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Validation of Advanced Computational Methods for Multiphase Flow

## Series in Thermal & Fluid Physics & Engineering

Editor: G.F. Hewitt

# Validation of Advanced Computational Methods for Multiphase Flow

Hervé Lemonnier  
Didier Jamet  
Olivier Lebaigue



New York • Wallingford, U.K.

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Multiphase Flow***

**Editors:**

**Hervé Lemonnier, Didier Jamet, and Olivier Lebaigue**

Commissariat à l'Energie Atomique  
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**VALIDATION OF ADVANCED COMPUTATIONAL METHODS FOR MULTIPHASE FLOW**

EDITORS: HÉRVE LEMONNIER, DIDIER JAMET AND OLIVIER LEBAIGUE

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# Contents

<b>Contents</b>	<b>iii</b>
<b>Foreword</b>	<b>ix</b>
<b>Preface</b>	<b>xi</b>
References . . . . .	xiv
<b>1 Test-case No 1: Rise of a spherical cap bubble in a stagnant liquid (PN)</b>	<b>1</b>
1.1 Practical significance and interest of the test-case . . . . .	1
1.2 Definitions and model description . . . . .	2
1.3 Summary of the requested calculations . . . . .	3
References . . . . .	4
<b>2 Test-case No 2: Free rise of a liquid inclusion in a stagnant liquid (PN, PE)</b>	<b>7</b>
2.1 Practical significance and interest of the test-case . . . . .	7
2.2 Definitions and model description . . . . .	8
2.3 A series of six numerical test-cases . . . . .	9
2.4 An experimental test-case . . . . .	12
References . . . . .	16
<b>3 Test-case No 3: Propagation of pure capillary standing waves (PA)</b>	<b>17</b>
3.1 Practical significance and interest of the test-case . . . . .	17
3.2 Definitions and model description . . . . .	18
3.3 A series of test-cases . . . . .	20
References . . . . .	21
<b>4 Test-case No 4: Rayleigh-Taylor instability for isothermal, incompressible and non-viscous fluids (PA)</b>	<b>23</b>
4.1 Practical significance and interest of the test-case . . . . .	23
4.2 Definitions and physical model description . . . . .	24
4.3 Test-case description . . . . .	25
References . . . . .	29
<b>5 Test-case No 5: Oscillation of an inclusion immersed in a quiescent fluid (PA)</b>	<b>31</b>
5.1 Practical significance and interest of the test-case . . . . .	31

5.2	Definitions and model description . . . . .	32
5.3	Numerical settings, initial and boundary conditions . . . . .	35
5.4	Requested calculations . . . . .	35
5.5	An illustrative example . . . . .	36
5.6	Additional information for 2D calculations. . . . .	37
	References . . . . .	39
<b>6</b>	<b>Test-case No 6: Two-dimensional droplet pinning on an inclined wall (PC)</b>	<b>41</b>
6.1	Practical significance and interest of the test-case . . . . .	41
6.2	Description of the model for the contact angle hysteresis and definition of the test-case . . . . .	42
6.3	Test procedure . . . . .	43
6.4	Comparison criteria . . . . .	44
	References . . . . .	44
<b>7</b>	<b>Test-case No 7a: One-dimensional phase change of a vapor phase in contact with a wall (PA)</b>	<b>45</b>
7.1	Practical significance and interest of the test-case . . . . .	45
7.2	General definitions and model description . . . . .	47
7.3	Steady state model . . . . .	48
7.4	Unsteady model for a phase initially uniformly superheated or under-cooled . . . . .	53
	References . . . . .	62
<b>8</b>	<b>Test-case No 7b: Isothermal vaporization due to piston aspiration (PA)</b>	<b>65</b>
8.1	Practical significance and interest of the test-case . . . . .	65
8.2	Definitions and model description . . . . .	67
8.3	Test-case description . . . . .	71
	References . . . . .	71
<b>9</b>	<b>Test-case No 10: Parasitic currents induced by surface tension (PC)</b>	<b>73</b>
9.1	Practical significance and interest of the test-case . . . . .	73
9.2	Definitions and physical model description . . . . .	74
9.3	Test-case description . . . . .	75
9.4	Example of comparison exercise . . . . .	75
	References . . . . .	79
<b>10</b>	<b>Test-case No 11a: Translation and rotation of a concentration disk (N)</b>	<b>81</b>
10.1	Practical significance and interest of the test-case . . . . .	81
10.2	Definitions and physical model description . . . . .	82
10.3	Test-case description . . . . .	82
10.4	Example of comparison exercise . . . . .	83
	References . . . . .	84

