a major role in convective heat and mass transfer. Yet an in-depth coverage of topics on turbulence structure and interactions, specifically related to their effects on transfer of heat and species, has seldom been the subject of specialized monographs or conferences proceedings.

This monograph provides recent achievements in the turbulence theory, experiments and computations, relevant to heat and mass transfer. It has been prepared in recognition of a growing interest both in fundamental issues and applications of turbulence related transport phenomena in fluids. The monograph emerged from the papers presented at the International Symposium on Turbulence, Heat and Mass Transfer, held in August, 1994, in Lisbon, under the sponsorship of the International Centre for Heat and Mass Transfer and Instituto Superior Tecnico (Technical University of Lisbon). The event was envisaged as a follow-up of the Joint ICHMT/IUTAM Symposium on Structure of Turbulence, Heat and Mass Transfer, held in Dubrovnik twelve years earlier, in 1982. Since then, many new developments and advances have been made, particularly related to the new experimental techniques, numerical simulation and modelling, as well as in the theory of turbulence, opening new prospects for understanding and resolving problems of turbulence controlled momentum, heat and mass transfer.

The volume contains 92 contributions, selected from among more than 145 papers presented at the symposium. They were chosen on the basis of originality, novelty of approach and topical relevance. All papers were reviewed and revised and many of them were completely rewritten.

The monograph covers a broad range of topics. It begins with reviews of recent advances in some key areas of current activities: similarity analysis, direct numerical and large eddy simulations, and turbulence modelling. This is followed by a series of chapters, covering specific topics. The first provides fresh experimental information on mechanical and thermal turbulence, their structures and interactions in some classic, and more exotic flow situations, as well as a few interesting analytical contributions. Recent advances in turbulence closure modelling, particularly in thermal problems are discussed in the next chapter. Subsequent sections deal with specific issues, such as impingement, separation and reattachment, turbulence related to bulk flow unsteadiness, thermal buoyancy, chemical reactions and combustion, multiphase fluids, heat and mass transfer augmentation, and applications in turbomachinery. The last chapter gives a selection of numerical computations in a variety of complex situations relevant to industrial and environmental heat and mass transfer problems.

The editors believe that the monograph will serve many readers as a source of valuable information and reference, as well as an inspiration for new advancements in the field of turbulence and related problems of heat and mass transport.

We wish to express our thanks and appreciation to the members of the Advisory Committee and Organizing Committee of the Symposium who helped in reviewing and selecting the papers. We also acknowledge the invaluable technical contribution of Ms. Sharmila Sewmar, who skillfully assisted in preparing and editing this monograph.

Editors:

K. Hanjalic

J.C.F. Pereira

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MICROSCALES OF TURBULENCE, MASS TRANSFER CORRELATIONS

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ABSTRACT

For buoyancy driven flows, a fundamental dimensionless number involving a combination of Prandtl and Rayleigh numbers,

$$\Pi_I \sim \frac{Ra}{1 + Pr^{-1}}$$

and a thermal Kolmogorov scale,

$$\eta_\theta \sim (1 + Pr)^{1/4} (a^3 / \mathcal{P}_\beta)^{1/4},$$

are reviewed. Here a and \mathcal{P}_β respectively denote the thermal diffusivity and the buoyant production of thermal energy.

In terms of Π_I ,

$$\eta_\theta / \ell \sim \Pi_I^{-1/4},$$

ℓ being an integral scale. A two-layer turbulence model based on Π_I ,

$$Nu \sim \frac{\text{Sublayer}}{1 - \text{Core}} \sim \frac{\Pi_I^{-1/4}}{1 - \Pi_I^{-1/12}}$$

for internal energy generated buoyancy driven turbulent flow between two horizontal plates correlates well with the experimental data.

For pool fires, a similar dimensionless number involving a combination of flame Prandtl and flame Rayleigh numbers,

$$\Pi_\beta \sim \frac{Ra_\beta}{1 + Pr_\beta^{-1}},$$

and a flame Kolmogorov scale,

$$\eta_\beta \sim (1 + Pr_\beta)^{1/4} (D_\beta^3 / B)^{1/4}$$

are reviewed. Here D_β and B respectively denote the flame diffusivity and the buoyant production of Schvab-Zeldovich energy. In terms of Π_β ,

$$\eta_\beta / \ell \sim \Pi_\beta^{-1/3},$$

ℓ being an integral scale. For fuel consumption in turbulent pool fires, a sublayer turbulence model based on Π_β ,

$$\frac{m'}{\rho D} \sim B \Pi_\beta^{1/3},$$

ρ , D and B respectively being density, diffusivity and transfer number, correlates well with the experimental data.

KEYWORDS

Turbulence, microscales, buoyancy, flame, fire, combustion

INTRODUCTION

Eliminating the time-dependence between two kinematic concepts, the dissipation of the rate of mean kinetic energy per unit mass of turbulent fluctuations

$$[\epsilon] \equiv L^2/T^3,$$

and the kinematic viscosity

$$[\nu] \equiv L^2/T,$$

Kolmogorov (1941) introduced about five decades ago, the length scale

$$\eta = \left(\frac{\nu^3}{\epsilon} \right)^{1/4},$$

now called the microscale of isotropic turbulent flows. About one decade later, extension of this idea to a thermal scale by Oboukhov (1949) and Corrsin (1951),

$$\eta_c = \left(\frac{a^3}{\epsilon} \right)^{1/4}, \quad Pr \rightarrow 0$$

a being thermal diffusivity and Pr Prandtl number, and about

another decade later, extension of the same idea to another thermal scale by Batchelor (1959),

$$\eta_B = \left(\frac{\nu a^2}{\epsilon} \right)^{1/4}, \text{ Pr} \rightarrow \infty$$

followed similar intuitive arguments. Although these scales have been extensively used in the development of some energy and entropy spectra, their extension to the scales of complex (buoyancy driven, reacting, pulsating, etc.) flows, as well as their relevance to turbulent heat and mass transfer correlations have been apparently overlooked except for the recent studies by Arpaci (1986a,b,1990,1992), and Arpaci and co-workers (1991,1993). The objectives of this lecture are two-fold: first, a general approach by which the microscales of complex flows are constructed; second, an interpretation of heat and mass transfer correlations (the latter also including reacting flows) in terms of these scales.

A DIMENSIONLESS NUMBER

As is well-known, the independent dimensionless numbers characterizing buoyancy driven flows are the Rayleigh and Prandtl numbers, Ra and Pr, respectively. A dimensionless number recently proposed by Arpaci (1986,1990) explicitly describes these flows by a combination of Ra and Pr. A review of this dimensionless number is needed for the microscales of buoyancy driven flows.

