SEMI-EMPIRICAL TURBULENCE MODELS: THEORY AND EXPERIMENT

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CONTENTS

READER'S NOTE

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics and the other is the turbulent motion of fluids. And about the former I am rather optimistic ."

Sir H. Lamb, 1932 [99]

This monograph present a review of methods that exist for describing the structure of a turbulent flow and the behavior in the flow under various conditions of the average velocity, variance of the velocity pulsations, one-point and two-point correlation moments of various orders, integral scales and microscales, spectral distributions. The turbulent motion often demonstrates a lot of peculiarities because of the reason that the integral correlation scale is comparable to the characteristic size of the flow.

There is a problem of closure of equation sets that describe the turbulent flow. Methods of solving the closure problem in the continuum mechanics are considered: when the free path is short, and especially in transport process descriptions with a finite free path. Analogies are found for many approximations introduced to describe the turbulent flow; the fruitfulness of the kinetic approach is advocated.

Methods of statistical modeling of various level of complexity are considered as the more detailed investigation of the turbulent flow structure is being pursued. The book elaborates on models of the mixing path theory; those are models that use equations for energy and the turbulence dissipation, and equations of transport for the components of a second-order correlation tensor.

It is demonstrated that the nonlocal nature of the transport is caused by a relatively large ratio of the integral correlation scale to the characteristic size of the flow. Taking into account the diffusion of the two mentioned values permits to derive a more flexible relationship between the stress tensor and the strain rate tensor the principal axes of which may be of different directions. This makes the transport processes strongly anisotropic, where lateral gradients may cause diffusion flows to appear in the field homogeneity direction. The effects of the nonlocal transport are demonstrated by a number of problems:

- development of a planar wake after a cylinder, where an "apparent negative turbulent viscosity" effect appears;
- a diffusion stage of the development of a turbulent wake behind a body that moves in a stratified medium;
- peculiarities of turbulent transport in a vortex core;

- turbulent diffusion of momentum and heat in a planar flow for an atmosphere layer of a constant friction stress and heat flux.

The construction of a transport equation for determining the integral scale of turbulence is discussed. In this regard, a model of the spectrum for a homogeneous isotropic field is identified and rules of similarity for it are formulated.

A set of kinetic equations for the probability densities in one-point and two-point description is used to derive an equation set for one-point and two-point second-order correlation moments. A set of Fokker-Planck-type approximate equations is employed. A method for introduction of relaxation times is described. A connection is established between constants in semi-empiric equations for the moments with the Kolmogorov spectral constant. An example is discussed of attenuation of a homogeneous isotropic turbulence.

The book presents experimentation results on the behavior of a turbulent flow under the influence of a vorticity created by rotation of a tube with respect to its longitudinal axis; on the behavior of spectral distributions which correspond to the second, third, and fourth-order moments; on the behavior of one-dimension and joint probability densities. The applicability of the assumption of the fourth-order cumulants being equal to zero is discussed (a Millionschikov approximation). Peculiar features of the behavior of the joint probability densities at abnormal values of the pulsations are addressed. Results obtained in Moscow Physical and Technical Institute are used.

Although some problems receive only a cursory attention, this approach seems even useful for the reader in order to get a more thorough comprehension of the behavior of a highly dynamic turbulent flow under various changing conditions. Bibliographic references are given for further studies on problems of choice. The book provides a list of issues which are important for the improvement of the semi-empiric theory of turbulent transport but not yet investigated to a sufficient extent.

In short, the objective of this book is a propaganda of usefulness of the kinetic approach in the construction of methods for describing turbulent flow properties.

First chapter discusses general models of continua used in the turbulence theory. Analogies with a molecular transport in a continuous medium are developed. A stress tensor is introduced, balance equations for the mass, momentum and energy fluxes are presented. This set of equations is not yet closed. Its closure is based on models of the medium: a perfect fluid, a viscous fluid, a stress-relaxation medium.

Second chapter presents a description of the behavior of a gas medium based on the kinetic theory, a Boltzmann equation. This chapter is important from the standpoint of an analogy between a turbulent and a free molecular flow. Equations of transport for various order moments of the velocity field are given (Maxwell's equations): mass, momentum, energy, the components of the stress tensor. A kinetic equation model in the relaxation approximation is discussed. The short free path means a nearly equilibrium state. An appropximate solution of the kinetic equation is provided. Expressions for diffusion fluxes of a "gradient type" are presented.

Third chapter is dedicated to ε-models of turbulence. It analyzes a dissipation rate equation and suggests new approximations of terms in the equation for ε. Further, equations for second and third moments are given where the effect of volume forces is taken into account directly, which has not been done in other publications. This makes it possible to derive new equations of the turbulent motion for rotating flows and employ them to describe experimental results. Third part of the chapter deals with applications of the developed semi-empiric theory to descriptions of rotating flows in channels. General equations in cylindrical coordinates are derived for the second-order and third-order moments. General formulations for differential and algebraic models are discussed.

Fourth chapter deals with a homogeneous isotropic turbulence based on the approach of a local-isotropy theory by Kolmogorov and Obukhov. The chapter presents contemporary results on a generalized model of spectral distribution for an isotropic field. There is a description of a spectral transport model, of a von Karman model approximation for the shape of the spectrum, an extended von Karman (EVK) model for the spectrum in a homogeneous isotropic field, its similarity properties, and an EVK model for a scalar field. The spectral models are a basis for the theory of spectral models of inhomogeneous turbulence, which is currently being developed by Harlow and Orzaga schools in USA.

Fifth chapter discusses semi-empiric turbulence models based on a set of equations for the pulsation moments of velocities. These equations are the basis for the method of statistical moments: SM-1 of first order, SM-2 of second order, and SM-3 of third order. Approximate models of closure based on an hierarchy of relaxation times are discussed. The natural method of closure is the Millionschikov hypothesis that fourth moments can be expressed via second moments. A local-balance hypothesis is used; under this assumption, algebraic equations can be used for second-order and third-order moments. Those form the basis for a differential model (DRSM) and an algebraic model (ARSM). A problem of a parallel flow with a constant shift of velocity is under consideration; the object of study is the behavior of the second-order moments depending on the magnitude of the shift. The ARSM model is used; Rodi's approximation is employed to take account of the rapid convection and diffusion processes. The Langevin equation for a one-point probability density function takes into consideration the effect of a lift in the shear flow that acts on a pulsating "mole". The solution is in good qualitative accordance with experimental data and Launder's solution. Further, similar approximations are used to consider turbulent diffusion. Approximations are used that have been introduced for the description of the diffusivity, based on its being proportional to the structural function and the two-point relaxation time. If the distance between two points is small, the diffusivity value is proportional to the squared distance, and if the distance is big, it is proportional to the product of the integral scale and the pulsation velocity. In the inertial interval of the distances, it follows the "4/3" law by Richardson.

In addition to the method of moments that follows from the Navier–Stokes equations, an equation for the probability density function (PDF) can be also derived. Therefore sixth chapter discusses the derivation and applications of a model equation for the probability density in the semi-empiric theory of turbulent transport. This equation plays the role of a kinetic equation in the turbulence theory for the probability density of pulsation fields (Monin, Lundgren, Ievlev). In addition to the one-point density, a model equation for the two-point pulsation probability density is considered, which has the form of a Fokker–Planck equation. Methods for closure of the moment equation set are discussed, together with the example of attenuation of a homogeneous isotropic turbulence in a flow behind a grate. The PDF equations contain the relaxation times for which approximations and relationships between the model's constants are discussed and a link is established to Kolmogorov's spectral constant. Results are presented on the behavior of the joint probability density of the pulsation field in a turbulent flow. An isotropic flow behind a grate is investigated; a comparison is made to the Gaussian distribution; the behavior of the pulsation's joint probability density is analyzed. Discussed examples include flows in a jet and in a tube; an analysis of intermittency effects is given by the example of a jet flow. An example is presented of the behavior of spatial correlation functions; a comparison is made with Millionschikov's hypothesis. There is a discussion on the importance of a nonlocal description of transport processes and some unresolved problems in both theory and experiments.

The next chapter deals with cases of anisotropy of the turbulent transport under external actions on the turbulent flow; the discussed example is a turbulent diffusion wake in a stratified medium. This case is a wake limited vertically; similarity rules are under consideration. Another example is a turbulent motion in the bottom layer of the atmosphere, in a layer of constant friction stress and constant heat flux. The problem is based on a set of equations for the second-order moments. Asymptotic laws for an unstable, neutral, and stable stratification are investigated; the role played by the transport's anisotropy is analyzed.

The non-local nature of the turbulent transport is studied by the example of the development of turbulence in a planar wake where the phenomenon of "negative" viscosity manifests itself. The numerical modeling is based on transport equations for the third-order moments.