<b>11 Test-case No 11b: Stretching of a circle in a vortex velocity field (N)</b>	<b>85</b>
11.1 Practical significance and interest of the test-case . . . . .	85
11.2 Definitions and physical model description . . . . .	86
11.3 Test-case description . . . . .	86
11.4 Example of comparison exercise . . . . .	87
References . . . . .	89
<b>12 Test-case No 12: Filling of a cubic mould by a viscous jet (PN, PE)</b>	<b>91</b>
12.1 Practical significance and interest of the test-case . . . . .	91
12.2 Definitions and physical model description . . . . .	92
12.3 Test-case description . . . . .	92
12.4 Figures, tables, captions and references . . . . .	93
References . . . . .	96
<b>13 Test-case No 13: Shock tubes (PA)</b>	<b>97</b>
13.1 Introduction . . . . .	97
13.2 The mathematical model and the solution of the corresponding Riemann Problem . . . . .	98
13.3 The shock tube . . . . .	101
References . . . . .	107
<b>14 Test-case No 14: Poiseuille two-phase flow (PA)</b>	<b>109</b>
14.1 Practical significance and interest of the test-case . . . . .	109
14.2 Definitions and physical model description . . . . .	110
14.3 Test-case description . . . . .	110
References . . . . .	111
<b>15 Test-case No 15: Phase inversion in a closed box (PC)</b>	<b>113</b>
15.1 Practical significance and interest of the test-case . . . . .	113
15.2 Definitions and physical model description . . . . .	114
15.3 Test-case description . . . . .	114
15.4 Illustrations of the problem . . . . .	115
References . . . . .	117
<b>16 Test-Case No 16: Impact of a drop on a thin film of the same liquid (PE, PA)</b>	<b>119</b>
16.1 Practical significance and interest of the test case . . . . .	119
16.2 Definitions and physical model description . . . . .	120
16.3 Test-case description . . . . .	122
References . . . . .	122
<b>17 Test-case No 17: Dam-break flows on dry and wet surfaces (PN, PA, PE)</b>	<b>125</b>
17.1 Practical significance and interest of the test-case . . . . .	125
17.2 Definitions and physical model description . . . . .	126
17.3 Test-case description . . . . .	127
References . . . . .	130

<b>18 Test-case No 19: Shock-Bubble interaction (PN)</b>	<b>131</b>
18.1 Introduction . . . . .	131
18.2 Description . . . . .	131
References . . . . .	133
<b>19 Test-case No 21: Gas bubble bursting at a free surface, with jet formation (PN-PE)</b>	<b>135</b>
19.1 Practical significance and interest of the test-case . . . . .	135
19.2 Definitions and physical model description . . . . .	136
19.3 Test-case description . . . . .	138
References . . . . .	141
<b>20 Test-case No 22: Axisymmetric body emerging through a free surface(PE)</b>	<b>143</b>
20.1 Practical significance and interest of the test-case . . . . .	143
20.2 Experimental setup description . . . . .	144
20.3 Test-case description . . . . .	145
References . . . . .	148
<b>21 Test-case No 23: Relative trajectories and collision of two drops in a simple shear flow (PA)</b>	<b>149</b>
21.1 Practical significance and interest of the benchmark . . . . .	149
21.2 Definitions and physical model description . . . . .	150
21.3 The description of the benchmark . . . . .	153
21.4 Conclusion . . . . .	154
References . . . . .	156
<b>22 Test-case No 24: Growth of a small bubble immersed in a superheated liquid and its collapse in a subcooled liquid (PE, PA)</b>	<b>157</b>
22.1 Practical significance and interest of the test-case . . . . .	157
22.2 Model and assumptions . . . . .	158
22.3 Bubble collapse: case 24-1 (PA) . . . . .	160
22.4 Initial stage of the growth of a vapor bubble, case 24-2 (PA) . . . . .	160
22.5 Thermally controlled growth of a vapor bubble (24-3) . . . . .	163
References . . . . .	171
<b>23 Test-case No 26: Droplet impact on hot walls (PA)</b>	<b>173</b>
23.1 Practical significance and interest of the test-case . . . . .	173
23.2 Definitions and physical model description . . . . .	174
23.3 Test-case description . . . . .	175
23.4 Relevant results for comparison . . . . .	176
References . . . . .	178
<b>24 Test-case No 27: Interface tracking based on an imposed velocity field in a convergent-divergent channel (PN)</b>	<b>179</b>
24.1 Practical significance and relevance of the test-case . . . . .	179
24.2 Definitions and model description . . . . .	180
24.3 Test-case description . . . . .	182
References . . . . .	184