Let the buoyancy driven momentum balance be

$$F_B \sim F_I + F_V, \quad (1)$$

where F_B , F_I and F_V denoting respectively the buoyant, inertial and viscous forces. Also, let the thermal energy balance be

$$Q_H \sim Q_K, \quad (2)$$

where Q_H and Q_K denoting respectively the enthalpy flow and conduction. Then, from Eq.(1),

$$\frac{F_B}{F_I + F_V} \sim \frac{F_B/F_V}{F_I/F_V + 1}, \quad (3)$$

and from Eq.(2),

$$Q_H/Q_K, \quad (4)$$

the numeral 1 in Eq.(3) implying order of magnitude. Although the force ratios of Eq.(3) and the energy ratio of Eq.(4) are dimensionless, they are usually expressed in terms of velocity which is a dependent variable in buoyancy driven flows:

$$\frac{F_B}{F_V} \sim \frac{g (\Delta \rho) \ell^2}{\mu V}, \quad \frac{F_I}{F_V} \sim \frac{\rho V \ell}{\mu}, \quad \frac{Q_H}{Q_K} \sim \frac{\rho c V \ell}{k}, \quad (5)$$

where ℓ is a characteristic length, and the rest of the notation is conventional. Now, combine Eqs. (3) and (4) for a result independent of velocity:

$$\Pi_N \sim \frac{(F_B/F_V) (Q_H/Q_K)}{(F_I/F_V) (Q_K/Q_H) + 1}, \quad (6)$$

or,

$$\Pi_N \sim \frac{Ra}{1 + \text{Pr}^{-1}} = \frac{\text{Pr} Ra}{1 + \text{Pr}} \quad (7)$$

which is the appropriate dimensionless number for buoyancy driven flows. Here,

$$\sigma = \text{Pr} = \frac{\nu}{a}, \quad Ra = \frac{g}{\nu a} \left(\frac{\Delta \rho}{\rho} \right) \ell^3$$

respectively denote the Prandtl and Rayleigh numbers. The two limits of Eq.(7) are

$$\lim_{\text{Pr} \rightarrow 0} \Pi_N \rightarrow \text{Pr} Ra = \text{Pe}_N,$$

Pe_N being a Peclet number. and

$$\lim_{\text{Pr} \rightarrow \infty} \Pi_N \rightarrow Ra.$$

For a specified temperature difference, the definition of the coefficient of isobaric expansion,

$$\beta = \frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_P,$$

gives

$$\frac{\Delta \rho}{\rho} \sim \beta \Delta T,$$

and Π_N now depends on the usual form of

$$Ra = \frac{g \beta \Delta T \ell^3}{\nu a}. \quad (8)$$

Although the existence of Π_N has never been directly shown, the integral solution for the laminar natural convection near a vertical plate given by Squire (1938) in terms of the Grashof number leads for heat transfer to

$$\text{Nu} = 0.508 \text{Pr}^{1/2} (\text{Pr} + 20/21)^{-1/4} \text{Gr}^{1/4}$$

which can be rearranged in terms of Π_N ,

$$\text{Nu} = 0.508 \Pi_N^{1/4},$$

where Nu is the Nusselt number, and

$$\Pi_N = \frac{Ra}{0.952 + \text{Pr}^{-1}}.$$

Since then the explicit role of Π_N in studies on buoyancy driven flows is usually ignored.

Computational literature (see, for example, Bertin and Ozoe 1986 and Lage, Bejan and Georgiadis 1991) recognize the dependence of the Benard transition on the Prandtl number,

$$(Ra_c)_I = f(\text{Pr})$$

without realizing the fact that

$$\lim_{\text{Pr} \rightarrow \infty} \Pi_N \rightarrow [Ra_c(\infty)]_I$$

which yields

$$(Ra_c)_I = [Ra_c(\infty)]_I \left(1 + \frac{C}{Pr}\right),$$

where C is a constant.

Experimental literature (see, for example, Krishnamurti 1973) does not recognize the dependence of the Benard transition on $Pr \ll 1$. However, it demonstrates the Pr -dependence of higher transitions. Any two successive transitions, illustrated here in terms of the first two, can be qualitatively related by a simple model depending on Π_N ,

$$(Ra_c)_{II} = (Ra_c)_I + \frac{(\Delta Ra_c)_I^2}{1 + Pr^{-1}},$$

or

$$(Ra_c)_{II} = (Ra_c)_I + (\Delta \Pi_N)_I^2,$$

where

$$(\Delta \Pi_N)_I^2 = \frac{(\Delta Ra_c)_I^2}{1 + Pr^{-1}} \quad (9)$$

and

$$(\Delta Ra_c)_I^2 = (Ra_c)_{II} - (Ra_c)_I, \quad Pr \rightarrow \infty.$$

For liquid metals, $Pr \ll 1$ and Eq.(9) is reduced to

$$(\Delta \Pi_N)_I^2 \rightarrow (\Delta Ra_c)_I^2 Pr.$$

For viscous oils, $10^2 < Pr < \infty$, and Eq.(9) is reduced to

$$(\Delta \Pi_N)_I^2 \rightarrow (\Delta Ra_c)_I^2$$

which is independent of Pr because of the negligible inertial effect.

The analytical literature, as well, overlooks the significance of Π_N . Beginning with Malkus and Veronis (1958) for free boundaries, and continuing with Schluter, Lortz and Busse (1965), Gough, Spiegel and Toomre (1975) and Busse (1985) for rigid boundaries, a first order inertial effect is incorporated into heat transfer by an expansion in powers of Pr^{-1} ,

$$\frac{Nu - 1}{Ra - Ra_c} = (C_1 + C_2 Pr^{-1} + C_3 Pr^{-2} + \dots)$$

which can be rearranged, in view of

$$1 - Pr^{-1} + Pr^{-2} - Pr^{-3} + \dots \equiv (1 + Pr^{-1})^{-1},$$

as

$$Nu - 1 \sim \frac{Ra - Ra_c}{1 + Pr^{-1}}$$

or,

$$Nu - 1 \sim \Delta \Pi_N.$$

Some of the empirical correlations show the dependence of Nu on Pr , as well as on Ra , but continue to overlook the significance of Π_N . For example, Catton (1978) suggests for a vertical rectangular cavity,

$$Nu = 0.18 \left(\frac{Pr}{0.2 + Pr} Ra \right)^{0.29}$$

which can be rearranged as

$$Nu = 0.18 \left(\frac{Ra}{1 + 0.2/Pr} \right)^{0.29},$$

or,

$$Nu = 0.18 \Pi_N^{0.29}.$$

For a specified energy generation, the energy balance

$$k \frac{\Delta T}{\ell^2} \sim u'''',$$

rearranged in terms of

$$\Phi = u'''' / \rho c_p,$$

yields

$$\Delta T \sim \Phi \ell^2 / a,$$

and Π_N , now identified with Π_I , depends on

$$Ra_I = \frac{g \beta \Phi \ell^5}{\nu a^2}. \quad (10)$$

The next section is devoted to the development of microscales for buoyancy driven turbulent flows in terms of Π_I .

