MOMENTUM AND HEAT TRANSFER IN TURBULENT GAS-SOLID FLOWS

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To authors' parents

Clara M. and Rafael M. Gorbis
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PREFACE

This book is concerned with simultaneous transfer of momentum, heat and particulate mass in turbulent gas flows containing a relatively small (by volume) fraction of solid particles. Multiple-transfer processes of this type appear in natural phenomena and in a wide range of industrial systems for drying, heat exchange and pneumatic conveying, as well as in more recent applications such as gas-cooled nuclear reactors with naturally-occurring or added entrained particles, MHD and EGD devices, etc.

Interest in this subject [1] has caused a number of publications to appear. The problem is dealt with only in passing in handbooks on heat transfer fundamentals and applications [2–6], attention therein being focused primarily on packed and fluidized beds or gas-water systems. Some of the available references analyze specific transfer processes. The book by Beddow [7], for instance, covers the formation, mixing, storage and separation of particles. Classic reference books by Fortier [8] and Soo [9], and recent publications [10–15] deal principally with particle transport in turbulent flows. Books by Gorbis [16–18] chiefly describe heat transfer by convection in disperse flows. Nigmatulin [19] is concerned with the mechanics of heterogeneous media. Boothroyd [20] provides introduction to features specific to suspension flows. Friedlander [21] focuses on mass transfer in aerosol flows.

Separate analysis of different transfer processes in such a flow is fundamentally incomplete when applied to a situation in which they occur simultaneously. To present a methodology for analysis of simultaneous transport processes on a unified basis this book is based on the following principles:

1. An approach developed by the authors is used to incorporate the actual heterogeneous structure of the flows under study. This allows gaining insight into specific features of the flows and establishing the effects associated with them.

2. The various physical and mathematical models, corresponding analytic solutions and available experimental data for each form of transport phenomena are analyzed.

3. The different problems are taken in order of increasing difficulty. Thus, for example, taking each of the transport modes (momentum, mass, and heat trans-

fer), transport within the components is first analyzed, then the interaction between components, and then interactions with the entire disperse flow. Similarly, momentum transport may be first analyzed, followed by transport of particle mass with allowance for transport of momentum, and then analysis of heat and mass transfer based on all the previous sections. Some examples of utilization of turbulent gas-suspension flows are given in the final chapter.

The overwhelming majority of applications of particle-laden gas streams are turbulent, and this book therefore deals exclusively with turbulent transport phenomena. It is assumed that the components are not subject to phase transitions and do not interact chemically.

Many software packages have become commercially available for implementation of computational fluid dynamic (CFD) methods for modeling gas-particle flows. Experience with CFD has shown that it is invaluable for evaluating overall flow patterns, especially in complex geometries. These capabilities make it important that computation be based on models which reflect the nature of the transport phenomena under study with sufficient completeness. Hopefully the results presented in this book will be instrumental in refinement of computer models, increasing accuracy and reliability of CFD predictions for gas-solid flows (especially for non-dilute and non-isothermal gas suspensions).

The contents of this book are based primarily on results obtained and compared with published data by the authors.

It is hoped that the information set forth here will be helpful to those engineers, physicists, environmentalists, and others involved in the study, design, interpretation or operation of systems utilizing turbulent gas-solid flows.

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NOMENCLATURE

a – thermal diffusivity; absorptivity;

B – velocity-profile constant; particle relative concentration; dimensionless complex;

c – specific heat;

c_f – coefficient of aerodynamic drag of particles;

D – channel diameter; diffusion coefficient;

d – diameter;

E – electric-field strength; radiation density; energy spectrum of turbulence;

F – force; surface area;

f – shape factor; frequency; focal distance;

G – flowrate; radial temperature distribution;

g – acceleration of gravity;

H – height of reactor;

I – intensity of radiation; dimensionless particle mass flux;

j – particle mass flux density; emissivity;

K – transport coefficients; Gasterstädt coefficient; overall heat transfer coefficient; acceleration parameter;