Tenth chapter is dedicated to processes in a viscous sublayer and intermittency in the wall area. Approaches are discussed where a Loytsiansky influence function is introduced to express the dependence of the turbulent viscosity coefficient: a relationship is introduced where the intermittency coefficient depends on the value of the local Reynolds number. The possibility of introducing an "hierarchical" model is discussed, which could produce approximate estimations for the anisotropy coefficients, the spectrum and the scales based on a solution for velocity, energy of pulsation motion K, and dissipation rate ε. Ways to develop and improve a kinetic model for the description of turbulent transport are also discussed. The standard k-ε model has a singularity on the wall in the boundary layer. To remove the singularity, it is suggested to take into account the important parameters of the viscous sublayer: dynamic velocity v_* and viscosity ν. This approach means essentially the derivation of a new form for the wall influence functions. Further from the wall the equations become standard k-ε equations. An attempt is made to identify the wall influence functions with a function for the intermittency coefficient. Next, the development of an "hierarchical" model of turbulent transport is discussed. We try to use an expression suggested by Loytsiansky for the turbulent viscosity coefficient, which takes into account the molecular and molar interaction in the flow, as a relationship for the intermittency coefficient that depends on the local Reynolds number (the value ν_T/ν). A comparison is made to data available for the external part of the boundary layer, the external part of a round free jet, and a wake behind a cylinder. There seems to be a qualitative conformance. Equations derived by using the one-point intermittency PDF are in accordance with those given in publications.

Eleventh chapter elaborates on models to describe the turbulent motion in cylindrical tubes and vortex flows. The object of study includes mechanisms of turbulent transport in the vortex core and the role played by the anisotropy of the transport that causes a reduction of radial diffusion flows; in particular, the development of a thermal wind in atmosphere is investigated. Unique experimental results are presented: the behavior of characteristic parameters of a turbulent flow in the conditions of vorticity appearing in a rectilinear round tube that rotates with respect to its longitudinal axis; and a reduction of the pulsation energy and the friction stress along the flow. Results of hot-wire experimental studies are presented; they concern second, third, and fourth moments of spectral distributions. Anm approximate similarity in the energy-containing interval is demonstrated. The theoretical description is based on a semi-empiric ARSM model for a turbulent flow where the effect of rotation is taken into account. Using the one-point PDF equations, equations of the ARSM model were derived in the boundary (thin) layer approximation taking into account the effect of rotation on the coefficients of "turbulent viscosity". The effect of rotation is also taken into consideration on the values of fluxes of the pulsation motion energy and the dissipation rate. Simplifying assumptions are made in order to make the model feasible. The wave motion in some turbulent flows is also under study. Results of experimentations on the equilibration of fields in a turbulent flow inside a tube with strongly asymmetric data at the inlet are presented. A comparison is made to the concept of wave motion of perturbations in a turbulent flow.

Twelfth chapter discusses the numerical modeling of turbulent flows with rotation in channels. The derived ARSM model in the boundary layer approximation is used to analyze the behavior of a rotating flow of two mixing jets in a cylindrical mixing chamber, and so on. In order to stabilize the calculation process, a re-calculation of the longitudinal velocity field is performed on the basis of the flow rate conservation condition; a boundary condition that corresponds to the "law of wall" is used. The initial conditions (at the inlet) are set on the basis of experimental data for the longitudinal and azimuth components of the velocity and the pulsation motion energy; Prandtl's formula is used to set the conditions for the dissipation rate. An approximate solution has been obtained by setting the longitudinal pressure gradient to zero; all functions are in good qualitative correspondence with the relationships obtained experimentally. By using the dissipation rate as a parameter and varying the constant factors at the rotation influence functions we can achieve a stable calculation process for a given length of the flow and a better correspondence between the experimental and calculated results. In addition, rotating flows in a conical diffuser and in a tube with an elbow are considered.

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INTRODUCTION

Turbulent flows are a fundamental phenomenon as they can be widely encountered in both the nature and engineering: nebulae of stars, motion of water in the rivers, atmospheric fronts, motion of air as the aircraft fly or vehicles move on the roads, motion of gas and oil in pipelines, motion of the air-fuel mix in internal combustion engines of vehicles and aircraft, compressors, fans, vacuum cleaners – all demonstrate turbulent motion. In spite of the many centuries of the usage and studies of the phenomena, we are still far from an adequate description and understanding. In some cases the turbulent motion is a desired result that facilitates the process; the examples comprise a range of phenomena from stirring tea in a cup to flow control systems and devices that make the flow laminar when the turbulent motion is undesirable.

The latter cases include craft that move at a great speed but with a small resistance of the medium. The nature demonstrates examples where fish have developed devices not yet reproducible by human technology. Some fish swim at a great speed which corresponds to a hypercritical Reynolds number where turbulence ought to take place but the resistance is nearly as low as laminar. The reason why the resistance of the fish is low seems to be their integument that reduces the resistance due to an effect similar to Toms' and to "feedback" effects where the integument of a moving fish adjusts its shape dynamically so as to suppress vortices which agitate the water flowing around. This example shows how much is yet to understand and apply in the control of the turbulent flow.

The goal of the turbulent flow studies can be formulated as **turbulence flow control**. The first step is to understand the nature of turbulence. In spite of a long history of research in fluid and gas mechanics and its seeming simplicity in comparison with microphysics, first cornerstones in the foundation of understanding of the turbulence phenomena were laid nearly at the same time when quantum mechanics, special relativity theory, quantum electrodynamics and laser physics were created, all the latter being acknowledged achievements of XX century. The science of turbulent motion in continua is based on a combination of stochastic and determinate phenomena. On one hand, at the first glance a turbulent flow seems to be totally stochastic: no two equal snapshots of the flow can ever be made. On the other hand, some integral properties remain the same with a high degree of accuracy: the flow rate, the logarithmic velocity profile etc. Sophisticated mathematical descriptions of turbulent flows make some researchers think the problem is mathematical in the first place, therefore more complicated equations, closure methods, computational schemes must be involved, more computing power needs to be employed etc. To the great surprise of the followers of this opinion, there are results beyond understanding when a more complicated model has a worse correspondence with the experiment than a simpler one. Speaking generally of the correspondence between experiments and theories, it should be noted that the experimentation usually involves geometries and conditions other than numerical modeling. To remove this contradiction, the conference of 1999 at Karlsruhe decided to define five classic cases of turbulent motion (a flow with a sudden expansion, a flow in an elbow etc.) so that further experiments and theories of turbulence were first to be tested on these models and only then, after proper conclusions are made, to be applied to other cases.

We would like to emphasize a few ideas which actually formed a basis for this book. First: the understanding of turbulence should be based on ideas of physics. Second: most conservative values are moments of stochastic functions. For example, first moments define average values such as the average flow velocity and the respective flow rate, second moment defines the friction in the turbulent flow which is responsible for the integral balance of forces, say, in a pipe or in a channel. Those are actually observable physical quantities. Higher moments, not accidentally, have less significant physical meanings. The similar roles are played by the moments in the kinetic gas theory. However, in the latter the presence of a small parameter, Knudsen number, helps identify two characteristic times: hydrodynamic and kinetic one. This allows, in most cases, to use a hydrodynamical description of phenomena via equations for first moments of the velocities. Turbulence has no explicit parameter like this; however, analysis of experimental works and a "direct computational modeling" of turbulence show that

1. moments of the same order have similar transient and relaxation times;

2. there exists an hierarchy of times: the greater the moment's order, the shorter the relaxation time;

3. in order to study the processes on characteristic hydrodynamical time intervals, we can assume the higher moments are in local equilibrium with the flow;

4. due to the equilibrium, of special significance are expressions of the higher moments via the lower ones, or their derivatives with respect to the coordinates; such relations include differential expressions of second moments via the derivatives of the first ones (a Boussinesq assumption) and algebraic cumulant expressions of fourth moments via second ones (a Millionschikov approximation);

5. the turbulent flow is closer to the motion of a dilute gas in the kinetic theory of gases when the free path is not much shorter than the characteristic size of the area; an anisotropy in the turbulent transport appears due to this – the gradient of the thermodynamic force in one direction causes a flux in the other, and a "negative viscosity" arises – the signs of the Reynolds stresses do not correspond to those of the local vorticities of the flow, i.e. the relations between the gradients and the forces become nonlocal.

The mathematical description of the turbulent motion of a medium has much in common with the kinetic description of dilute and solid gases. Just as the kinetic theory of gases, the turbulence theory has two general approaches to the description of motion. First approach is based on the Liouville equation for the particle ensemble distribution function. Its counterpart in the statistical theory of turbulence is based on an equation for the probability density function. Second approach (being the first historically) is based on equations for the moments of pulsation values. In this approach, the PDF (Probability Distribution Function) equation is a counterpart of Boltzmann's equation in the kinetic theory of gases. Navier–Stokes equations can be used to derive equations for the moments of second, third and higher orders. They are a chain of connected equations. The procedure of closure, i.e. getting a closed equation set, is based on an hierarchy of the characteristic times, similarly to the closure in the kinetic theory of gases. It is known that the Boltzmann kinetic equation describes a kinetic phase of processes with the characteristic times of the order of the mean free time. When averaged, the Boltzmann equation produces a motion equation (Euler or Navier–Stokes) that describes the hydrodynamical phase of the process, i.e. the phase that has the characteristic time of the order of the hydrodynamical time. It means first-order moments have a slow characteristic time. Higher-order moments become steady-state in a significantly shorter time. This idea, based on the hierarchy of times for higher-order moments, manifests itself also in turbulence: higher-order moments come to their steady state faster than lower-order moments do, and after a fast time there appears a parametric dependence of them on some integral quantities such as the local vorticity of the flow, average values of external forces etc. Further below we will deal with both approaches to the description of turbulence.