<b>25 Test-case No 28: The lock-exchange flow (N, PA)</b>	<b>187</b>
25.1 Practical significance and interest of the test-case . . . . .	187
25.2 Definitions and physical model description . . . . .	188
25.3 Test-case description . . . . .	189
References . . . . .	191
<b>26 Test-case No 29a: The velocity and shape of 2D long bubbles in inclined channels or in vertical tubes (PA, PN) Part I : in a stagnant liquid</b>	<b>193</b>
26.1 Practical significance and interest of the test-case . . . . .	193
26.2 Definitions and model description . . . . .	194
26.3 Motion in horizontal channel . . . . .	195
26.4 Motion in inclined channel . . . . .	197
26.5 Motion in vertical channel and in tube . . . . .	199
26.6 Acknowledgements . . . . .	200
References . . . . .	205
<b>27 Test-case No 29b: The velocity and shape of 2D long bubbles in inclined channels or in vertical tubes (PA, PN) Part II: in a flowing liquid</b>	<b>207</b>
27.1 Practical significance and interest of the test-case . . . . .	207
27.2 Definitions and model description . . . . .	208
27.3 Motion in horizontal and inclined channel . . . . .	210
27.4 Motion in vertical channel and in tube . . . . .	212
27.5 Acknowledgements . . . . .	213
References . . . . .	222
<b>28 Test case No 30: Unsteady cavitation in a Venturi type section (PN)</b>	<b>223</b>
28.1 Practical significance and interest of the test-case . . . . .	223
28.2 Definitions and physical model description . . . . .	224
28.3 Geometry and boundary conditions . . . . .	226
28.4 Comparison with experiments . . . . .	231
References . . . . .	233
<b>29 Test-case No 31: Reorientation of a Free Liquid Interface in a Partly Filled Right Circular Cylinder upon Gravity Step Reduction (PE)</b>	<b>235</b>
29.1 Practical significance and interest of the test-case . . . . .	235
29.2 Definitions and model description . . . . .	237
29.3 Experimental setup and procedure . . . . .	239
29.4 Results . . . . .	241
29.5 Proposed calculations . . . . .	248
References . . . . .	252
<b>30 Test-case No 33: Propagation of solitary waves in constant depths over horizontal beds (PA, PN, PE)</b>	<b>255</b>
30.1 Practical significance and interest of the test-case . . . . .	255
30.2 Definitions and model description . . . . .	256
30.3 A series of three test-cases . . . . .	258



30.4 Summary of the required calculations for propagations of solitary waves	262
References . . . . .	268
<b>31 Test-case No 34: Two-dimensional sloshing in cavity - an exact solution (PA)</b>	<b>269</b>
31.1 Practical significance and interest of the test-case . . . . .	269
31.2 Definitions and physical model description . . . . .	270
31.3 Test-case description . . . . .	273
References . . . . .	275
<b>32 Test-case No 35: Flow rate limitation in open capillary channels (PE)</b>	<b>277</b>
32.1 Practical significance and interest of the test case . . . . .	277
32.2 Definitions and model description . . . . .	278
32.3 The Experimental Test Case . . . . .	280
32.4 Results . . . . .	284
References . . . . .	290
<b>33 Test-case No 36: Kelvin-Helmholtz instability (PA)</b>	<b>291</b>
33.1 Practical significance and interest of the test-case . . . . .	291
33.2 Experiment description . . . . .	292
33.3 Inviscid linear analysis . . . . .	294
33.4 Experimental results to be predicted by the simulation . . . . .	296
References . . . . .	298
<b>Index</b>	<b>299</b>

## Foreword

I am very pleased to introduce to the Series what I believe will become a standard reference in multiphase flow in future years. Research in this area has increasingly been focussed on the prediction of the position and temporal movement of phasic interfaces, using the so-called interface tracking methods. The simplest example of this type of system is the rise of a single bubble in a stationary fluid. However, systems of ever-increasing complexity are being predicted and, as with all numerical methods, it is difficult to establish the accuracy and efficacy of the techniques being employed, particularly when there is a dearth of experimental data for validation.

One way of increasing confidence in predictions is to compare the methods used with standard "benchmark" examples. In this book, no less than 33 separate test cases are presented in sufficient detail to allow extensive testing of interface tracking methods. The Editors (Drs. Jamet, Lebaigue and Lemonnier) have drawn together an impressive team for this Herculean task and the outcome will, I am sure, be immensely influential in progressing this important activity.