K^* – flow-through number; thermal accommodation factor;

k – correction factors; adiabatic exponent; coefficient of particle interaction with radiation; turbulent kinetic energy;

L – reference dimension; length in direction of flow;

l – molecular mean free path; radiation wavelength; flow-cell size;

M – moment of force; Mach number;

m – mass; coefficient;

N – number of particles; number of transfer units; power;

n – rotational frequency; index of refraction; normal to a surface;

P – thermal resistance; volumetric radiation density;

p – pressure;

Q – heat flux;

q – heat flux density; charge of particle;

R – dimensionless radius; correlation coefficient;
r – radius;
S – shearing-rate tensor; distance between two points;
s – entropy;
T – temperature; time scale;
U – dimensionless velocity;
u – velocity;
V – volume, dimensionless transverse velocity;
v – transverse velocity;
v_{ff} – terminal setting velocity (free fall);
v^* – friction velocity;
W – flow heat capacity rate; dimensionless relative velocity;
w – relative velocity;
X – dimensionless coordinate;
x – coordinate;
Y – shift; dimensionless coordinate;
y – transverse coordinate;
Z – heat capacity rate ratio;
z – coordinate;
α – heat transfer coefficient; amplification factor;
α_β – coefficient of mass transfer in transport of particles;
β – particle concentration by volume;
Γ – strength of vortex;
γ – probability of capture of particles on the wall; scattering function;
Δ – relative thickness of boundary layer;
δ – thickness of boundary layer; scattering-function shape factor;
ε – porosity ($\varepsilon = 1 - \beta$); viscous dissipation of the energy of turbulence; emissivity factor; fouling factor; dimensionless eddy viscosity;
ζ – momentum loss factor;
η – dynamic viscosity; particle collection efficiency; thermodynamic efficiency;
θ – polar angle; relative temperature;
ϑ – excess temperature; dimensionless temperature;
æ – radiation absorption or attenuation factor; contribution of particle to eddy thermal conductivity;
Λ – macroscale of turbulence; dimensionless eddy thermal conductivity;
λ – thermal conductivity; microscale of turbulence; friction factor;
μ – solids loading ratio (solids mass flow rate ratio);
ν – kinematic viscosity;
ξ – flow resistance coefficient;
Π – group of geometric characteristics;
ρ – density; diffraction parameter; distance from point to coordinate origin;
σ – Boltzman constant; tensor of viscous stresses; thermal effectiveness;
τ – time; shear stresses; optical thickness;
τ_{rel} – relaxation time;

Φ – relative contribution of particles to momentum transfer; dimensionless force; light flux;

φ – slip coefficient;

ϕ – angular coordinate;

χ – velocity profile constant; absorption factor; configuration factor;

Ψ – stream function;

ψ – phase shift; nonuniformity coefficient; dimensionless relative velocity; configuration factor;

Ω – albedo; channel cross section;

ω – angular velocity; frequency of fluctuations, solid angle.

SUBSCRIPTS

0 – value in particle-less flow; non-disturbed value; value at start;

i – component corresponding to Cartesian coordinate;

max, min – maximal and minimal values;

a – value on flow axis; adhesion;

r – radial distribution;

R – quantity of radiative nature;

conv – quantity of convective nature;

ce – cell in suspension flow;

c – flow core; circulation;

t – tangential component; quantity of thermal nature;

w – value on the wall;

x – lengthwise local value;

fin – final value;

cr – critical value;

l – local value; layer of deposit;

in – initial value;

opt – optimal value;

f – suspension flow;

inc, *abs*, *res*, *em* – incident, absorbed, resultant and emitted radiant fluxes;

b – boundary layer;

s – solid particle;

imp – impact-related quantity;

sph – sphere;

e – effective or equivalent value; quantity of electrical nature;

* – quantity of turbulent nature; effective value;

$'$ – fluctuating component; inlet value of quantity; $''$ – exit value of quantity;

$\langle\ \rangle$ – averaging;

$\bar{\ }$ – averaged by volume;

