INELASTICITY

VARIANTS OF THE THEORY

V. S. BONDAR

Dedicated to Aleksei Antonovich Ilyushin and Valentin Valentinovich Novozhilov, the distinguished mechanical scientists of our times

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The simplest applied variants of the inelasticity theory that can be used to investigate the laws governing deformation and destruction of a material subjected to complex nonisothermal loading as well as to calculate the kinetics of the stressed-strained state and to predict the resource of light-parameters constructions are presented

The book is intended for specialists of design organizations, scientific-research institutes, as well as for post-graduates and students dealing with computations and investigations of high-loaded structures.

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INTRODUCTION

The problems of reliable functioning of the contemporary technological constructions, operating under the conditions of high-level power and temperature loadings, and of reduction of the amount of materials required per unit of output of them makes the problem of mathematical modeling of their inelastic behavior and failure extremely urgent. The increase of the working parameters of modern machines and devices leads to an increase in both the general and local strength of constructions. Real processes of loading such constructions lead to development of inelastic (viscoplastic) deformations in their material. The loading itself is a complex nonisothermal process, and the character of its change can be most arbitrary under the conditions of recurrence and duration of the effect of the thermal and force loadings and of ionizing radiation.

The currently used theories of plasticity, creep, and inelasticity generalized to nonisothermal loading can lead to reliable results only under restricted conditions, i.e., under loadings close to simple and stationary ones. Separate consideration of the processes of plasticity, creep, and damage accumulation without account for their mutual influence is practically typical of all the theories applied in calculations. Such important aspects that influence damage accumulation as embrittlement and healing are practically not considered. All the aforesaid substantially limits the fields of application of the theories of plasticity, creep, and of the kinetic equations of damage accumulation (failure criteria) used in calculations.

At the present time, the theory of inelasticity has been developed [1–3] which results from the generalization and evolution of the ideas contained in different variants of the theories of plasticity, creep, and inelasticity based on the concept of microstresses which was put forward by V. V. Novozhilov and his school [4–6]. The theory of inelasticity belongs to the class of single-surface theories of flow with combined hardening. The validity of the developed theory of inelasticity was justified in [3, 7–15] on a wide spectrum of structural materials (steels and alloys) and various programs of experimental investigations. Comparisons of calculations carried out according to various theories of plasticity, creep, and inelasticity have shown that the results obtained with the use of the developed theory of inelasticity correspond best to experimental data. Based on the above-mentioned studies, the conclusion has been drawn that the developed theory of inelasticity can be used in practical calculations of inelastic behavior of the materials of constructions and damage accumulation in them in an arbitrary process of complex nonisothermal loading, and that based on this theory one can carry out a reliable prediction of the service life of high-parameters structure materials under recurrent and prolonged effect of thermal power loadings and ionizing radiation.

The range of application of the theory of inelasticity is limited by small strains of homogeneous and initially isotropic metals at temperatures, when there are no phase transformations, and deformation rates, when dynamic effects can be neglected.

Worthy of note are some of the characteristic features of the developed theory of inelasticity:

- deformation has both an elastic and inelastic components (there is no conditional division of inelastic deformation into plastic deformation and creep);
- the loading surface can be displaced, expanded or narrowed and can change shape;
- the trinomial structure of the equations for loading surface displacement stipulates the removal of limitations on the trajectories of complex loading;
- the kinetic equations of damage accumulation are based on the energy principle with account for the processes of embrittlement and healing;
- the equations of inelastic behavior and damage accumulation are interconnected, i.e., a damage influences the behavior, and the history of loading affects the process of damage;
- inelastic behavior and failure can depend on the type of stressed state;
- in the case of complex (disproportionate) cyclic loading an extra isotropic hardening is possible;
- analytical integration of the inelasticity theory equations for elementary stationary loading regimes leads to the well-known criteria of low-cycle and long-term strength;
- the processing of experimental curves which is not connected with determination of yield stress and other quantities with any allow-

ances serves as a basis of the experimental-computational method of determining the material functions of the inelasticity theory;

- standard testings of a material for low-cycle and long-term strength, and also for deformation under the conditions of plasticity and creep are the basic experiments underlying the experimental-computational method.

The present book sets out the simplest applied variants of the inelasticity theory which can be used in researching the laws governing deformation and destruction of a material on complex nonisothermal loading, in calculating the kinetics of a stressed-strained state and predicting the service-time of high-parameter structures.

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PART 1

ELASTICITY

CHAPTER 1

THEORY OF STRESSES AND STRAINS

1.1. Stress Tensor and Its Invariants

An element of a solid is chosen to describe the stressed state at a point of the solid. This element is represented as a rectangular parallelepiped the three edges of which do match with the coordinate axes (Fig. 1.1) [16, 17]. This is the way in which the stressed state at a point of a solid is determined by a symmetrical tensor of the 2nd order:

$$T_\sigma = \begin{Bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{Bmatrix} = \sigma_{ij}, \qquad (i, j = 1, 2, 3). \tag{1.1}$$

According to the pairing law of the tangential stresses

$$\sigma_{ij} = \sigma_{ji} \qquad (i, j = 1, 2, 3; \quad i \neq j) . \tag{1.2}$$

The stress tensor has the following invariants:

$$I_1\left(T_\sigma\right) = \sigma_{ii}, \tag{1.3}$$

$$I_2\left(T_\sigma\right) = \frac{1}{2}\sigma_{ij}\sigma_{ij} - \frac{1}{2}\left(\sigma_{kk}\right)^2, \tag{1.4}$$

$$I_3\left(T_\sigma\right) = \frac{1}{3}\sigma_{ij}\sigma_{jk}\sigma_{ki} - \frac{1}{2}\sigma_{ij}\sigma_{ij}\left(\sigma_{kk}\right) + \frac{1}{6}\left(\sigma_{kk}\right)^3. \tag{1.5}$$

Here and below, the same indices in the monomial point to summation over all the values of these indices.