MICROSCALES

Following the usual practice, decompose the instantaneous velocity and temperature of a buoyancy driven turbulent flow into a temporal mean (denoted by capital letters) and fluctuations

$$\hat{u}_i = U_i + u_i \quad \text{and} \quad \hat{\theta} = \Theta + \theta$$

and let U_i and Θ be statistically steady. Then, the balance of the mean kinetic energy of velocity fluctuations

$$K = \frac{1}{2} \overline{u_i u_i}$$

yields (see, for example, Tennekes and Lumley 1972)

$$U_j \frac{\partial K}{\partial x_j} = - \frac{\partial \mathcal{D}_j}{\partial x_j} - \mathcal{P}_\beta + \mathcal{P} - \epsilon \quad (11)$$

where

$$\mathcal{D}_j = \frac{1}{2} \overline{p u_j} + \frac{1}{2} \overline{u_i u_i u_j} - 2 \nu \overline{u_i s_{ij}}$$

is the transport,

$$\mathcal{P}_\beta = - \overline{g_j u_j} \theta / \Theta_0 \quad (12)$$

is the buoyant production, g_j being vector acceleration of gravity and Θ_0 a characteristic temperature for isobaric ambient,

$$\mathcal{P} = - \overline{u_i u_j} s_{ij} \quad (13)$$

is the inertial production, and

$$\epsilon = 2 \nu \overline{s_{ij} s_{ij}} \quad (14)$$

is the viscous dissipation of turbulent energy.

Also, the balance of the root mean square of temperature fluctuations

$$K_\theta = \frac{1}{2} \overline{\theta^2}$$

gives

$$U_j \frac{\partial}{\partial x_j} (K_\theta) = -\frac{\partial}{\partial x_j} (\mathcal{D}_\theta)_j + \mathcal{P}_\theta - \epsilon_\theta, \quad (15)$$

where

$$(\mathcal{D}_\theta)_j = \frac{1}{2} \overline{\theta^2} u_j - \alpha \frac{\partial}{\partial x_j} \left(\frac{1}{2} \overline{\theta^2} \right)$$

is the thermal transport,

$$\mathcal{P}_\theta = -\overline{u_j \theta} \frac{\partial \Theta}{\partial x_j} \quad (16)$$

is the thermal production, and

$$\epsilon_\theta = \alpha \frac{\partial \theta}{\partial x_j} \frac{\partial \theta}{\partial x_j} \quad (17)$$

is the thermal dissipation.

For a homogeneous pure shear flow (in which all averaged quantities except U_i and Θ are independent of position and in which S_{ij} and $\frac{\partial \Theta}{\partial x_j}$ are constant), Eqs. (11) and (15) reduce to

$$\mathcal{P}_\beta = \mathcal{P} + (-\epsilon) \quad (18)$$

and

$$\mathcal{P}_\theta = \epsilon_\theta. \quad (19)$$

Eq.(18) states that the buoyant production is partly converted into inertial production and partly into viscous dissipation.

On dimensional grounds, assuming $S_{ij} \sim u/\ell$ and $\partial \Theta / \partial x_j \sim \theta/\ell$, Eqs. (18) and (19) may be written as

$$\mathcal{P}_\beta \sim \frac{u^3}{\ell} + \nu \frac{u^2}{\lambda^2}, \quad (20)$$

and

$$u \frac{\theta^2}{\ell} \sim \alpha \frac{\theta^2}{\lambda_\theta^2}, \quad (21)$$

where u and θ respectively denote the rms values of velocity and temperature fluctuations, ℓ is an integral scale, λ and λ_θ are Taylor scales (1935). Eqs. (20) and (21) imply isotropic mechanical and thermal dissipations. Note that the isotropic dissipation is usually a good approximation for any turbulent flow (see for example, Tennekes and Lumley 1972).

To proceed further, invoke the Squire postulate and let

$$\lambda \sim \lambda_\theta \quad (22)$$

in Eq.(20). This is an often misinterpreted pivotal assumption. It postulates the secondary importance of $\lambda \neq \lambda_\theta$ for heat transfer rather than suggesting equal thickness for these scales. Now, elimination of velocity between Eqs. (20) and (21) results in a thermal Taylor scale arranged relative to viscous dissipation

$$\lambda_\theta \sim \ell^{1/3} \left(1 + \frac{1}{\sigma} \right)^{1/6} \left(\frac{\nu \alpha^2}{\mathcal{P}_\beta} \right)^{1/6}, \quad (23)$$

or, arranged relative to inertial production,

$$\lambda_\theta \sim \ell^{1/3} (1 + \sigma)^{1/6} \left(\frac{\alpha^3}{\mathcal{P}_\beta} \right)^{1/6}, \quad (24)$$

where Eq.(23) explicitly includes the limit for $\sigma \rightarrow \infty$ and is

convenient for fluids with $\sigma \geq 1$, and Eq.(24) explicitly includes the limit for $\sigma \rightarrow 0$ and is convenient for fluids with $\sigma \leq 1$.

For the isotropic flow, replacing both ℓ and λ_θ with one scale, say η_θ ,

$$\left(\frac{\lambda_\theta}{\ell} \right) \rightarrow \eta_\theta, \quad (25)$$

Eqs. (23) and (24) are respectively reduced to a thermal Kolmogorov scale for buoyancy driven flows,¹

$$\eta_\theta \sim \left(1 + \frac{1}{\sigma} \right)^{1/4} \left(\frac{\nu \alpha^2}{\mathcal{P}_\beta} \right)^{1/4}, \quad (26)$$

or

$$\eta_\theta \sim (1 + \sigma)^{1/4} \left(\frac{\alpha^3}{\mathcal{P}_\beta} \right)^{1/4}. \quad (27)$$

For $\sigma \gg 1$, Eq.(27) is reduced to

$$\lim_{\sigma \rightarrow \infty} \eta_\theta \rightarrow \left(\frac{\nu \alpha^2}{\mathcal{P}_\beta} \right)^{1/4}. \quad (28)$$