D – diffusion;

v – volume; velocity; vibration;

sl – slip flow regime;

d – drag force; diffraction;

int – internal;
ext – external;
mol – molecular;
inj – injection;
lim – limiting;
n – normal;
sed – sedimentation;
T – turbulent;
M – modified value;
act – actual;
g – gravity;
ac – acceleration;
st – stabilized; steady;
fr – friction; flow rate;
sh – shear;
j – jet;
un – unsteady;
rot – rotation;
Σ – resultant value;
id – ideal;
ap – aperture;
red – reduced;
m – melting;
rel – relaxation;
L – laminarization;
noz – nozzle;
sc – scattering;
itc – intercomponent;
d.p – dew point;
g – flue gas;
tu – tube;
sp – specific.

CHAPTER
ONE

PRINCIPAL CHARACTERISTICS OF SUSPENSION FLOWS AND MODELS OF TRANSPORT PROCESSES

1.1 PRINCIPAL CHARACTERISTICS AND CLASSIFICATION OF SUSPENSION FLOWS

Systems exhibiting significant inhomogeneity exist in nature and technology side by side with homogeneous systems. Below we consider systems in which the inhomogeneity arises because one of the system's components is in a macrodiscrete state. This component is known as the dispersed medium, and may differ by its nature, state and size of the particles making it up. The second component is the continuous (dispersion) medium, and occupies the entire system volume with the exception of the space taken up by the dispersed medium. This medium also may differ by its state, kind of substance and other parameters. Such systems are known as disperse systems. A major feature of such systems, steming from the fact that they are heterogeneous, is the presence of a developed interface between the system's constituents, which are frequently termed phases or macrocomponents. Each phase may in general be a homogeneous mixture of substances of different chemical nature (i.e., a mixture of constituents). Henceforth we are concerned with systems in which both phases consist of different components for which reason phase transitions cannot occur in such systems. In order to distinguish between such heterogeneous systems and two-phase mixtures with phase transition (for example, boiling liquids, condensing vapors, etc.) one should apply the concept of macrocomponents, which are in fact phases which are prevented from undergoing mutual transitions. It is precisely in this sense that the dispersion and the dispersed media are macrocomponents of a disperse system, and the interaction between them is of intermacrocomponent (or, for brevity, intercomponent) nature.

Conçentration, which represents the fraction (denumerable, volumetric, mass) of the dispersed medium, is a fundamentally new parameter which distinguishes heterogeneous from homogeneous systems. Another parameter which is particularly important for different transport processes is the system "through-flow" feature [1]. Flows in which all the components pass once completely through the apparatus are treated in [1, 2] over the entire concentration range as a single class of disperse through-flow systems. Mutual transitions are possible between this class of disperse systems and others (nonflowing, i.e., nonmoving systems, semi-through-flowing – with one of the components flowing through and the other not; fluidized). However, it is precisely through-flow systems in which the component velocities do not have a physical upper limit; technical limitations obviously, do exit – wear of particles, surface erosion, etc. The lower limit on the velocities consists only of conditions $\bar{u} \neq 0$ and $\bar{u}_s \neq 0$. This is one of the fundamental differences between this class of disperse systems and all others, allowing to effectively organize various continuous transport processes while continuously varying the operating conditions and geometric parameters. The structure of the system, and accordingly, the mechanisms of transport processes, in such flows may undergo abrupt changes under certain, critical conditions. These conditions are estimated approximately [2] from the value of the solids concentration, whereas in [1] – this was additionally done on the basis of the through-flow criterion. Disperse flows may take the following forms with increasing concentration: suspension (volumetric particle concentration $0 < \beta \leq 0.03$), dense flows ($0.03 \leq \beta \leq 0.3$) and moving dense packed bed ($0.3 \leq \beta \leq 0.7$).