This book presents methods for describing average one-point and two-point, correlation and spectral characteristic functions of a developed turbulent flow in an incompressible medium. There is a discussion on the analogy with the molecular transport at a finite mean free path, on methods of closure and descriptions of flows based on the kinetic approach. Statistical models are presented for moments of various orders. A semi-empiric set of kinetic equations for one-point and two-point probability densities is employed; the equations are similar to those of Fokker–Planck. A discussion is provided on a nonlocal nature of transport in the turbulent diffusion process and on the anisotropy of transport caused by a difference in the directions of the principal axes of the stress tensor and of the strain rate tensor. Methods of solution are demonstrated by the example problems of the development of a turbulent wake in a homogeneous or stratified medium, and of peculiarities of the diffusion in a vortex core or in a rotating flow.

The following monographs are recommended for reading together with this book: Monin A.S. and Yaglom A.M. [136]; Hinze J.O. [70]; Batchelor G. [11]; Landau L.D., Lifshitz E.M. [100]; all those contain detailed derivations for many of the relationships found in this book.

CHAPTER

ONE

MODELS OF CONTINUOUS MEDIA

The turbulence can take place in various media described by various models. This book deals with turbulent motions in continuous media only, but the volume and surface media can be different. This chapter discusses various models of continua; it is based on monographs [75, 100, 115, 174]. It is essential for understanding how turbulent phenomena manifest themselves in media described by various models.

1.1 NOTATION

Hydrodynamics is a branch of continuum mechanics that studies spatial and time variations of fields, therefore basics of tensor algebra will be necessary for the presentment.

The following designations will be used in connection with hydromechanics:

$\boldsymbol{ab} \Leftrightarrow a_i b_k$	is a second-rank tensor;
$\boldsymbol{\sigma} \leftrightarrow \sigma_{ik}$	is a second-rank tensor;
$\mathcal{T} = \mathcal{T}^a + \mathcal{T}^s$	is a division of a tensor into symmetric and asymmetric parts;
$\nabla \cdot \boldsymbol{a}$	means div $\boldsymbol{a}$;
$\nabla \cdot \boldsymbol{\sigma}$	means div $\boldsymbol{\sigma}$;
$\boldsymbol{v}$	is velocity or specific momentum;
ρ	is density of the medium (gas or fluid);
T	is temperature;
u	is specific internal energy of the medium;
h	is specific enthalpy of the medium;
s	is specific entropy of the medium;
$c_m = \rho_m/\rho$	is specific concentration of component m;
$n_m = N_m/V$	is concentration of component m;
$\boldsymbol{i}_k$	is mass flux of component k;
$\boldsymbol{q}$	is heat flux;
$\boldsymbol{\sigma}$	is viscous stress tensor;
ν	is kinematic viscosity;
$\mu = \nu\rho$	is dynamic viscosity;

λ	is heat conductivity factor;
k_r	is rate of chemical reaction $r = 1 \dots R$.

Dimensionless parameters are:

$\mathrm{Re} = UL/\nu$	is the Reynolds number;
$\mathrm{Pr} = \eta c_p/\lambda$	is the Prandtl number;
$\mathrm{Le} = \rho D/\lambda$	is the Lewis number;
$\mathrm{Sm} = \nu/D$	is the Schmidt number.

The CGS system of units is used principally, though it can be easily converted into SI when electromagnetic processes are not taken into consideration.

1.2 BALANCE EQUATIONS FOR MASS, MOMENTUM AND ENERGY

The continuous medium model assumes that an infinitesimal volume of the medium is much larger than the characteristic structural sizes of the matter (the size of particles, the distance between the particles, the mean free path, the effective range of the particle interaction potential).

The quantities sought for in hydromechanics are local mass-average velocity of the medium, temperature, density, pressure, specific internal energy, specific enthalpy, specific entropy, specific concentrations of components, volume concentrations of particles. Other sought-for values include fluxes which comprise mass fluxes of components, the heat flux, the radiation heat flux, the viscous stress tensor. Transport coefficients such as viscosity, diffusivity, electric conductivity, heat conductivity, and constants of chemical processes are assumed to be given functions of temperature and pressure.

A full set of hydromechanics equations contains a continuity equation, component diffusion equations, motion equations, and an energy equation. In addition, the equation set must be complemented by expressions of thermodynamical functions, a state equation, expressions of constants of chemical reaction rates, component mass fluxes, an energy flux, a momentum flux, and expressions of transport coefficients.

It is most convenient to use specific quantities in order to describe hydrodynamical fields, i. e., quantities per unit of mass.

The behavior of a continuous medium is defined by dynamic forces and an extent to which the medium is compressible. From the hydromechanics viewpoint, the behavior of equally compressible media is the same in spite of different physical states. For example, both fluid and gas behave in the same way at velocities much less than the sonic speed.

1.2.1 Continuity equation

Let us imaginably bound a fixed volume in a medium, through which a gas is flowing (see Figure 1.1). The amount of mass in this volume is equal to the integral of the gas density over the volume:

$$M = \int \rho dV.$$

The variation of the mass with time in this bounded volume is caused only by inflow and outflow of the matter. Both processes are defined by the surface integral of the mass flux:

$$\frac{d}{dt}\int\limits_V \rho dV = -\oint \rho \boldsymbol{v} \cdot d\boldsymbol{S}.$$

REFERENCES

1. Abramovich, G. N., Krasheninnikov, S. Yu., and Sekundov, A. N., *Turbulent Flows Under the Influence of Body Forces and Nonself-Similarity*, Mashinostroenie, Moscow, 1975 [in Russian].
2. Ahmetov, D. G. and Kisarov, O. P., A Hydrodynamic Structure of the Ring Vortex, *J. Appl. Mech. Tech. Phys. (Prikladnaya Mekhanika i Tekhnicheskaya Fizika)*, No. 4, pp. 120–123, 1966 [in Russian].
3. Anderson, D. A., Tannehill, J. C., and Pletcher, R. H., *Computational Fluid Mechanics and Heat Transfer*, McGraw-Hill, New York, 1984.
4. Arina R., Domain Decomposition Approaches for Turbulent flow Simulations, in: *3-rd ECCOMAS CFD Conf.*, Paris, France, pp. 859–865, 1996.
5. Armfield, S. W., Nam-Hyo Cho, and Fletcher, C. F. J., Prediction of Turbulence Quantities for Swirling Flow in Conical Diffusers, *AIAA J.*, **28**, No. 3, pp. 453–460, 1990.
6. Baldwin, L. V. and Sandborn, V. A., Intermittency of Far Wake Turbulence, *AIAA J.*, **6**, No. 6, pp. 1163–1164, 1968.
7. Balescu, R., *Equilibrium and Non-Equilibrium Statistical Mechanics (in 2 vols.)*, Wiley, New York, 1975.
8. Barenblatt, G. I., On the Scaling Laws (Incomplete Self-Similarity with Respect to Reynolds Numbers) for the Developed Turbulent Flows in Tubes, *C. R. Acad. Sci. Paris*, 1991, pp. **313**, Ser. 2, 307–312.
9. Barenblatt, G. I., *Scaling, Self-similarity, and Intermediate Asymptotics*, Cambridge Univ. Press, Cambridge, UK, 1996.
10. Batchelor, G. K., Note on Free Turbulent Flows with Special Reference to the Two-Dimensional Wake, *J. Aeronaut. Sci.*, **17**, No. 7, pp. 441–445, 1950.
11. Batchelor, G. K., *The Theory of Homogeneous Turbulence*, Cambridge Univ. Press, Cambridge, UK, 1953.
12. Belotserkovskii, O. M., *Computational Mechanics: Modern Problems and Results*, Nauka, Moscow, 1991 [in Russian].
13. Belotserkovskii, O. M. (Ed.), *Studies on Turbulence*, Nauka, Moscow, 1994 [in Russian].
14. Belotserkovskii, O. M. and Ginevskii, A. S., *Modeling of Turbulent Jets and Wakes of the Basis of the Discrete Vortices Technique*, Fizmatgiz, Moscow, 1995 [in Russian].
15. Belotserkovskii, O. M., Numerical Experiment in Turbulence: From Order to Chaos, *Int. J. Fluid Mech. Res.*, **23**, No. 5-6, pp. 321–488, 1996.
16. Belotserkovskii, O. M., *Turbulence and Instabilities*, Edwin Mellen, Lewinston, 2000.
17. Belov, I. A. and Isaev, S. A., *Modeling of Turbulent Flows*, Baltic State Eng. Univ. "Voenmekh", St.-Petersburg, 2001 [in Russian].
18. Belov, I. A., Isaev, S. A., and Korobkov, V. A., *Problems and Methods of Calculation of Separated Flows of Incompressible Fluid*, Sudostroenie, Leinngrad, 1989 [in Russian].
19. Besnard, D. C., Harlow, F. H., Rauenzahn, R. and Zemach, C., *Spectral Transport Model for Turbulence*, Los Alamos Nat. Lab. Rep., LA-UR-92-1666, 1992.
20. Birkgoff, G. and Zarantonello, E. H., *Jets, Wakes and Cavities*, Academic Press, New York, 1957.
21. Bishaev, A. M. and Rykov, V. A., On Longitudinal Heat Flux in the Couette Flow, *Fluid Dynamics (Izvestiya AN SSSR, Mekhanika Zhidkosti i Gaza)*, No. 3, pp. 162–166, 1980 [in Russian].