Also,

$$\lim_{\sigma \rightarrow \infty} \mathcal{P} \rightarrow 0 \quad (29)$$

and, in view of Eq.(18),

$$\mathcal{P}_\beta \sim \epsilon. \quad (30)$$

Eq. (28) becomes the scale introduced by Batchelor (1959),

$$\lim_{\sigma \rightarrow \infty} \eta_\theta \rightarrow \eta_B \sim \left(\frac{\nu \alpha^2}{\epsilon} \right)^{1/4}. \quad (31)$$

For $\sigma \ll 1$, Eq.(26) is reduced to

$$\lim_{\sigma \rightarrow 0} \eta_\theta \rightarrow \left(\frac{\alpha^3}{\mathcal{P}_\beta} \right)^{1/4}. \quad (32)$$

Also

$$\lim_{\sigma \rightarrow 0} \epsilon \rightarrow 0 \quad (33)$$

and, in view of Eq.(18),

$$\mathcal{P}_\beta \rightarrow \mathcal{P}. \quad (34)$$

Then, in a viscous layer order of magnitude thinner than η_θ ,

$$\mathcal{P} \rightarrow \epsilon. \quad (35)$$

Now, the inner limit of Eq.(34) matched to the outer limit of Eq.(35) leads to Eq.(30), and Eq.(32) becomes the scale proposed by Oboukhov (1949) and Corrsin (1951),

$$\lim_{\sigma \rightarrow 0} \eta_\theta \rightarrow \eta_C \sim \left(\frac{\alpha^3}{\epsilon} \right)^{1/4}. \quad (36)$$

Finally, for $\sigma \sim 1$, because of (an order of magnitude) equipartition of the buoyant production into inertial production and viscous dissipation, Eq.(18) becomes

$$\mathcal{P}_\beta \sim 2\epsilon, \quad (37)$$

and, Eqs. (26) and (27) are reduced to the scale originated by Kolmogorov (1941),

$$\lim_{\sigma \rightarrow 1} \eta_\theta = \eta \sim \left(\frac{\nu^3}{\epsilon} \right)^{1/4}. \quad (38)$$

¹The first numeral 1 in the right hand side of Eqs. (23), (24), (26) and (27) is related to the numeral 1 of Eq.(3) and implies order of magnitude.

The relation between the thermal microscales and the integral scale may now be obtained by eliminating the factor $(1 + 1/\sigma)$ ($\nu a^2/\mathcal{P}_\beta$) between Eqs. (23) and (26). This readily yields

$$\left(\frac{\eta_\theta}{\lambda_\theta}\right)^2 = \frac{\lambda_\theta}{\ell}. \quad (39)$$

Eqs. (24) and (27) lead to the same relation, as expected. The foregoing scales are utilized in the next section on the development of a heat transfer correlation for buoyancy driven flows. Before this development, however, the relations between these scales and the dimensionless number Π_I (recall Eq. 10), need to be shown.

Note that \mathcal{P}_β usually depends on velocity, and Eq.(26) or (27) expressed in terms of velocity cannot be ultimate forms of the Kolmogorov scale for buoyancy driven flows. To eliminate any velocity dependence, reconsider Eq.(12). On dimensional grounds,

$$\mathcal{P}_\beta \sim g u \theta / \Theta_0. \quad (40)$$

Noting

$$\Theta_0^{-1} \sim \beta,$$

β being the coefficient of thermal expansion, rearrange Eq.(40) as

$$\mathcal{P}_\beta \sim g \beta u \theta, \quad (41)$$

or, with the isotropic velocity

$$u \sim a / \eta_\theta \quad (42)$$

obtained from Eqs. (21) and (25), as

$$\mathcal{P}_\beta \sim g a \beta \theta / \eta_\theta. \quad (43)$$

Now, assume θ across η_θ of volume $(\eta_\theta \ell^2)$ be a result of the rate of internal energy u''' generated per unit of ℓ^3 volume,

$$k \frac{\theta}{\eta_\theta^2} (\eta_\theta \ell^2) \sim u''' \ell^3 \quad (44)$$

which gives

$$\theta \sim \left(\frac{\eta_\theta \ell}{a}\right) \Phi, \quad (45)$$

where $\Phi = u''' / \rho c_p$. Elimination of θ between Eqs. (43) and (45) yields

$$\mathcal{P}_\beta \sim g \beta \Phi \ell. \quad (46)$$

Then, Eqs. (26) and (27) respectively lead to

$$\eta_\theta \sim \left(1 + \frac{1}{\sigma}\right)^{1/4} \left(\frac{\nu a^2}{g \beta \Phi \ell}\right)^{1/4} \quad (47)$$

and

$$\eta_\theta \sim (1 + \sigma)^{1/4} \left(\frac{a^3}{g \beta \Phi \ell}\right)^{1/4}, \quad (48)$$

or,

$$\frac{\eta_\theta}{\ell} \sim \Pi_I^{-1/4}, \quad (49)$$

where

$$\Pi_I \sim \frac{Ra_I}{1 + Pr^{-1}} = \frac{Pr Ra_I}{1 + Pr}. \quad (50)$$

and

$$Ra_I = \frac{g \beta (\Phi \ell^2)}{\nu a} \ell^3 = \frac{g \beta \Phi \ell^5}{\nu a^2} \quad (10)$$

is the Rayleigh number based on Φ .

The thermal intermittency given by Eq.(39) continues to hold. Then, from Eqs. (47) and (48),

$$\frac{\lambda_\theta}{\ell} \sim \Pi_I^{-1/6}. \quad (51)$$

In the next section a heat transfer model based on the foregoing microscales is proposed for buoyancy driven turbulent flows.

A HEAT TRANSFER MODEL

Consider a buoyant flow driven by internal energy generated between two horizontal plates. Assume large enough energy generation resulting in fully developed turbulent conditions. This is an ideal problem for a test on the proposed microscales because of the availability of some experimental and analytical literature. In a manner similar to the Prandtl-Taylor two-layer turbulence model for forced convection, let the buoyancy driven turbulent flow be described by a sublayer next to each plate and a core between these layers. Assume each sublayer be characterized by the Kolmogorov scale, and the diffusion in the core by the Taylor scale.