This means that suspension flows exhibit low particle concentrations at which transport processes are not yet significantly affected by the collective nature of particle behavior. Conditions under which not only the interaction between particles, but their effect on the dispersion medium (the single-particle approximation) are insignificant, exist in dilute, low-concentration flows ($0 < \beta \lesssim 4 \cdot 10^{-4}$). The values of concentration, all other conditions remaining equal, control the degree of heterogeneity of the flow structure (in the limiting case of $\beta = 0$ the flow becomes homogeneous). The degree of inhomogeneity of the fields of other flow variables (velocity, temperature, etc.) is estimated by the applicable slip coefficients ($\varphi_u \equiv u_s / u$ and $\varphi_t = T_s / T$). All other conditions remaining equal, the ratio of the particle size d_s to the reference dimension of the flow L serves as a characteristic of the flow constraint. From the phenomenological point of view all the manifestations of heterogeneity are associated with the presence of the dispersed phase. However, at certain value of the Knudsen number Kn_s (ratio of the mean free path of the gas molecules and the dimension of macroparticles) the phenomenological approach cannot be applied to analysis of intercomponent transport and it becomes necessary to make allowance not only for the macroheterogeneous nature of the dispersed medium, but also for the microheterogeneity of the carrier medium.

Dilute "gas-particle" flows were defined by Crowe [3] on the basis of the effect on the particle motion of only local aerodynamic forces, whereas dense flows are

Table 1.1 Classification of through disperse flows

Kind of through disperse flow	β, m^3/m^3	μ, $\frac{kg/s}{kg/s}$	K^*
1. Single-phase flow	0	0	10^6
2. Gas suspension flow:			
2.1. Dilute gas suspension (single-particle approximation)	$\leq 4 \cdot 10^{-4}$	$\leq 1 \div 3$	$\sim 1.4 \cdot 10^6$
2.2. Dense gas suspension	$4 \cdot 10^{-4} - 3 \cdot 10^{-2}$	$5 - 50$	$\sim 3 \cdot 10^7$
3. Highly concentrated disperse flow	$3 \cdot 10^{-2} - 0.3$	$50 - 500$	$\sim 10^8$
4. Ventilated moving particle bed	$0.3 - 0.65$	$1 - 50$	$\sim 7 \cdot 10^6$

characterized by the motion of particles, primarily due to their collisions. This rather extensively employed classification [3–10, etc.] is based on the ratio of the time constant for interaction between the particles and the gas and the time between particles collisions [3]. Subsequently the definition of the difference between dilute and dense gas suspension flows was defined by analogy with the behavior of single-phase flows as a function of the Knudsen number *Kn* [4].

The "dilute" mode is thus taken to be analogous to free-molecular flow and the "dense" mode – to continuous flow. If $Stk \cdot Re^{0.5} < 0.1$, then the particle flux should be treated as a fluid without slip at the wall [4]. This interesting approach, however, leaves out of consideration a number of important factors such as effects of nonisothermicity and of the wall; presence of nonaerodynamic forces (electrical, phoretic, etc.), the particle size distribution and their concentration profile, continuous effect of the carrier gas on the free-molecular motion of the particles, etc.

The additional fact that the particles exert their effect on the average and turbulent local flow of the gas requires special analysis.

The claim that an analogy can be drawn between dense gas suspension flow and a continuous fluid raises very fundamental objections. By virtue of the inevitable macroheterogeneity, these flows, up to the limiting solids concentrations (dense moving beds with up to $\beta_{max} \simeq 0.7$ m^3/m^3 [1, 2]) do not satisfy the "nonslip" conditions for particles at the wall (zero equality of the velocity at the wall), which defines the acceptability of continuum description for single-phase media. This situation is not changed by the presence of deposited particles. In any case, macroheterogeneity in the flow at the wall or at the core persists and hence cannot be disregarded.

The tentative boundaries of different kinds of through gas particles flows can be defined on the basis of the volumetric particle concentration β, ratio of mass flowrates of macrocomponents μ and the flow through criterion K^*, suggested as the ratio of inertia forces of the flow components to the forces of their viscous and "dry" friction [1] (Table 1.1).

It is assumed here that particles in dilute flows do not affect the cartrier fluid and do not collide, whereas in dense flows collisions can be neglected only at $\beta < \beta_{cr} \simeq 3\%$ for large particles and $\simeq 0.5$ to 1% for Stokesian particles (Sec. 2.1).