22. Bonnin, J. C., Buchal, T., and Rodi, W., ERCOFTFC Workshop on Data Bases and Testing of Calculation Methods for Turbulent Flows, *Eur. Res. Com. Flow Turb. Combust. Bul.*, No. 28, pp. 48–54, 1996.
23. Bradshaw, P., The Analogy Between Streamline Curvature and Buoyancy in Turbulent Shear Flow, *J. Fluid Mech.*, **36**, pp. 177–191, 1969.
24. Bradshaw, P., *An Introduction to Turbulence and its Measurement*, Pergamon Press, Oxford, UK, 1971.
25. Bradshaw, P., *Effects of Streamline Curvature on Turbulent Flow*, AGARDograph, No. 169, 1973.
26. Bradshaw, P., Cebeci, T. and Whitelaw, J. H., *Engineering Calculation Methods for Turbulent Flow*, Academic Press, New York, 1981.
27. Bukreev, V. I., *An Experimental Checking of Modern Imagination about the Turbulent Flow of Incompressible Fluid*, Doct. Phys.-Math. Sci. Dissertation, Inst. of Hydrodynamics, Acad. Sci. USSR, Novosibirsk, 1984 [in Russian].
28. Bukreev, V. I., Zykov, V. V., and Kostomaha, V. A., One-Dimensional Laws for Probability Distribution of Velocity Fluctuations at Turbulent Flow in a Circular Pipe, *Proc. Sib. Dep. Acad. Sci. USSR, Ser. Eng. Sci. (Izvestiya Sibirskogo Otdeleniya AN SSSR, Ser. Terhnicheskih Nauk)*, **13**, No. 3, pp. 3–9, 1975 [in Russian].
29. Buleyev, N. I., *Spatial Model for Turbulent Exchange*, Nauka, Moscow, 1990 [in Russian].
30. Cantwell, B., Coles, D., and Dimotakis, P. E., Structure and Entrainment in the Plane of Symmetry of a Turbulent Spot, *J. Fluid Mech.*, **87**, pp. 641–672, 1978.
31. Chahine, M. T., Narasimha, R., The Integral $\int_0^\infty v^n \exp(-(v-u)^2 - x/v)dv$., *J. Math. Phys.*, **43**, No. 2, pp. 163–167, 1964.
32. Champagne, F. H., Harris, V. G., and Corrsin, S., Experiments on Nearly Homogeneous Turbulent Shear Flow, *J. Fluid Mech.*, **41**, pp. 81–133, 1970.
33. Chapman, S. and Cowling, T. G., *The Mathematical Theory of Non-Uniform Gases*, Cambridge Univ. Press, Cambridge, UK, 1952.
34. Chien, K. Y., Predictions of Channel and Boundary-Layer Flows with a Low-Reynolds-Number Turbulence Model, *AIAA J.*, **20**, No. 1, pp. 33–38, 1982.
35. Chung, P. M., A Simplified Statistical Model of Turbulent Chemically Reacting Shear Flows, *AIAA J.*, **7**, No. 10, pp. 1982–1991, 1969.
36. Chusov, M. A., Relaxation Processes in a Developed Turbulent Flow, in: *Turbulent Flows*, Nauka, Moscow, pp. 100–110, 1974 [in Russian].
37. Clarke, J. F. and McChesney, M., *The Dynamics of Real Gases*, Butterworths, London, 1964.
38. Complex Turbulent Flows, in: *Proc. 1980-81 Stanford-AFQSR-HTTM Conf. on Complex Turbulent Flows. Vol.1–3*, Ed. by S. J. Kline, B. J. Cantwell, and G. M. Lilley, Dept. of Mech. Eng., Stanford, 1981.
39. Craft, G., Launder, B. E., and Suga, K. Extending the Applicability of Eddy Viscosity Models Through the Use of Deformation Invariants and Non-Linear Elements Proc. 5-th IAHR Conf. of Refined Flow Modelling and Measurements Paris125–1321993
40. Cytovich, V. N., *A Turbulence of High-Temperature Plasma*, Nauka, Moscow, 1971 [in Russian].
41. Daly, B. J. and Harlow, F. H., Transport Equations in Turbulence, *Phys. Fluids*, **13**, pp. 2634–2649, 1970.
42. Darintsev, A. P., Zabavin, V. N., Zamyshlyaev, B. V., Onufriev, A. T., Khristianovich, S. A., and Shcherbin, M. D., Features of Motion of Heated Air, Initially Located in Spherical Volume, in the Atmosphere, in: *Modern Problems of Continuum Mechanics*, Moscow Inst. Phys. Technol.

Publications, Moscow, pp. 126–135, 1985 [in Russian].

43. Davydov, B. I., On a Statistical Theory of Turbulence, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **127**, No. 5, pp. 980–982, 1959 [in Russian].

44. Donaldson, J. W., Calculation of Turbulent Shear Flows for Atmosphere and Vortex Motions, *AIAA J.*, **10**, No. 1, pp. 4–12, 1972.

45. Donaldson, C. du P., Atmospheric Turbulence and the Dispersal of Atmospheric Pollutatants, in: *AMS Workshop on Micrometeorology*, Science Press, Boston, pp. 313–390, 1973.

46. Driscall, R. J. and Kennedy, L. A., A Model for the Turbulent Energy Spectrum, *Phys. Fluids*, **26**, pp. 1228–1233, 1983.

47. Driscall, R. J. and Kennedy, L. A, A Model for the Spectrum of Passive Scalars in an Isotropic Turbulence Field, *Phys. Fluids*, **28**, pp. 72–80, 1985.

48. Driver, D. M. and Seegmiller, H. L., Features of a Reattaching Turbulent Shear Layer in Divergent Channel Flow, *AIAA J.*, **23**, pp. 163–171, 1985.

49. Eisenschitz, R., *Statistical Theory of Irreversible Processes*, Oxford Univ. Press, London, 1958.

50. Fedyaevskii, L. L., Ginevskii, A. S., and Kolesnikov, A. V., *Calculation of the Boundary Layer of Incompressible Fluid*, Sudostroenie, Leningrad, 1973 [in Russian].

51. Frenkiel, F. N. and Klebanoff, P. S., Two-Dimensional Probability Distribution in a Turbulent Field, *Phys. Fluids*, **8**, pp. 2291–2293, 1965.

52. Frenkiel, F. N. and Klebanoff, P. S., Higher-Order Correlations in a Turbulent Field, *Phys. Fluids*, **10**, pp. 507–520, 1967.

53. Frisch, U. and Orszag, S. A., Turbulence: Challenges for Theory and Experiment, *Phys. Today*, No. 1, pp. 24–32, 1990.

54. Frost, V. A., The Mathematical Model of Turbulent Combustion, in: *Proc. 3-rd All-Union Meet. Theory of Combustion*, Acad. Sci. USSR Publications, Moscow, pp. 121–125, 1960 [in Russian].

55. Frost, W. and Molden, T. H. (Eds.), *Handbook of Turbulence. Vol. 1: Fundamentals and Applications*, Plenum Press, New York, 1977.

56. Garbaruk, A. V., Lapin, Yu. V. and Strelets, M. Kh., Simple Algebraic Model of Turbulence for the Calculation of Turbulent Boundary Layer with Adverse Pressure Gradient, *High Temp.*, **37**, pp. 82–86, 1999.

57. Garbaruk, A. V., Lapin, Yu. V. and Strelets, M. Kh., An Algebraic Model of Turbulence for Flows with a Maximum of Tangential Stress Inside the Boundary Layer, *High Temp.*, **39**, pp. 548–557, 2001.

58. Gibson, M. M. and Launder, B. E., Ground Effects on Pressure Fluctuations in the Atmospheric Boundary Layer, *J. Fluid Mech.*, **86**, pp. 491–511, 1978.

59. Ginevskii, A. S., *Turbulent Jet and Wake Flow Theory: Engineering Methods for Calculation*, Mashinostroenie, Moscow, 1969.