G. F. Hewitt  
Series Editor



## Preface

### Why collect test-cases for interface tracking methods?

Several years ago, Hewitt *et al.* (1986) suggested that refined modelling of two-phase flow was a major key in meeting some complex industrial challenges associated with nuclear energy production. In particular, they reported that the understanding of the intricate heat transfer and fluid mechanics phenomena that control the critical heat flux or the pressurized thermal shock could not be easily reached within the frame of area-averaged or time-averaged models. In addition, the understanding of the interaction of probes with two-phase flows or the local conditions controlling wall boiling could benefit from a refined analysis of two-phase flows describing both the motion of each phase and that of the interfaces.

Beyond the energy field, the oil industry is also facing new challenges related to two-phase production of oil and gas. Sizing pipelines, separators and predicting the hydrate formation requires a flow description at a scale which is not covered by the existing 1D area-averaged models. From these examples arising from nuclear engineering and the oil industry, there is clearly a *definite need* to refine the models scale of analysis of two-phase flow with or without phase change.

Modeling requires characterizing flow and heat transfer at a scale consistent with the scales described by the models. When refining the scales of observation, instrumentation may become unacceptably intrusive or mere observation may become impossible without hampering the flow features by modifying the boundary conditions. An example of this situation is forced convective boiling. Details of the high pressure water flows close to the walls are probably beyond the reach of existing experimental techniques for several years and refined modeling was identified as a possible breakthrough to progress towards an in-depth physical understanding (Delhay & Garnier, 1999).

Another objective favouring the development and the use of local models of two-phase flows is the tremendous difficulty in providing appropriate models (closure relations) to local-time-averaged models. Solving at a refined scale and analyzing the results by averaging them at the scale relevant to the time-averaged model is a possible way to identify appropriate closure relations to these averaged models.

However, *local modeling* of two-phase flow must not be confused with *simulation* as it is for example understood when solving the Navier-Stokes equations for

single-phase flow. Indeed, wetting phenomena, coalescence physics or heat transfer along a moving contact line cannot be described and must be modeled. As a consequence and as usual when modeling is involved, validation is necessary to gain confidence in the model predictions.

## A historical view

In 1994, CEA started studying new methods to describe local two-phase flows and heat transfer with phase change. At that time, several two-phase CFD modeling methods or computational multiphase flow dynamics models (CMFD) were developed, but the inclusion of phase change into them was still a real challenge. Among all tracks which were identified, two main paths were explored to account for phase change phenomena: improving front tracking algorithms to account for the normal velocity discontinuity at the interface, and modifying the thermodynamic description of the interface within a single fluid approach. It was readily demonstrated that various physical and numerical problems were to be solved and that the evaluation of the potential solutions required reference situations where both the physics and the numerical techniques were precisely controlled. This was the basic idea of the test-cases.

In France and in Europe, several groups were also interested in these problems and during two meetings on January and June 2000 in Grenoble, France, it was realized that the need for test-cases was merely universal in this community and that exchanging, or better sharing, a common set of well tried and tested benchmarks could benefit to the progress of CMFD development. A first set of nearly 20 test-cases were then decided. Originally written in French, it was thought useful to invite European colleagues to contribute and to select English as a common language. Next, collecting these test-cases into a book was the sound and logical follow-up of the basic idea to provide worldwide developers with this previously scattered information.

Multiphase Science and Technology traditionally fosters this type of activity. Hewitt *et al.* (1986) edited a selection of reference data sets for validating 1D area-averaged two-phase flow and later, Hewitt *et al.* (1991) provided a forum to discuss the merits of various systems codes based on their ability to describe the physical situations relative to the collected data. The editors of this book thank Multiphase Science and Technology for its constant help in suggesting contributors and organizing the internal review of the proposed test-cases.

## Organization of the test-cases collection

Test-cases were initially collected rather randomly and as they became more and more numerous, it was deemed important to provide the reader with the primary interest of the test-case and the targeted part of the CMFD model. This is indicated by capital letters directly following the title of the test-case. Two main categories are proposed:

- **N**: Purely numerical test-case to check for example some discretization methods,
- **P**: Physical test-case to verify a selected physical model or phenomenon controlled by the balance of selected effects.

In this latter category, further subdivision was considered:

- **PN**: Physical test-case compared to a reference numerical method regarded as more accurate
- **PA**: Physical test-case compared to an analytical solution possibly evaluated numerically
- **PE**: Physical test-case compared to an experiment
- **PC**: Test of coherence

A tentative sorting may be proposed according to the type or number of competing physical mechanisms involved in the test-case.