The mean heat flux in the sublayer, in view of the assumed isotropy (recall Eq. 42), is

$$\theta \sim k \frac{\theta}{\eta_\theta} \sim \rho c_p u \theta \quad (52)$$

which shows the same order of magnitude contributions from conduction and convection. The mean heat flux in the core is

$$q_c \sim k \frac{\theta_c}{\lambda_\theta} + \rho c_p u_c \theta_c \quad (53)$$

which, in view of Eq.(21), or,

$$\frac{1}{\lambda_\theta} \sim \left(\frac{\lambda_\theta}{\ell}\right) \frac{u_c}{a}, \quad (54)$$

may be rearranged as

$$q_c \sim \rho c_p \left(1 + \frac{\lambda_\theta}{\ell}\right) u_c \theta_c \quad (55)$$

and, in view of $\lambda_\theta/\ell \ll 1$, is reduced to

$$q_c \sim \rho c_p u_c \theta_c, \quad (56)$$

where the subscript c indicates to the core. At the interface between the sublayer and core

$$q \sim q_c. \quad (57)$$

There is conclusive evidence about a temperature reversal in

the core of the turbulent Benard problem demonstrated experimentally by Thomas and Townsend (1957), Gille (1967), Chu and Goldstein (1973), and numerically by Herring (1963) and Elder (1969) (Fig. 1). Some of the Kulacki and

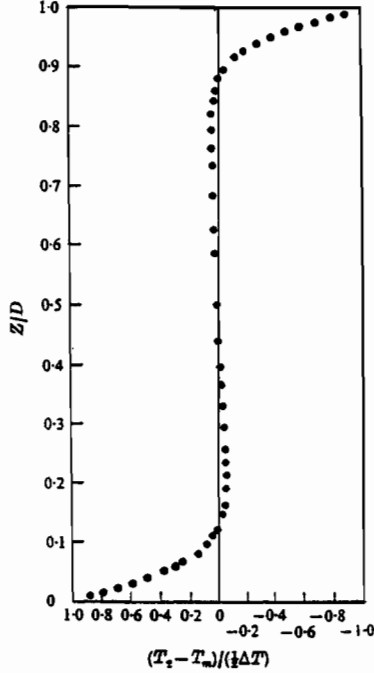


Fig.1 Core temperature reversal. Chu and Goldstein (1973)

Emara (1977) data on electrolytically heated water indicates also to a similar trend for the present case. Accordingly, let

$$\theta - \theta_c \sim \Delta T, \quad (58)$$

where ΔT is the temperature difference across the plates. Inserting θ of Eq.(52) and θ_c of Eq. (56) into Eq.(58), noting Eq.(58),

$$q(1 - \alpha/u_c \eta_\theta) \sim k \Delta T / \eta_\theta \quad (59)$$

which may be rearranged in terms of the Nusselt number,

$$Nu = \frac{q}{k(\Delta T/\ell)}, \quad (60)$$

as

$$Nu \sim \frac{\ell/\eta_\theta}{1 - (\ell/\eta_\theta)(u_c \ell/\alpha)^{-1}}, \quad (61)$$

where the numerator shows the contribution of the sublayer and the denominator shows that of the core on heat transfer. To express Eq.(61) in terms of the length scales alone, reconsider Eq.(21) for velocity of the core,

$$u_c \sim \alpha \frac{\ell}{\lambda_\theta^2}, \quad (62)$$

which may be rearranged as

$$\frac{u_c \ell}{\alpha} \sim \left(\frac{\ell}{\lambda_\theta} \right)^2. \quad (63)$$

In terms of this relation, Eq.(61) becomes

$$Nu \sim \frac{\ell/\eta_\theta}{1 - (\ell/\eta_\theta)(\ell/\lambda_\theta)^{-2}} \quad (64)$$

which, in view of Eqs. (49) and (51), yields a model for any Prandtl number

$$Nu \sim \frac{\Pi_1^{1/4}}{1 - \Pi_1^{-1/12}}. \quad (65)$$

The two limits of this result,

$$\lim_{Pr \rightarrow 0} Nu \sim \frac{(Pr Ra_1)^{1/4}}{1 - (Pr Ra_1)^{-1/12}} \quad (66)$$

and

$$\lim_{Pr \rightarrow \infty} Nu \sim \frac{Ra_1^{1/4}}{1 - Ra_1^{-1/12}}, \quad (67)$$

are identical to the models already proposed by Cheung (1980). Thus, the present review generalizes, via microscales appropriate for buoyancy driven flows, two Cheung correlations into Eq.(65) which is valid for fluids of any Prandtl number. Now, Eq.(65) may be written as an equality in terms of three constants

$$Nu = \frac{C_1 \Pi_1^{1/4}}{1 - C_2 \Pi_1^{-1/12}}, \quad \Pi_1 = \left(\frac{Pr}{C_0 + Pr} \right) Ra_1. \quad (68)$$

Eq.(68) provides a heat transfer correlation for turbulent natural convection driven by internal energy generation between two parallel plates. Although the values of C_0 , C_1 , and C_2 must be determined from experimental data, they are expected to be numerical constants.

The experimental literature on the buoyant turbulent flow driven by volumetric internal energy generation is confined to the studies of Tritton and Zarraga (1967), Fiedler and Wille (1971), Kulacki and Goldstein (1972), Kulacki and Nagle (1975), and Kulacki and Emara (1977). These studies employ electrolytically heated water for which Pr remains within the narrow range of 6-7. If one assumes $C_0 \ll 1$ indicating to a small inertial effect (see Arpaci 1990), the numerical value of

$$\left(\frac{Pr}{C_0 + Pr} \right)^{1/4}$$

can be very closely approximated by unity. Then

$$\Pi_1 \rightarrow Ra_1, \quad Pr > 1$$

and Nu given by Eq.(68) is reduced to

$$Nu = \frac{C_1 Ra_1^{1/4}}{1 - C_2 Ra_1^{-1/12}}. \quad (69)$$

Cheung employs the data of Kulacki and Emara and proposes

$$Nu = \frac{0.206 Ra_1^{1/4}}{1 - 0.847 Ra_1^{-1/12}}. \quad (70)$$

Figure 2 taken from Cheung shows the correlation of the

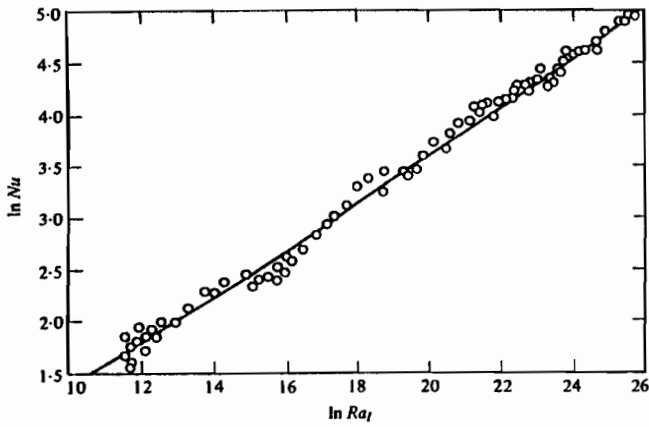


Fig.2 $\ln Nu$ vs. $\ln Ra_l$. —, Eq.(70); O, Kulacki-Emara (1977)

experimental data by Eq.(70). A correlation for any Prandtl number involving the numerical values of C_0 and C_1 in Eq.(68) needs data for another Prandtl range (preferably for liquid metals) which is not presently available. However, for buoyant turbulent flows between two horizontal plates kept at different temperatures, there is extensive data for a variety of fluids (including liquid metals, gases, water and viscous oils). A recently proposed model by Arpaci (1990),

$$Nu = \frac{0.0471 \Pi_N^{1/3}}{1 - 1.734 \Pi_N^{1/9}}, \quad \Pi_N = \frac{Ra}{1 + 0.0414 Pr^{-1}},$$

correlates this data over the range of $10^6 - 10^{11}$. The rest of the review is on mass transfer illustrated in terms of laminar, buoyancy-driven diffusion flames and turbulent pool fires.