60. Glushko, G. S. and Traskovskii, V. D., A Method for Calculation of Power Spectral Density of a Uniform Isotropic Turbulence, *Fluid Dynamics (Izvestiya AN SSSR, Mekhanika Zhidkosti i Gaza)*, No. 1, pp. 13–19, 1978 [in Russian].

61. Gorbunov, S. I., Gordeichik, B. N., Darintsev, A. P., Zabavin, V. N., Zamyshliayev, B. V., Zaslavskii, B. I., Onufriev, A. T., and Shcherbin, M. D., A Structure of the Buoyant Thermic, *J. Appl. Mech. Tech. Phys. (Prikladnaya Mekhanika i Tekhnicheskaya Fizika)*, No. 5, pp. 70–76, 1992 [in Russian].

62. Gore, R. A., Harlow, F. H., and Zemach, C., Computation of Mixing Layers by a Spectral Transport Model, in: *Proc. 8-th Int. Sympos. on Free Shear Flows*, Munich, Germany, Sec. 29-

2, pp. 1–6, 1991.

63. Grigoryev, Yu.R., *Organized Structures in the Developed Near-Wall Turbulence*, Inst. Comput. Technol., Siber. Dep. Rus. Acad. Sci. Publications, Novosibirsk, 1993 [in Russian].
64. Haken, H., *Synergetics*, Springer Verlag, Berlin, 1978, Chap. 6.
65. Hall, V. G., Vortex Breakdown, in: *Ann. Rev. Fluid Mech.*, **4**, pp. 195–218, 1972.
66. Hanjalic, K. and Launder, B. E., Fully Developed Asymmetric Flow in a Plane Channel, *J. Fluid Mech.*, **51**, pp. 301–335, 1972.
67. Harlow, F. H. and Nakayama, P. I., Turbulence Transport Equations, *Phys. Fluids*, **10**, pp. 2323–2332, 1967.
68. Harris, V. C., Graham, I. K. H., and Corrsin, S., Further Experiments in Nearly Homogeneous Flow, *J. Fluid Mech.*, **81**, pp. 657–687, 1977.
69. Hill, R. J., Models of the Scalar Spectrum for Turbulent Advection, *J. Fluid Mech.*, **88**, pp. 541–561, 1978.
70. Hinze, J. O., *Turbulence*, McGraw-Hill, New York, 1959.
71. Hogg, S. and Leschziner, M. A., Computation of Highly Swirling Confined Flow with a Reynolds Stress Turbulence Model, *AIAA J.*, **27**, No. 1, pp. 37–63, 1989.
72. Hossain, M. S., *Mathematische Modellierung von turbulenten Auftriebsströmungen*, Ph. D. Thesis, Univ. of Karlsruhe, 1980.
73. Huang, K., *Statistical Mechanics*, Wiley, New York, 1963, Chap. 5.
74. Ievlev, V. M., Equations for Finite-Dimensional Probability Distributions of Pulsing Parameters in a Turbulent Flow, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **208**, No. 5, pp. 1044–1047, 1973 [in Russian].
75. Ievlev, V. M., *A Turbulent Motion of High-Temperature Medium*, Nauka, Moscow, 1975 [in Russian].
76. Ievlev, V. M., *Numerical Modeling of Turbulent Flows*, Nauka, Moscow, 1990 [in Russian].
77. Ievlev, V.M. and, Son, E. E., *Gas, Liquid and Plasma Turbulence*, Moscow Inst. Phys. Technol. Publications, Moscow, 1982 [in Russian].
78. Ievlev, V.M. and, Son, E. E., A Half-Empiric Theory for Turbulence of Inhomogeneous Flows with Bulk Forces, *Fluid Dynamics (Izvestiya AN SSSR, Mekhanika Zhidkosti i Gaza)*, No. 3, pp. 33–40, 1985 [in Russian].
79. Jaugashtin, K. E., Calculating the Turbulent Motion in Surface Layer of the Atmosphere, in: *Mechanics of Inhomogeneous Medium*, Inst. Theor. Appl. Mech, Siber. Dep. Acad. Sci. USSR Publications, Novosibirsk, pp. 104–113, 1987 [in Russian].
80. Johnson, D. V. and Bennet, J. C., *Mass and Momentum Turbulent Transport Experiments with Confined Coaxial Jets*, NASA Contractor Rep., No. 165574, 1981.
81. Kadomtsev, B. B. and Kostomarov, D. P., Turbulent Layer in an Ideal Two-Dimensional Fluid, *Phys. Fluids*, **15**, pp. 1–3, 1972.
82. Kadomtsev, B. B. and Kontorovich, V. M., The Theory for a Turbulence in Hydrodynamics and Plasma, *Radiophys. (Izvestiya VUZov, Radiofizika)*, **17**, No. 4, pp. 511–540, 1974.
83. Keller, L. V. and Friedman, A. A., Differentialgleichung für die turbulent Bewegung einer Kompressibltn Flüssigkeit, in: *Proc. 1-st Int. Congr. Appl. Mech.*, Delft, pp. 395–405, 1924.
84. Kharkevish, A. A., *Spectra and Analysis*, Gosizdat, Moscow, 1952 [in Russian].
85. Klein, F., Review: Turbulent Developing Pipe Flow, *J. Fluids Eng.*, **103**, pp. 243–249, 1981.
86. Kochin, N. E., Kibel', I. A., and Roze, N. V., *Theoretical Hydromechanics (in 2 vols.)*, Fizmatgiz, Moscow, 1963 [in Russian].

87. Kogan, M. N., *The Dynamics of Rarefied Gas*, Nauka, Moscow, 1967 [in Russian].

88. Kolmogorov, A. N., A Local Turbulence Structure in an Incompressible Viscid Liquid at Very High Reynolds Numbers, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **30**, No. 4, pp. 299–303, 1941 [in Russian].

89. Kolmogorov, A. N., Equations for Turbulent Motion of an Incompressible Liquid, *Proc. Acad. Sci. USSR, Ser. Phys. Sci. (Izvestiya AN SSSR, Ser. Fizicheskie Nauki*, **6**, No. 1-2, pp. 56–58, 1942 [in Russian].

90. Konyuhov, A. V., Holodov, A. S., Onufriev, A. T., Son, K. E., Son, E. E., and Utyuzhnikov, S. V., *A Half-Empiric Theory and Numerical Modeling of Turbulent Flows with High Streamline Curvature and Contrgradient Diffusion*, MIPT-ABB Contractor Rep. 1996-23, Moscow Inst. Phys. Technol., Dolgoprudny, 1996 [in Russian].

91. Kraichnan, R. H., Preliminary Calculation of the Kolmogorov Turbulence Spectrum, *Phys. Fluids*, **8**, pp. 995–997, 1965.

92. Kurbatskii, A. F., *Modeling of the Non-Local Turbulent Pulse and Heat Transfer*, Nauka, Novosibirsk, 1988 [in Russian].

93. Kurbatskii, A. F. and Onufriev, A. T., Modeling of Turbulent Pulse Transfer in Wake Past the Cylinder Involving the Equations for the Third Moments, *J. Appl. Mech. Tech. Phys. (Prikladnaya Mekhanika i Tekhnicheskaya Fizika)*, No. 6, pp. 99–107, 1979 [in Russian].

94. Kutateladze, S. S., *The Essentials of Theory of Heat Exchange*, Mashgiz, Moscow/Leningrad, 1962 [in Russian].

95. Kuznetsov, V. R. and Sabel'nikov, V. A., *Turbulence and Combustion*, Nauka, Moscow, 1986 [in Russian].

96. Lai, Y. G. and So, R. M. C., On Near-Wall Turbulent Flow Modelling, *J. Fluid Mech.*, **221**, pp. 641–673, 1990.

97. Lakshminarayana, B., Turbulence Modelling for Complex Shear Flows, *AIAA J.*, **24**, No. 12, pp. 1900–1917, 1986.

98. Lam, C. K. G. and Bremhorst K.A., A Modified Form of the k-ε Model for Predicting Wall Turbulence, *J. Fluids Eng.*, **103**, pp. 456–460, 1981.

99. Lamb, H., *Hydrodynamics*, Dover, New York, 1932.

100. Landau, L. D. and Lifshitz, E. M., *Continuum Mechanics*, Gostekhizdat, Moscow, 1953 [in Russian].

101. Landau, L. D. and Lifshitz, E. M., *Statistical Physics*, Elsevier Butterworth-Heinemann, Burlington, MA, 1980.

102. Laufer, J., The Structure of Turbulence in Fully Developed Pipe Flow, *NACA Tech. Rep.*, No. 1174, 1954.

103. Launder, B. E., Turbulence Modelling for Flow in Arbitrarily Complex Domains, in: *3-rd ECCOMAS CFD Conf.*, Paris, France, pp. 65–76, 1996.

104. Launder, B. E. and Spalding, D. B., The Numerical Computation Of Turbulent Flows, *Comp. Meth. Appl. Mech. Eng.*, **3**, pp. 269–289, 1974.

105. Launder, B. E., Priddin, C. H. and Sharma, B. I., The Calculation of Turbulent Boundary Layers on Spinning and Curved Surfaces, *ASME J. Fluids Eng.*, **99**, pp. 1977, 231–239.