Based on Eqs. (1.3)–(1.5), the following expressions can be obtained for the invariants of the stress tensor:

$$\mathrm{I1}(T_\sigma) = \sigma_{11} + \sigma_{22} + \sigma_{33} , \tag{1.6}$$

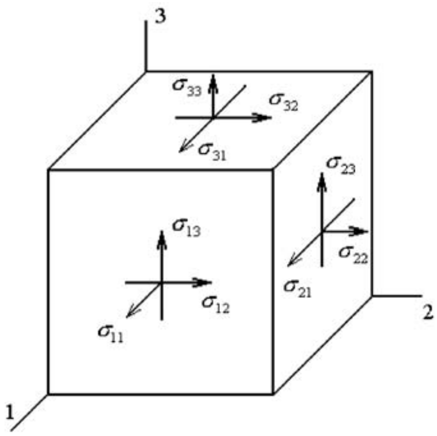

Fig. 1.1. Stressed state model

$$I_2(T_\sigma) = -\sigma_{11}\sigma_{22} - \sigma_{22}\sigma_{33} - \sigma_{33}\sigma_{11} + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2, \tag{1.7}$$

$$I_3(T_\sigma) = \sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{11}\sigma_{23}^2 - \sigma_{22}\sigma_{31}^2 - \sigma_{33}\sigma_{12}^2 + 2\sigma_{12}\sigma_{23}\sigma_{31}. \tag{1.8}$$

Three different cases of loading a thin-walled tubular model are considered as an example: by an axial force (P tests), by a torsional moment (M tests), and by an axial force, torsional moment, and inner pressure simultaneously (P, M, and q tests). In the first two cases, uniaxial stretching (compression) and torsion are realized, and a generalized plane state is realized in the third case. For all the three cases, the stress tensor and its invariants have the following form (axis 1 is directed along the specimen axis):

1) uniaxial stretching (compression)

$$T_\sigma = \begin{Bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{Bmatrix}, \tag{1.9}$$

$$I_1(T_\sigma) = \sigma_{11}, \quad I_2(T_\sigma) = 0, \qquad I_3(T_\sigma) = 0, \tag{1.10}$$

2) torsion

$$T_\sigma = \begin{Bmatrix} 0 & \sigma_{12} & 0 \\ \sigma_{21} & 0 & 0 \\ 0 & 0 & 0 \end{Bmatrix}, \tag{1.11}$$

$$I_1(T_\sigma) = 0, \quad I_2(T_\sigma) = \sigma_{12}^2, \quad I_3(T_\sigma) = 0, \tag{1.12}$$

3) generalized plane state

$$T_\sigma = \begin{Bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{Bmatrix} , \tag{1.13}$$

$$\begin{aligned} I_1(T_\sigma) &= \sigma_{11} + \sigma_{22} + \sigma_{33} , \\ I_2(T_\sigma) &= -\sigma_{11}\sigma_{22} - \sigma_{22}\sigma_{33} - \sigma_{33}\sigma_{11} + \sigma_{12}^2 , \\ I_3(T_\sigma) &= \sigma_{11}\sigma_{22}\sigma_{33} - \sigma_{33}\sigma_{12}^2 . \end{aligned} \tag{1.14}$$

1.2. Stress Deviator and Its Invariants

The stress tensor T_σ can be represented as a sum of two tensors: the spherical tensor T_S and stress deviator D_σ [16, 17]:

$$T_\sigma = T_S + D_\sigma = \begin{Bmatrix} \sigma_o & 0 & 0 \\ 0 & \sigma_o & 0 \\ 0 & 0 & \sigma_o \end{Bmatrix} + \begin{Bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{Bmatrix} = \sigma_o \delta_{ij} + s_{ij} . \tag{1.15}$$

Here δ_{ij} is the Kronecker symbol ($\delta_{ij} = 1$ at $i = j$, $\delta_{ij} = 0$ at $i \neq j$), $\sigma_o = \frac{1}{3}\sigma_{ii}$ is the mean stress, and s_{ij} is the stress deviator component.

Thus, the stress deviator components are defined by

$$s_{ij} = \sigma_{ij} - \sigma_o \delta_{ij} . \tag{1.16}$$

The stress deviator D_σ has the following invariants:

$$I_1(D_\sigma) = s_{ii} = 0 , \tag{1.17}$$

$$I_2(D_\sigma) = \frac{1}{2} s_{ij} s_{ij} , \tag{1.18}$$

$$I_3(D_\sigma) = \frac{1}{3} s_{ij} s_{jk} s_{ki} . \tag{1.19}$$

or, expanding Eqs. (1.18) and (1.19), one can obtain the following expressions for the 2nd and 3rd stress deviator invariants:

$$I_2(D_\sigma) = \frac{1}{2}\left(s_{11}^2 + s_{22}^2 + s_{33}^2\right) + s_{12}^2 + s_{23}^2 + s_{31}^2 , \tag{1.20}$$

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