LAMINAR DIFFUSION FLAME

A brief dimensional review of laminar flames will prove convenient for the next section on turbulent flames. Accordingly, reconsider the pioneering work of Spalding (1954).

The balance of momentum integrated over the boundary layer thickness δ is

$$\frac{d}{dx} \int_0^\delta \rho u^2 dy + \left(\mu \frac{\partial u}{\partial y} \right)_w = g \int_0^\delta (\rho_\infty - \rho) dy, \quad (71)$$

where ρ is the density, u the longitudinal velocity, μ the dynamic viscosity, and subscripts w and ∞ denote wall (fuel surface) and ambient conditions. Also, the balance of the first Schvab-Zeldovich (heat+oxidizer) property integrated over the boundary layer thickness δ_β is

$$\frac{d}{dx} \int_0^{\delta_\beta} \rho u (b_\infty - b) dy - B (\rho v)_w = \left(\rho D \frac{\partial b}{\partial y} \right)_w, \quad (72)$$

where v_w is the velocity normal to the fuel surface; $Le = \alpha/D = 1$, α and D being thermal and mass diffusivities, respectively; b and the transfer number B (Busemann 1933) are defined as,

$$b = (Y_O Q / \nu_O M_O + h) / h_{fg}, \quad (73)$$

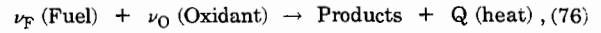
and

$$B = b_\infty - b_w, \quad (74)$$

or, in terms of Eq.(73), explicitly,

$$B = (Y_{O\infty} Q / \nu_O M_O - h_w) / h_{fg}. \quad (75)$$

Here, Y_O is the mass fraction of the oxidizer; $Y_{O\infty}$ its ambient value; Q is the heat released according to single global chemical reaction



where

$$\frac{Q}{\nu_O M_O} = \left(\frac{Q}{\nu_F M_F} \right) \left(\frac{\nu_F M_F}{\nu_O M_O} \right) \quad (77)$$

$Q/\nu_F M_F$ being the lower heating value (heat released per kg of fuel), $\nu_F M_F / \nu_O M_O$ the stoichiometric fuel to oxidant ratio (kg fuel/kg oxidant); ν_F , ν_O , and M_F and M_O being the fuel and oxidant stoichiometric coefficients and molecular weights, respectively; h the specific enthalpy relative to ambient temperature; $h_w = c_p (T_w - T_\infty)$, c_p the specific heat, T_w and T_∞ fuel surface and ambient temperatures, respectively; h_{fg} the heat of evaporation.

On dimensional grounds, Eq.(71) yields

$$U \frac{U}{\ell} \delta + \nu \frac{U}{\delta} \sim g \left(\frac{\Delta \rho}{\rho} \right) \delta, \quad (78)$$

U being a characteristic longitudinal velocity, and ℓ a length scale characterizing the direction of flow. Similarly, Eq.(72) yields

$$U \frac{B}{\ell} \delta_\beta - v_w B \sim D \frac{B}{\delta_\beta}. \quad (79)$$

In terms of the surface mass balance,

$$\rho v_w \sim \rho D \frac{B}{\delta_\beta}, \quad (80)$$

Eq.(79) may be rearranged as

$$U \frac{B}{\ell} \sim D (1 + B) \frac{B}{\delta_\beta^2}, \quad (81)$$

and, in terms of the Squire postulate for buoyancy-driven flows,

$$\delta \sim \delta_\beta, \quad (82)$$

Eq.(78) becomes

$$U \frac{U}{\ell} + \nu \frac{U}{\delta_\beta^2} \sim g \left(\frac{\Delta \rho}{\rho} \right). \quad (83)$$

The Squire postulate has been well-tested in natural convection even for δ/δ_β differing considerably from unity. Also, because of the same b-gradient involved with Eqs. (79) and (80), the factor $(1+B)$ is independent of the dimensional arguments leading to Eq.(81). For notational convenience, let

$$D_\beta = D (1 + B). \quad (84)$$

Then, Eq.(81) is reduced to

$$U \frac{B}{\ell} \sim D_\beta \frac{B}{\delta_\beta^2}. \quad (85)$$

Clearly, Eqs. (83) and (85) can be directly obtained from the corresponding differential formulations, provided D_β is assumed for diffusivity in the latter.

A dimensionless number that describes buoyancy-driven diffusion flames may now be obtained by coupling Eqs. (83) and (85). Since velocity is a dependent variable for any buoyancy-driven flow, its elimination yields

$$\frac{\ell}{\delta_\beta^2} \left(1 + \frac{D_\beta}{\nu}\right) \sim \frac{g}{\nu D_\beta} \left(\frac{\Delta\rho}{\rho}\right) \quad (86)$$

which, in terms of a flame Schmidt number,

$$\sigma_\beta \sim \frac{\nu}{D_\beta} \quad (87)$$

and a flame Rayleigh number,

$$Ra_\beta = \frac{g}{\nu D_\beta} \left(\frac{\Delta\rho}{\rho}\right) \ell^3, \quad (88)$$

may be rearranged as

$$\frac{\ell}{\delta_\beta} \sim \Pi_\beta^{1/4}, \quad (89)$$

where

$$\Pi_\beta \sim \left(\frac{\sigma_\beta}{1 + \sigma_\beta}\right) Ra_\beta \quad (90)$$

is a fundamental dimensionless number for diffusion flames. Actually, the numeral one in Eq.(90) is an unknown constant because of the dimensional nature of the foregoing arguments. Eq.(87) reflects this fact by its proportionality sign. Also, in view of

$$\frac{\Delta\rho}{\rho} \sim \frac{\rho_\infty - \rho_f}{\rho_f} = \frac{T_f - T_\infty}{T_\infty}, \quad (91)$$

the Rayleigh number may be more appropriately written as

$$Ra_\beta = \frac{g (T_f - T_\infty) \ell^3}{\nu D_\beta T_\infty}. \quad (92)$$