106. Launder, B. E., Reece, G. J., and Rodi, W., Progress in the Development of a Reynolds Stress Turbulence Closure, *J. Fluid Mech.*, **68**, Pt. 3, pp. 537–566, 1975.

107. Lavrentiev, M. A. and Shabat, B. V., *Problems in Hydrodynamics and Their Mathematical Models*, Nauka, Moscow, 1977 [in Russian].

108. Lawn, C. I., The Determination of the Rate of Dissipation in Turbulent Pipe Flow, *J. Fluid*

Mech., **48**, Pt. 3, pp. 477–505, 1971.

109. Leibovich, S., The Structure of Vortex Breakdown, *Ann. Rev. Fluid Mech.*, **10**, pp. 221–246, 1978.

110. Leschziner, M. A. and Rodi, W., Calculation of Annular and Twin Parallel Jets Using Various Discretization Schemes and Turbulence-Model Variations, *ASME J. Fluid Eng.*, **103**, pp. 352–360, 1981.

111. Leslie, D. C., Analysis of a Strongly Sheared, Nearly Homogeneous Turbulent Shear Flow, *J. Fluid Mech.*, **98**, Pt. 2, pp. 435–443, 1984.

112. Liepman, H. W., Free Turbulent Flows, in: *Coll. Int. du CNRS Marseille*, Paris, pp. 211–226, 1962.

113. Liboff, R. L., *Introduction to the Theory of Kinetic Equations*, Willey, New York, 1969.

114. Lifshitz, E. M. and Pitaevskii, L. P., *Physical Kinetics*, Nauka, Moscow, 1979 [in Russian].

115. Loitsyanskii, L. G., *Mechanics of Liquids and Gases*, Begell House, New York, 1996.

116. Loitsyanskii, L. G., A Half-Empiric Theory for Interaction of Molecular and Molar Exchange Processes in the Turbulent Flow of a Liquid , in: *Proc. 2-nd All-Union Congr. Theor. Appl. Mech.*, Acad. Sci. USSR Publications, Moscow/Leningrad, 1962 [in Russian].

117. Lumley, J. L., Modelling Turbulent Flux of a Passive Scalar Quantities in Inhomogeneous Flows, *Phys. Fluids*, **18**, pp. 619–621, 1975.

118. Lumley, J. L. and Khajeh-Nouri, B., Computational Modeling of Turbulent Transport, *Adv. Geophys.*, **18A**, pp. 169–193, 1974.

119. Lumley, J. L. and Newman, G. R., The Return to Isotropy of Homogeneous Turbulence, *J. Fluid Mech.*, 1977, pp. **82**, 161–178.

120. Lundgren, T. S., Distribution Functions in the Statistical Theory of Turbulence, *Phys. Fluids*, **10**, pp. 969–975, 1967.

121. Lundgren, T. S., Model Equation for Nonhomogeneous Turbulence, *Phys. Fluids*, **12**, pp. 485–497, 1969.

122. Lundgren, T. S., A Closure Hypothesis for the Hierarchy of Equations for Turbulent Probability Distribution Functions, in: *Statistical Models and Turbulence,* Ed. by M. Rosenblatt and C. Van Atta, Springer, Berlin, pp. 70–100, 1972.

123. Lushchik, V. G., Pavieliev, A. A., and Yakubenko, A. E., The Three-Parameter Model of Shear Turbulence, *Fluid Dynamics (Izvestiya AN SSSR, Mekhanika Zhidkosti i Gaza)*, No. 3, pp. 13–24, 1978 [in Russian].

124. Lushchik, V. G., Pavieliev, A. A., and Yakubenko, A. E., Transfer Equations for Characteristics of Turbulence: Models and Calculation Results, *Sci. Tech. Proc., Ser. Fluid Gas Mech. (Itogi Nauki i Tekhniki, Ser. Mekhanika Zhidkosti i Gaza)*, **22**, pp. 3–61, 1978 [in Russian].

125. Lushchik, V. G. and Regirer, S. A., The Mixing Layer of Turbulent Flows in Homogeneous Nonconducting and Conducting Fluids, *Fluid Dynamics*, **6**, No. 1, pp. 41–53, 1971.

126. Lytkin, Yu. M. and Chernyh, G. G., On One Method of Closure of the Karman – Hovart Equation, *Dynam. Cont. (Dinamika Sploshnoi Sredy)*, **27**, pp. 124–130, 1976 [in Russian].

127. Lytkin, Yu. M. and Chernyh, G. G., Calculating the Correlation Functions in Isotropic Turbulence, *Dyn. Probl. Cont. Mech. (Dinamicheskie Zadachi Mekhaniki Sploshnyh Sred)*, **35**, pp. 74–88, 1978 [in Russian].

128. Markov, Yu. B., *An Experimental Study of Some Probability and Correlation Characteristics in a Turbulent Jet*, Cand. Phys.-Math. Sci. Dissertation, Moscow Inst. Phys. Technol., Moscow, 1978 [in Russian].

129. Marvin, J. G. and Coakley, T. J., *Turbulence Modeling for Hypersonic Flows. The Third Joint*

Europe/US Short Course In Hypersonics, RWTH Aachen Univ. Technol., Aachen, FRG, 1990.

130. Mathieu, J. and Jaendel, D., *Pathological Cases in Turbulent Field and Spectral Approach*, Von Kármán Institute for Fluids Dynamics, Lect. Ser. 1979-2, 1979.

131. Menter, F. R., Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications, *AIAA J.*, **32**, pp. 1598–1605, 1994.

132. Merritt, G. E., Wake Growth and Collapse in a Stratified Flow, *AIAA J.*, **12**, No. 7, pp. 940–949, 1974.

133. Millionschcikov, M. D., On the Theory of Homogeneous Isotropic Turbulence, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **32**, No. 9, pp. 611–614, 1941 [in Russian].

134. Monin, A. S., The Equations for Finite-Dimensional Probability Distributions of Turbulent Field, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **177**, pp. 1036–1038, 1967 [in Russian].

135. Monin, A. S., On the Nature of Turbulence, *Soviet Physics – Uspekhi*, **21**, No. 5, pp. 429–442, 1978.

136. Monin, A. S. and Yaglom, A. M., *Statistical Hydromechanics (in 2 vols.)*, Gidrometeoizdat, St.-Petersburg, 1992, 1996 [in Russian].

137. Naudasher, E., Flow in the Wake of Self-Propelled Bodies and Related Sources of Turbulence, *J. Fluid Mech.*, **22**, Pt. 4, pp. 625–656, 1965.

138. Nee, V. W. and Kovasznay, L. S. G., Simple Phenomenological Theory of Turbulent Shear Flows, *Phys. Fluids*, **12**, pp. 473–484, 1969.

139. Novikov, E. A., The Kinetic Equation for Vortex Field, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **177**, pp. 299–301, 1967 [in Russian].

140. O'Brien, E. E. and Dopazo, C., Behavior of Conditioned Variables in Free Turbulent Shear Flows, in: *Structure and Mechanisms of Turbulence II*, Springer-Verlag, Berlin, pp. 124–137, 1978.

141. Obuhov, A. M., On Turbulence Scale Distribution in Pipes of Arbitrary Section, *J. Appl. Math. Mech. (Prikladnaya Matematika i Mekhanika)*, **6**, No. 2-3, pp. 209–220, 1942 [in Russian].

142. Obuhov, A. M., Description of Turbulence in Terms of Lagrangian Variables, in: *Advances in Geophysics.* Vol. 6: Atmospheric Diffusion and Air Pollution, Ed. by F. N. Frenkiel and P. A. Sheppard, Academic Press, New York, pp. 1959, 113–115.

143. Onufriev, A. T., On Equations of Half-Empiric Theory of Turbulent Transfer, *J. Appl. Mech. Tech. Phys. (Prikladnaya Mekhanika i Tekhnicheskaya Fizika)*, No. 2, pp. 62–71, 1970 [in Russian].

144. Onufriev, A. T., On the Model Equation for Probability Density in Half-Empiric Theory of Turbulent Transfer, in: *Turbulent Flows*, Nauka, Moscow, pp. 110–117, 1977 [in Russian].

145. Onufriev, A. T., On Features of a Turbulent Motion in Core of the Vortex Ring, in: *Physical Mechanics – 4: The Dynamic Processes in Gases and Solids*, Leningrad Univ. Publications, Leningrad, pp. 31–70, 1980 [in Russian].

146. Onufriev, A. T., *Description of the Turbulent Transfer*, Moscow Inst. Phys. Technol., Moscow, 1995 [in Russian].

147. Onufriev, A. T. and Khristianovich, S. A., On Features of Turbulent Motion in a Vortex Ring, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **229**, No. 1, pp. 42–44, 1976 [in Russian].

148. Onufriev, A. T. and Son, K. E., *A Half-Empiric Theory for Turbulent Flow in Channels*, Moscow Inst. Phys. Technol., Dolgoprudny, 1998 [in Russian].