It should be emphasized that the above boundaries classify through gas-

Table 1.2 Classification of suspension flows

Nos.	Classification attribute	Kinds of suspensions
1	Origin	Natural, industrial
2	Use	Heat-transfer agent, materials-transporting fluid, working fluid, mass-transfer agent
3	Kind of components	Gas-solids, gas-droplets, liquid-solids
4	Particle size	Colloid; aerosol; smoke; fog ($d_s < 0.1\,\mu$m); suspension ($d_s > 0.1 \div 1\,\mu$m)
5	Dispersity of particles	Monodisperse, two particle-size ranges, multiple particle-size ranges, polydisperse
6	Particle shape	Spherical, non-spherical (ellipsoid, flake-like, acicular, irregular)
7	Particle properties	Different thermal, electromagnetic, optical, radiative and other properties
8	Particle concentration	Dilute suspension flows; dense suspension flows
9	Through-flowability of components	Nonflow (equal-density slurry, non-ventilated non-moving bed of particles); semithrough flow (precipitation of particles in liquid at rest – sedimentation; fluidization of particles by a moving fluid without particle carry-over); throughflow (through gas suspension flows)
10	Conveying-flow mode	Laminar, transition, turbulent
11	Mode of flow over the particles	
	a) macro (based on Re_s)	Separationless; with separation (laminar or turbulent boundary layer)
	b) molecular (based on Kn_s)	Continuum, slip flow, transitional, free-molecular
12	Gas suspension flow pattern	Channel flow (rectilinear, curved, swirled, pulsating flow); flow over bodies (longitudinal, transverse); jet flows (free, submerged, cross, counterdirected)
13	Relative motion of components	Cocurrent flow (upflow, downflow, horizontal), counterflow, crossflow
14	Forces acting in the gas suspension flow	
	a) by their type	Internal and applied; volume and surface
	b) by their nature	Mechanical, hydrodynamic, electromagnetic, radiometric

particles systems primarily by their fluid mechanical conditions. The fact that such systems are nonisothermal may supplement or modify these conditions.

The different kinds of suspensions can be classified on the basis of the attributes listed in Table 1.2. In keeping with the intent of this book, attention is primarily given to suspensions arising in engineering equipment (Chapter 7), with most emphasis on gas-solids flows. The relationships derived for these conditions are to a large extent applicable also to other types of suspensions although the behavior of liquids with suspended particles (slurries) is affected by closeness of the component densities, and in the case of gas-droplets flows consideration must additionally be given to deformation, coalescence of droplets and to other effects. Gas-solids flows are of interest due to their prevalence under various natural and industrial conditions. In addition, investigation of these flows is extremely useful from the scientific point of view, since under these conditions the heterogeneous nature of suspension flows manifests itself strongest in various transport processes.

Transport processes in such flows depend to a large degree on the parameters of the dispersed phase – in the first place, on the average size and shape of particles, their granulometric composition, physico-mechanical, thermophysical and other properties of particles. The extensively used terminology of "small" and "large" particles has up to present not been precisely defined. For example, particles which can be regarded as small – "fine" in the thermal respect – from the point of view of conduction of heat within the particles (gradientless temperature field in a particle at $Bi_s \leq 0.1$, Sec. 4.1) may be regarded as large from the point of view of interaction between the gas molecules and the surface of particles ($Kn_s < 10^{-2}$, Sec. 1.5) or of the scattering of light ($\rho_d > 20$, Sec. 6.1). What actually happens is that the so-called "coarseness" is in each case estimated on the basis of the degree of manifestation of heterogeneity of the flow structure.

It is thus obvious that the boundary between small and large particles is different, depending on the nature of the transport process. This boundary will in any case be approximate, but must be determined with consideration of the specifics of the given transport process (Table 1.3). It is important to emphasize that in all the cases listed in Table 1.3 the "size" of the solids in a suspension is not controlled by the absolute geometric dimensions of the particles, but by the applicable dimensionless groups.