Now, in terms of a (fuel) mass transfer coefficient h_β ,

$$Sh_\beta = \frac{h_\beta \ell}{D} \sim \frac{\ell}{\delta_\beta}, \quad (93)$$

Sh_β being a flame Sherwood number. Then, the fuel consumption in a laminar diffusion flame of size ℓ ,

$$\frac{m'}{\rho D} = \frac{m_w'' \ell}{\rho D} = Sh_\beta B \sim B \frac{\ell}{\delta_\beta}, \quad (94)$$

(m_w'' being the fuel consumption per unit area) may be written

in terms of Eq.(89) as

$$\frac{m'}{\rho D} \sim B \Pi_\beta^{1/4}, \quad (95)$$

or, explicitly,

$$\frac{m'}{\rho D} \sim B \left(\frac{\sigma_\beta}{1 + \sigma_\beta}\right)^{1/4} Ra_\beta^{1/4}, \quad (96)$$

or, in terms of the usual Rayleigh number for mass transfer,

$$Ra = \frac{g}{\nu \alpha} \left(\frac{\Delta\rho}{\rho}\right) \ell^3, \quad (97)$$

as

$$\frac{m'}{\rho D Ra^{1/4}} \sim B \left(\frac{\sigma_\beta}{1 + \sigma_\beta}\right)^{1/4} \left(\frac{D}{D_\beta}\right)^{1/4}. \quad (98)$$

Now, introduce the definition of the usual Schmidt number,

$$\sigma = \frac{\nu}{D}, \quad (99)$$

and combine Eqs. (84) and (87) for

$$\frac{\sigma_\beta}{\sigma} \sim \frac{D}{D_\beta} = \frac{1}{1 + B}. \quad (100)$$

Then, noting the proportionality and equality relations of Eq.(100), an equality replacing Eq.(98) may be written as

$$\frac{m'}{\rho D Ra^{1/4}} = \frac{C_1 B}{(C_0 + B)^{1/4} (1 + B)^{1/4}}, \quad (101)$$

where C_0 and C_1 remain to be determined from a computer/laboratory experiment, or, from an analytical solution. Arpaci and Selamet (1991) evaluates C_0 and C_1 by the numerical work of Kim, deRis and Kroesser (1971). The foregoing dimensional arguments are extended in the next section to turbulent flames.

TURBULENT DIFFUSION FLAME. POOL FIRE

Following the usual practice, decompose the instantaneous velocity and the first Schvab-Zeldovich (heat+oxidizer) property of a buoyancy-driven, turbulent diffusion flame into a temporal mean (denoted by capital letters) and fluctuations

$$\tilde{u}_i = U_i + u_i \text{ and } \tilde{b} = B + b.$$

For a homogeneous pure shear flow (in which all averaged except U_i and B are independent of position and in which S_{ij} is a constant), the mean kinetic energy of velocity fluctuations and the root mean square of the first Schvab-Zeldovich property yield

$$B = \mathcal{P} + (-\epsilon) \quad (102)$$

and

$$\mathcal{P} = \epsilon_\beta, \quad (103)$$

where

$$B = -g_i \overline{u_i \theta} / \Theta_0 \quad (104)$$

is the buoyant production (imposed),

$$\mathcal{P} = -\overline{u_i u_j} S_{ij} \quad (105)$$

is the inertial production (induced),

$$\epsilon = 2\nu \overline{s_{ij} s_{ij}} \quad (106)$$

is the dissipation of turbulent energy, and

$$\mathcal{P}_\beta = - \overline{u_i b} \frac{\partial B}{\partial x_i} \quad (107)$$

and

$$\epsilon_\beta = D_\beta \overline{\left(\frac{\partial b}{\partial x_i} \right) \left(\frac{\partial b}{\partial x_i} \right)} \quad (108)$$

are the production and dissipation of the first Schvab-Zeldovich property, respectively. Note that the incorporation of the boundary mass transfer into the b-balance is taken into account by considering the b-dissipation in terms of D_β . For buoyancy-driven flows, kinetic dissipation retains its usual form.

On dimensional grounds, Eqs. (102) and (103) lead to

$$B \sim \frac{u^3}{\ell} + \nu \frac{u^2}{\lambda^2}, \quad (109)$$

and

$$u_\beta \frac{b^2}{\ell} \sim D_\beta \frac{b^2}{\lambda_\beta^2}. \quad (110)$$

where λ and λ_β are the Taylor microscales associated with momentum and the first Schvab-Zeldovich property.

Now, for a buoyancy-driven turbulent diffusion flame, following the Squire postulate, assume

$$u \sim u_\beta, \quad \lambda \sim \lambda_\beta. \quad (111)$$

Then, elimination of the velocity between Eqs. (109) and (110), gives

$$\lambda_\beta \sim \ell^{1/3} (1 + \sigma_\beta)^{1/6} \left(\frac{D_\beta^3}{B} \right)^{1/6}. \quad (112)$$

Under conditions of isotropic flow,

$$\left(\frac{\lambda_\beta}{\ell} \right) \rightarrow \eta_\beta, \quad (113)$$

and, Eq.(112) leads to a Kolmogorov microscale

$$\eta_\beta \sim (1 + \sigma_\beta)^{1/4} \left(\frac{D_\beta^3}{B} \right)^{1/4}, \quad (114)$$

where, on dimensional grounds,

$$B \sim g u \theta / \Theta_0 \quad (115)$$

Θ_0 being the temperature of isobaric ambient. The foregoing microscale is identical in form to Eq.(27) introduced earlier. Furthermore, for $\sigma_\beta \rightarrow 0$, Eq.(114) is reduced in form to the microscale discovered by Oboukhov and Corrsin,

$$\eta_\beta \sim \left(\frac{D_\beta^3}{B} \right)^{1/4}. \quad (116)$$

Also, for $\sigma_\beta \rightarrow \infty$, Eq.(114) is reduced in form to the scale discovered by Batchelor,

$$\eta_\beta \sim \left(\frac{\sigma_\beta D_\beta^3}{B} \right)^{1/4} = \left(\frac{\nu D_\beta^2}{B} \right)^{1/4}. \quad (117)$$