149. Orszag, S. A., Numerical Methods for the Simulation of Turbulence, *Phys. Fluids*, **12**, Suppl. II, pp. 250–257, 1969.

150. Ozmidov, R. V., *Horizontal Turbulence and Turbulent Exchange in the Ocean*, Nauka,

Moscow, 1968 [in Russian].

151. Pao, J. H., Structure of Turbulent Velocity And Scalar Fields at Large Wavenumbers, *Phys. Fluids*, **8**, pp. 1063–1075, 1965.

152. Pettersson, B. A., Andersson, H. T., and Øyulvstad, S., Modelling System Rotation with an Anisotropic Eddy-Viscosity Closure, in: *3-rd ECCOMAS CFD Conf.*, Paris, France, pp. 884–889, 1996.

153. Pilipchuk, M. I., *Studying the Statistical Characteristics of Longitudinal Velocity Component of the Turbulent Flow in Rotating Pipe*, Cand. Phys.-Math. Sci. Dissertation, Moscow Inst. Phys. Technol., Moscow, 1986 [in Russian].

154. Pourahmadi, F. and Humphrey, J. A. C., Prediction of Curved Channel Flow with an Extended k-ε Model of Turbulence, *AIAA J.*, **21**, No. 10, pp. 1365–1373, 1983.

155. Prandtl, L., Einfluss stabilisierender Krafte auf die Turbulenz, in: *Gebiete der Aerodynamik und verwandter Gebiete,* Ed. by A. Gilles, L. Hopf, and T. von Karman, Berlin, pp. 1–7, 1930.

156. Reichardt, H., Heat Transfer Through Turbulent Friction Layers, *NACA TM*, No. 1047, 1943.

157. Ribeiro, M. and Whitelaw, J., Statistical Characteristics of a Turbulent Jet, *J. Fluid Mech.*, **70**, No. 1, pp. 1–15, 1975.

158. Rodi, W., A New Algebraic Relation for Calculating the Reynolds Stresses, *ZAMM*, **56**, pp. 219–221, 1976.

159. Rodi, W., *Turbulence Models for Environmental Problems*, Prediction Methods for Turbulent Flows, Von Kármán Institute for Fluids Dynamics, Lect. Ser. 1979-2, 1979.

160. Rodi. W., Examples of Turbulence Modeles for Incompressible Flow, *AIAA J.*, **20**, pp. 872–889, 1982.

161. Rodi, W. and Spalding, D. B., A Two-Parameter Model of Turbulence and its Application to Free Jets, *Wärme Stoffubertragung*, **3**, pp. 585–592, 1970.

162. Rotta, J. C., Statistische Theorie nichthomogener Turbulenz I, *Z. Phys.*, **129**, No. 5, pp. 547–572, 1951.

163. Rotta, J. C., Statistische Theorie nichthomogener Turbulenz II, *Z. Phys.*, **131**, No. 1, pp. 51–77, 1951.

164. Sadovskii, M. V., Spectral Studying of a Non-Isothermal Jet at the Outlet of a Circular Pipe by Termoanomometric Technique, in: *Thes. All-Union Workshop of Young Researchers "Modern Problems of Thermophysics"*, Inst. Thermophys., Siber. Dep. Acad. Sci. USSR Publications, Novosibirsk, pp. 22–24, 1990 [in Russian].

165. Sadovskii, M. V., *Termoanomometric Study of the Behavior of Power Spectral Densities, Velocity Pulsation Spectra and Temperature in a Turbulent Flow in the Rotating Pipe*, Cand. Phys.-Math. Sci. Dissertation, Moscow Inst. Phys. Technol., Moscow, 1991 [in Russian].

166. Saffman, P. G., Model Equations for Turbulent Shear Flow, *Stud. Appl. Math.*, **53**, No. 1, pp. 17–34, 1974.

167. Sakipov, Z., An Experimental Studying of Semirestricted Streams, in: *Applied Thermophysics*, Acad. Sci. Kazakh. SSR Publications, Alma-Ata, pp. 29–46, 1964 [in Russian].

168. Safarov, N. A., *Behavior of the Parameters of a Developed Turbulent Flow in Straight Cylindrical Channel Rotating with Respect to Longitudinal Axis*, Cand. Phys.-Math. Sci. Dissertation, Moscow Inst. Phys. Technol., Moscow, 1986 [in Russian].

169. Schildknecht, M., Miller, J. A., and Meier, G. E. A., The Influence of Suction on the Structure of Turbulence in Fully Developed Pipe Flow, *J. Fluid Mech.*, **90**, pp. 67–107, 1979.

170. Schlichting, H., Entstehung der Turbulenz, in: *Handbuch der Physik, Bd. VIII/1: Strömungsmechanik I,* Ed. by S. Flügge, Springer, Heidelberg, pp. 1959, 351–450.

171. Schlichting, H., *Boundary-Layer Theory*, McGraw-Hill, New York, 1955.

172. Schubauer, G. B. and Tchen, C. M., Turbulent Flow, in: *Turbulent Flows and Heat. Transfer,* Ed. by C. C. Lin, Princeton Univ. Press, pp. Princeton, NJ, 75–195.1959

173. Shchukin, V. K. and Khalatov, A. A., *Heat and Mass Transfer and Hydrodynamics of Twisted Flows in Axisymmetric Channels*, Mashinostroenie, Moscow, 1982 [in Russian].

174. Sedov, L. I., *Continuum Mechanics: in 2 vols.*, Nauka, Moscow, 1970 [in Russian].

175. Sharif, M. A. R. and, Wong, Y. K. E., Evaluation of the Performance of Three Turbulence Closure Models in the Prediction of Confined Swirling Flows, *Comput. Fluids*, **24**, No. 1, pp. 81–100, 1995.

176. Slezkin, N. A., *The Dynamics of a Viscid Incompressible Liquid*, Gostekhizdat, Moscow, 1955 [in Russian].

177. Sloan, D. G., *Modeling of Swirling and Heterogeneous Char Combustion in Pulverized Coal Systems*, Ph. D. Thesis, Brigham Young University, 1985.

178. Sloan, D. G., Smith, P. J., and Smart, L. D., Modeling of Swirl in Turbulent Flow Systems, *Progr. Energy Combust. Sci.*, **12**, pp. 163–250, 1986.

179. So, R. M. C. and Yoo, G. J., *On the Modeling of Low-Reynolds-Number Turbulence*, NASA Contractor Rep., No. 3994, 1986.

180. Son, E. E., *The Kinetic Theory of Gases, Liquids and Plasma*, Moscow Inst. Phys. Technol., Moscow, 1978 [in Russian].

181. Spalding, D. B., *GENMIX: A General Computer Program for Two-Dimensional Parabolic Phenomena*, Pergamon Press, Oxford, UK, 1977.

182. Speziale, C. G., On Nonlinear k-l and k-ε Models of Turbulence, *J. Fluid Mech.*, **178**, pp. 459–475, 1987.

183. Spiegel, E. A., Qualitative Model for Turbulent Intermittency, *Phys. Fluids*, **15**, pp. 1372–1376, 1972.

184. Stewart, R. W., Triple Velocity Correlations in Isotropic Turbulence, *Proc. Cambridge Philos. Soc.*, **47**, pp. 146–157, 1951.

185. Stratonovich, R. L., *The Non-Linear Non-Equilibrium Thermodynamics*, Nauka, Moscow, 1985 [in Russian].

186. Struminskii, V. V., Kinetic Equations of Turbulent Flows, in: *Problems of Turbulent Flows*, Nauka, Moscow, pp. 14–32, 1987 [in Russian].

187. Taganov, V. I. and Sadovskii, V. S., Mathematical Modeling of Transient Coherent Structures in Turbulent Shear Flows, in: *Abstr. V All-Union Congr. Theor. Appl. Mech.*, Acad. Sci. Kazakh. SSR Publications, Alma-Ata, pp. **312**, 1981 [in Russian].

188. Taylor, G. I., Statistical Theory of Turbulence, *Proc. Roy. Soc. London*, **A151**, No. 874, pp. 421–444, 1935.

189. Townsend, A. A., *The Structure of Turbulent Shear Flow*, Cambridge Univ. Press, Cambridge, UK, 1956.

190. Tretyakov, V. V. and Yagodkin, V. I., Numerical Study of a Twisted Turbulent Flow in a Pipe, *J. Eng. Phys. Trermophys. (Inzhenerno-Fizicheskii Zhurnal)*, **37**, No. 2, pp. 254–259, 1979 [in Russian].

191. Troshkin, O. V., *On Wave Theory of Turbulence*, Comput. Center Acad. Sci. USSR, Moscow, 1989 [in Russian].

192. [3.10] Tsarenko, V. M., The Model Calculation of Unimodular Functions in the Monin – Obuhov Theory, *Izvestiya, Atmosph. Ocean. Phys. (Izvestiya AN SSSR, Fizika Atmosfery i Okeana)*, **23**, No. 2, pp. 115–124, 1987 [in Russian].