Obviously, geometric dimensions should be taken into account when considering hydromechanical conditions, and geometric and hydromechanical dimensions should be employed in analyzing thermal conditions. Thus, for example, geometric conditions control to a large degree the role of inhomogeneity of particles in the course of heat conduction (the geometric dimensions of the particles control their "conductive size"). Similarly, the hydrodynamic dimensions control to a large degree the "convective size". Such an estimate of the size of particles can also be obtained on the basis of features of other transport processes (electrical, mass-transfer, chemical, etc.)

The absolute size of particles can be described by a single dimension only for particles of close to spherical shape, whereas two or even three dimensions are needed for defining anisometric particles. The deviation of the geometric shape factor from unity in spherical particles changes the dynamic shape factor (Sec. 2.1). Note that many of the parameters of particles (size, shape, dispersity) listed in Table 1.2 may change in the course of flow of the gas suspension, primarily due to wear of particles. For example, prolonged circulation of a gas-graphite suspension in a closed loop causes (irrespective of the starting parameters of the particles) the formation of virtually monodisperse suspensions of micron-size particles. The commonly-held belief that the shape of crushed particles succes-

Table 1.3 Kinds of estimates of the degree of inhomogeneity ("size" of particles)

Nos.	Conditions of estimation of the "size" of particles	Controlling quantity	Phenomenon described
1	Geometric	Shape factor, f	Deviation of the shape from the spherical (for approximate limits see Sec. 2.1)
		Variance of particle sizes σ_s	Degree of polydispersity of particles
		Relative dimension of particle conglomerate, d_c / d_s	Agglomeration of particles in the flow
		Ratio of reference dimension of flow to the mean diameter of particles L/d_s	Degree of geometric constraint – effect of walls (Sec. 2.1)
		Function of volumetric concentration of particles	Degree of geometric constraint – effect of concentration (Sec. 2.1)
		Knudsen number for particles $Kn_s = 2l_0 / d_s$	Ratio of scales for micro- and macro-heterogeneity in the flow
		Ratio of the diameter of pores in the deposited layer to the particle size	Describes the looseness of the structure of particle sediments
		Ratio of height of wall roughness to the particle diameter	Specifics of contact interaction between the particles and the surface
2	Hydrodynamic	Free-fall velocity, $v_{f.f}$	"Fall diameter" (terminal setting velocity of the particles) – Sec. 2.1
		Reynolds number, $Re_{ff} = v_{ff} d_s / \nu$	Dimensionless free-fall velocity (Sec. 2.1)
		Ratio of thickness of the boundary layer at the outer edge of the flow δ and the particle diameter d_s	Effect of particles or the boundary layer parameters; features specific to the behavior of particles near the wall
		Relative turbulence scale	Interaction of particles with the turbulence field of the gas (Sec. 2.6)
		Stokes number, $Stk = v_{f.f} u / gL$	Degree to which the inertia force of the particles affects their behavior (Sec. 2.1)
		Particle velocity slip ratios mean velocity $\varphi_u \equiv \langle u_s \rangle / \langle u \rangle$ fluctuating $\varphi_u' \equiv \sqrt{\frac{\langle u_s' \rangle}{\langle u' \rangle}}$	Kinematic inhomogeneity of components in mean and fluctuating motion (Secs. 2.1 and 2.6)

3	Thermal	Biot number for particles $Bi_s = \alpha_s d_s / \lambda_s$	Thermal conductivity within the particle, degree of uniformity of the temperature field in the particle (Sec. 4.1) in intercomponent convective heat transfer
		Minimum Nusselt number for particles $Nu_{s,\min} = \alpha_{s,\min} d_s / \lambda$	Conductive intercomponent heat transfer (Sec. 4.2)
		Diffraction parameter $\rho_d \equiv \pi d_s / l_u$ – measure of the ratio of the particle diameter and light wavelength	Interaction of particles with the radiant flux (Sec. 6.1)
		Radiation Biot number – $Bi_R \equiv \sigma T_0^3 d_s / \lambda_s$	Thermal conductivity in particles in the course of radiative transfer (sec. 6.1)

sively approaches the spherical as they are made smaller is not always correct. Thus, when graphite particles with $d_s < 0.5$ mm are circulated, their shape becomes increasingly irregular, flake-like, which can be attributed to the "layered" structure of the hexagonal crystal lattice of graphite [1]. This means that a change in the size of particles of this kind of materials is accompanied by a change in the shape factor, and consequently, in the relationships governing their hydromechanical and thermal interaction with the gas.