Now, assume

$$\theta \sim \Delta T, \quad (118)$$

ΔT being the imposed temperature difference, and note, for gaseous media,

$$\Theta_0^{-1} = \beta. \quad (119)$$

Then, Eq.(115) becomes

$$B \sim g \beta u \Delta T, \quad (120)$$

or, in view of Eq.(110),

$$B \sim g \beta D_\beta \ell \Delta T / \lambda_\beta^2. \quad (121)$$

Insertion of Eq.(121) into Eq.(112) leads to a Taylor microscale in terms of the buoyant force rather than buoyant energy,

$$\lambda_\beta \sim \ell^{1/4} (1 + \sigma_\beta)^{1/4} \left(\frac{D_\beta^2}{g \beta \Delta T} \right)^{1/4}, \quad (122)$$

or, under the isotropy stated by Eq.(113), to a Kolmogorov microscale,

$$\eta_\beta \sim (1 + \sigma_\beta)^{1/3} \left(\frac{D_\beta^2}{g \beta \Delta T} \right)^{1/3}. \quad (123)$$

Now, the Taylor and Kolmogorov scales for any σ_β may be rearranged in terms of Π_β as

$$\frac{\lambda_\beta}{\ell} \sim \Pi_\beta^{-1/4} \quad (124)$$

and

$$\frac{\eta_\beta}{\ell} \sim \Pi_\beta^{-1/3}. \quad (125)$$

Let the turbulent diffusion flame near a vertical fuel or the pool fire over a horizontal fuel be controlled by a turbulent sublayer.

Assume the thickness of this layer be characterized by η_β .

Then, the averaged fuel consumption is found to be

$$\frac{m'}{\rho D} = B \frac{\ell}{\eta_\beta} \sim B \Pi_\beta^{1/3}, \quad (126)$$

or, explicitly,

$$\frac{m'}{\rho D} \sim B \left(\frac{\sigma_\beta}{1 + \sigma_\beta} \right)^{1/3} Ra_\beta^{1/3}, \quad (127)$$

or, in terms of the usual Rayleigh number,

$$\frac{m'}{\rho D Ra^{1/3}} \sim B \left(\frac{\sigma_\beta}{1 + \sigma_\beta} \right)^{1/3} \left(\frac{D}{D_\beta} \right)^{1/3}. \quad (128)$$

Now, rearranging Eq.(128) in terms of Eqs. (84) and (87),

$$\frac{m'}{\rho D Ra^{1/3}} = \frac{C_1 B}{(C_0 + B)^{1/3} (1 + B)^{1/3}}, \quad (129)$$

where C_0 and C_1 are to be determined from the experimental literature. The 1/3-power law of the Rayleigh number in pool fires is supported experimentally (Kanury 1975, Lockwood and Corlett 1987, Alpert 1977).

Now, in a manner similar to the three regimes of laminar flames (see Arpaci and Selamet 1991), the regimes of turbulent flames may be identified. For small values of B ,

$$\lim_{B \rightarrow 0} \left(\frac{m'}{\rho D Ra^{1/3}} \right) \rightarrow B. \quad (130)$$

For $B > 1$, inertial effects are negligible and Eq.(129) is reduced to

$$\frac{m'}{\rho D Ra^{1/3}} \rightarrow B^{2/3}. \quad (131)$$

For $B \gg 1$,

$$\lim_{B \rightarrow \infty} \left(\frac{m'}{\rho D Ra^{1/3}} \right) \rightarrow B^{1/3}. \quad (132)$$

The experimental data on small fires (see Corlett (1968,1970), de Ris and Orloff (1972), Burgess *et al.* 1961) appears to correlate well with Eq.(129) as shown in Fig. 3. The original figure is taken from de Ris and Orloff who rearranged Fig. 11 of Corlett (1970) for ethane-nitrogen flames burning above a 10.16 cm diameter burner and compared with their model. The open symbols in the original figure for pure ethane are deleted here since they include a small radiative heat transfer component towards the burner surface. Remaining data from Corlett represents the dominant convective component of the surface heat transfer. Half-filled symbols indicate increasing heat transfer with increasing velocity of gases leaving the burner surface. Also included in Fig. 3 are two data points from Burgess *et al.* for liquid methanol and liquid butane as shown by open symbols. The low B-range is in the vicinity of extinction of the flames.

Arpaci (1990) has recently demonstrated, with a correlation on natural convection, the sensitivity of C_0 to experimental data. A preliminary attempt for the evaluation of C_0 and C_1 by a least-square fitting of Eq.(129) to Corlett's data demonstrates a similar sensitivity. Here, following the approach taken in the preceding section on laminar flames, the value of $C_1/C_0^{1/3} = 0.16$ is taken from the recent data of Fujii and Imura (1972). Then, at $B=5$, $1/C_0 = 0.05$ is evaluated by fitting Eq.(129) to Corlett's data. With these values,

$$\frac{m'}{\rho D Ra^{1/3}} = \frac{0.16 B}{(1 + 0.05B)^{1/3} (1 + B)^{1/3}} \quad (133)$$

which agrees very well with the correlation already given by de Ris and Orloff (1972)

$$\frac{m'}{\rho D Ra^{1/3}} = 0.16 B \left[\frac{\ln(1 + B)}{B} \right]^{2/3} \quad (134)$$

obtained from the stagnant film theory coupled with the empirically assumed 2/3-power law. The maximum difference between the two correlations remains less than 1.8% for the entire B-range. This agreement, despite the fact that they are developed by following quite different arguments, is remarkable. Both models predict only the region $B \geq 1$ beyond bifurcation (Arpaci and Selamet 1991).

So far, the proposed models for laminar and turbulent flames and fires exclude any effect of radiation. Because of different intrinsic nature of radiation and conduction (or any diffusion), the Schvab-Zeldovich transformation used in the present review no longer applies to radiation-affected flames. On intuitive grounds, the emission of radiation (hotness of flame) has been already incorporated into the heat of combustion and the latent heat of evaporation by fractional lowering (say γ and ψ) of

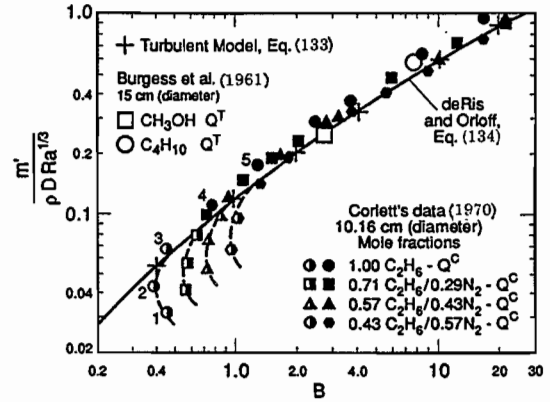


Fig.3 Correlation of turbulent data

these properties (see Kanury 1975). However, because of the lack of experimental data on the absorption effect (optical thickness), no attempt is made here to demonstrate its influence on γ and ψ .

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