193. Tsarenko, V. M., On Determining of a Tensor of the Turbulent Diffision Coefficients in the Stratified Surface Atmospheric Layer, *Izvestiya, Atmosph. Ocean. Phys. (Izvestiya AN SSSR, Fizika Atmosfery i Okeana)*, **23**, No. 3, pp. 257–265, 1987 [in Russian].

194. Tsarenko, V. M., On Half-Empiric Description of a Turbulence in Surface Atmospheric Layer, *Izvestiya, Atmosph. Ocean. Phys. (Izvestiya AN SSSR, Fizika Atmosfery i Okeana)*, **23**, No. 5, pp. 470–478, 1987 [in Russian].

195. Turbulent Shear Flows I (Durst, F., Launder, B. E., Schmidt, F. W., and Whitelaw, J. H., Eds.), *Selected Papers from the First International Symposium on Turbulent Shear Flows, The Pennsylvania State University, University Park, Pennsylvania, USA, April 18–20, 1977*, Springer, Berlin/Heidelberg, 2012.

196. Turbulent Shear Flows II (Bradbury, L. J. S., Ed.), *Selected Papers from the Second International Symposium on Turbulent Shear Flows, Imperial College, London, July 2–4, 1979*, Springer-Verlag, New York, 1980.

197. Uberoi, M. S., Qaudruple Velocity Correlations and Pressure Fluctuations in Isotropic Turbulence, *J. Aeronaut. Sci.*, **20**, No. 3, pp. 197–203, 1953.

198. Ulinich, F. R. and Lyubimov, B. Ya., The Statistical Theory for a Turbulence in an Incompressible Liquid at High Reynolds Numbers, *JETP (Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki)*, **55**, pp. 951–965, 1968 [in Russian].

199. Van Driest, E. R., On Turbulent Flow Near a Wall, *J. Aeronaut. Sci.*, **23**, No. 11, pp. 1007–1011, 1956.

200. Van Dyke, M., *An Album of Fluid Motion*, Parabolic Press, Stanford, CA, 1982.

201. Velikanov, M. A. and Shveikovskii, N. T. (Eds.), *Problems of Turbulence: Translated Papers*, ONTI, Moscow/Leningrad, 1936 [in Russian].

202. Veske, D. R. and Sturov, G. E., An Experimental Study of a Twisted Turbulent Flow in the Cylindrical Pipe, *Proc. Sib. Dep. Acad. Sci. USSR, Ser. Eng. Sci. (Izvestiya Sibirskogo Otdeleniya AN SSSR, Ser. Terhnicheskih Nauk)*, **13**, No. 3, pp. 3–7, 1972 [in Russian].

203. Voinov, V. V., Voinov, O. V., and Petrov, A. G., A Hydrodynamic Interaction of Bodies in the Ideal Incompressible Fluid and Their Motion in the Non-Homogeneous Flows, *J. Appl. Math. Mech. (Prikladnaya Matematika i Mekhanika)*, **37**, No. 4, pp. 680–689, 1973 [in Russian].

204. Von Kármán, T., Mechanische Ähnlichkeit und Turbulenz, *Nachr. Ges. Wiss. Göttingen, Math. Phys. Klasse*, **5**, pp. 58–76, 1930 [see NACA TM 611 for English translation].

205. Von Kármán, Th., Progress in the Statistical Theory of Turbulence, *Proc. Nat. Acad. Sci. US*, **34**, pp. 530–539, 1948.

206. Von Kármán, Th. and Howarth, L. On the Statistical Theory of Isotropic Turbulence Proc. Roy. Soc. LondonA164192–2151938

207. Voropaeva, O. F. and Chernyh, G. G., The Internal Waves Generated by a Non-Impact Turbulent Wake in Linearly Stratified Medium, *Math. Model. (Matematicheskoe Modelirovanie)*, **10**, No. 6, pp. 75–89, 1998 [in Russian].

208. Vu, B. T. and Gouldin, F. C., Flow Measurements in a Model Swirl Combustor, *AIAA J.*, **20**, No. 5, pp. 642–651, 1982.

209. Wilcox, D. C., Reassessment of the Scale-Determining Equation for Advanced Turbulence Models, *AIAA J.*, **26**, pp. 1299–1310, 1988.

210. Wolfshtein, M., On the Length-Scale of Turbulence Equation, *Isr. J. Technol.*, **8**, No. 1-2, pp. 87–99, 1970.

211. Wygnanski., I. and Fiedler, H., Some Measurements in the Self-Preserving Jet, *J. Fluid Mech.*, **38**, Pt. 3, pp. 577–612, 1969.

212. Yahot, V. and Orszag, S. A., Renormalisation Group Analysis of Turbulence. 1. Basic Theory,

J. Sci. Comput., **1**, No. 1, pp. 3–51, 1986.

213. Yanenko, N. N. and Novikov, V. A., On One Model for a Fluid with Sign-Changing Coefficient of Viscosity, *Numer. Meth. Cont. Mech. (Chislennie Metody Mekhaniki Sploshnoi Sredy)*, **4**, No. 2, pp. 142–147, 1973 [in Russian].

214. Yoshizawa, A. A., Governing Equation for the Small-Scale Turbulence. 2. Modified DIA Approach and Kolmogorov's $-5/3$ Power Law, *J. Phys. Soc. Jap.*, **45**, No. 5, pp. 1734–1740, 1978.

215. Zayec, P. G., *An Experimental Study of a Turbulence Spectrum in Flow in the Rotating Pipe*, Cand. Phys.-Math. Sci. Dissertation, Moscow Inst. Phys. Technol., Moscow, 1986 [in Russian].

216. Zayec, P. G., Kurbatskii, A. F., Onufriev, A. T., Poroseva, S. E., Safarov, N. A., Safarov, R. A., and Yakovenko, S. N., Experimental Study and Mathematical Modeling of Characeristics of the Turbulent Flow in Straight Circular Pipe Rotating with Respect to Longitudinal Axis, *J. Appl. Mech. Tech. Phys. (Prikladnaya Mekhanika i Tekhnicheskaya Fizika)*, No. 2, pp. 103–116, 1998 [in Russian].

217. Zayec, P. G., Onufriev, A. T., Pilipchuk, M. I., and Safarov, R. A., *The Fourth-Order Two-Point Correlation Functions for Longitudinal Velocity in a Turbulent Flow in the Rotating Pipe*, Moscow Inst. Phys. Technol., Moscow, VIVITI Deponent, No. 3831-84, 1984 [in Russian].

218. Zayec, P. G., Onufriev, A. T., Pilipchuk, M. I., Samusenko, V. A., Safarov, N. A., and Safarov, R. A., Using the Computer-Matched Thermoanemometric Complex for Measuring the Characteristics of Eddying Turbulent Flows, in: *Physical Methods for Studying the Transparent Inhomogeneities*, Znanie, Moscow, 1986 [in Russian].

219. Zayec, P. G., Onufriev, A. T., and Sadovskii, M. V., Spectral Study of Non-Isothermal Jet. A Model of Temperature Pulsation Spectrum, in: *Thes. Semin. "Measurement in Flows: Methods, Equipment, Application*, IVT and CLIS, Acad. Sci. USSR Publications, Moscow, 1990 [in Russian].

220. Zayec, P. G., Onufriev, A. T., Safarov, N. A., and Safarov, R. A., Experimental Study of Behavior of Energy Spectrum in the Turbulent Flow in the Pipe Rotating with Respect to Longitudinal Axis, *J. Appl. Mech. Tech. Phys. (Prikladnaya Mekhanika i Tekhnicheskaya Fizika)*, No. 1, pp. 33–38, 1992 [in Russian].

221. Zayec, P. G., Onufriev, A. T., Safarov, N. A., and Safarov, R. A., Experimental Studying of the Spectra Corresponding to Friction Stress and the Third-Order Moments in Developed Turbulent Flow in the Pipe, *J. Appl. Mech. Tech. Phys. (Prikladnaya Mekhanika i Tekhnicheskaya Fizika)*, No. 6, pp. 99–104, 1994 [in Russian].

222. Zayec, P. G., Safarov, N. A., and Safarov, R. A., An Experimental Studying of Behavior of the Turbulent Flow Characteristics at Channel Rotation with Respect to Longitudinal Axis, in: *The Modern Problems of Continuum Mechanics*, Moscow Inst. Phys. Technol., Moscow, pp. 136–142, 1986 [in Russian].

223. Zhigulyov, V. N., On Equations of Turbulent Gas Motion, *Trans. Acad. Sci. USSR (Doklady AN SSSR)*, **165**, No. 3, pp. 502–505, 1965 [in Russian].

224. Zhigulyov, V. N. and Tumin, A. M. Rise of a Turbulence NaukaNovosibirsk1987 [in Russian]

225. Zubarev, V. N., Morozov, Z. G., Troshkin, O. V., and Danilenko, A. Yu., *The Bifurcation Model of Simple Turbulent Flows*, Math. Inst. Acad. Sci. USSR Publications, Moscow, 1988 [in Russian].