Table 1.2 lists the properties of the material of particles as one of the classification attributes. The table lists basically only those properties which have a bearing on the subsequently analyzed transport processes. Strength, thermal, chemical and other properties may be of importance in other processes. However, in any case consideration must be given to the difference between the properties of the material as such and of particles of this material (this manifests itself most in electromagnetic, radiative properties).

Various disperse systems (including suspensions) may differ significantly by their through-flowability. Henceforth we consider primarily through gas suspension flows. The usually defined boundaries between different gas flow modes (for example, on the basis of the critical values of the Reynolds number) become unstable for suspension flows due to the effect of particle size, concentration and other parameters of the suspension. In nature and in the majority of engineering applications (due to the high rates at which the transport processes should occur) the flow of suspension is usually turbulent. Below we consider primarily this mode, and in several cases also transition flow. The effect of particles on the turbulence characteristics is analyzed in Chapter 3. Some of the specifics of transport processes in laminar and transition suspension flows are given in [11–17, etc.].

In the gas suspension flows usually analyzed in the literature, both the gas and the particles are moving in the same direction [3–8, 15, 18–27, etc.]. At the same time, the maximum difference in transport potentials in intercomponent interactions (for example, the driving temperature difference) does not occur in such flows, but in counterflow or (as an approach to counterflow) in multiply crossed flow. In such cases the particles usually move downward and the gas moves toward or across them.

Free convection in gas suspension flows is investigated, among others, in [28] and [29], and unsteady processes – in [30–32].

1.2 CLASSIFICATION AND SPECIFIC FEATURES OF TRANSPORT PROCESSES IN GAS SUSPENSIONS

a) Classification of Transport Processes

On the basis of the features in the flows of gas suspensions analyzed in the preceding section, we can identify the following fundamental differences between any transport process in gas suspensions and in homogeneous flows:

1. The fact that transport occurs simultaneously in the discrete and continuous flow components.

appearance of at least spatial inhomogeneous structures, which are so typical of such flows. It is clear here that the principal conditions for inception of the coherent structures (thermodynamic openness of the system, nonlinearity of the equations describing it, significant deviations from equilibrium, cooperative nature of behavior of the system's elements) may be satisfied in a gas suspension.

Obviously, turbulent gas suspension flows are a specific spatial structure, which feature distinguishes them from other kinds of disperse systems (Sec. 1.1). Such structure becomes converted into others at critical values of the characteristic parameters. Thus, for example, transition from moving bed structure to that of a suspension or back, of a gas suspension into a dense flow are controlled by the corresponding critical values of the flow-through criterion K^* [1], Froude, Stokes, etc., numbers. It is precisely the concept of the existence of a regular, cellular structure of the gas suspension flow which is the basis of the theoretical analysis of transport within it. It is known that the rate of all the transport processes is determined, in the first place, by specifics of the flow structure and all the critical phenomena in transport processes are primarily attributed to changes in this structure. However, virtually no attention is being paid to the study of various macroheterogeneous structures, conditions of their formation and mutual transitions. Such an analysis can be performed on the basis of methods under development in the theory of open dissipative systems, synergetics [97].

Set of Equations (1.25)–(1.32) obtained in this section is employed in subsequent chapters, with pertinent initial and boundary conditions, for analyzing various transport processes in gas suspension flows. The finite-difference operators have been subsequently replaced by differential ones for the sake of uniformity and convenience. However, these equations can be used only subject to restrictions analyzed in Sec. 1.4.

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