- Test with only one effect:
  - Pure transport (numerical scheme only: rotation, translation, stretching, etc.)
  - Interface deformation in a prescribed velocity fields
  - Shock jump conditions
- Two or three terms of the momentum balance equation:
  - Surface tension truncation errors against viscosity (spurious currents)
  - Buoyancy against surface forces (Rayleigh-Taylor)
  - Inertia against surface tension forces (oscillation of an inclusion)
  - Inertia against viscous forces (capillary standing waves, solitary capillary wave)
  - Buoyancy against drag forces (rise of an inclusion)
  - Viscosity contrast (two-phase Poiseuille flow)
  - Compressibility contrast (shock-bubble interaction)
- Phase-change:
  - Heat conduction and mass balance (1D Stefan problem, plane or spherical symmetry,)
  - Bubble growth in a superheated liquid
- Solid surface effects:
  - Contact angle (pinning)
  - Capillary rise upon gravity reduction
  - Droplet impact and bouncing on a hot wall

- Local mechanisms with more complex situations :
  - Drop impact on a liquid film
  - Gas bubble bursting at a free surface
  - Collision of two droplets (hydrodynamics)
  - Shape of long bubbles in a tube
- Complex mechanisms or situations:
  - Mould filling by a viscous jet (inertia, viscosity and gravity)
  - Phase inversion in a closed box (mass balance, with viscosity and surface tension)
  - 2D sloshing
  - Lock-exchange flow
  - Unsteady cavitation in a Venturi

Each test-case is self-supporting and focused. The interest and emphasis are developed in a first section. Next, the theory necessary to understand the physical situation and the reference model is shortly explained with a discussion of its validity domain. Necessary references are provided. Next, the details of the test-case are provided *i.e.* the definition of the computation domain, the boundary conditions and the physical properties. Finally some results in a form that allows an easy handling (analytical formula, arrays of figures) are proposed with a common method for evaluating the errors between the calculated results and the reference.

Finally to ensure each data set is complete, a referee has been selected within the group of contributors to play the role of a potential user of the test-case. When the input of this internal referee was deemed significant by the authors they usually included him as the last author of their test-case.

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## Chapter 1

### Test-case No 1: Rise of a spherical cap bubble in a stagnant liquid (PN)

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#### Abstract

The test case concerns the simulation of a fluid inclusion rising in another stagnant fluid. The transient rise velocity, and the steady-state rise velocity and shape should be compared to the results obtained with an accurate numerical technique.

#### 1.1 Practical significance and interest of the test-case

This test-case could usually be considered as a very preliminary one for a new numerical method. An extensive tester may want to reproduce most parts of the Clift, Grace and Weber map (Clift *et al.*, 1978). However, this selected case deserves special attention for the result not only consists in a final shape of the bubble (that is nevertheless a real criteria of comparison) but also in a precise build-up of the bubble velocity, starting from rest, exhibiting an overshoot before reaching its final asymptotic value.



To get the proper results, mainly the correct terminal velocity, and to reproduce the overshoot, a numerical method has to take accurately into account buoyancy, viscous stresses and surface tension effects. In particular, this test allows validating the numerical model that takes care of jump conditions at the interface (see *e.g.* (Scardovelli & Zaleski, 1999)). However, the test is less severe than the "Free rise of a liquid inclusion in a stagnant liquid", a test-case proposed by Lemonnier and Hervieu, presented in this volume.

## 1.2 Definitions and model description

The situation of the test-case is relative to a fluid inclusion rising in another fluid. The inclusion and the surrounding fluid are initially at rest. Gravity induced buoyancy is the only force inducing the motion. The test-case consists in the computation of the transient build-up of the velocity of the rising inclusion that finally reaches a constant value.

The physical model is reduced to Navier-Stokes equation in both phases, a constant surface tension at the interface. No phase-change takes place at the interface. As the solution does not depend on a possible compressibility of one or both of the phases, the test-case can be conducted in both cases (compressible / incompressible), depending on specific features of the numerical method to be evaluated.

Reference calculations are available in non-dimensional units; however, a typical set of dimensional physical parameter is suggested. The length scale of the problem is the initial diameter  $d_e$  of the inclusion whose initial shape is spherical. The velocity scale for the speed of displacement of the center of mass  $U$  is,

$$U_c = \sqrt{gd_e}, \quad (1.1)$$

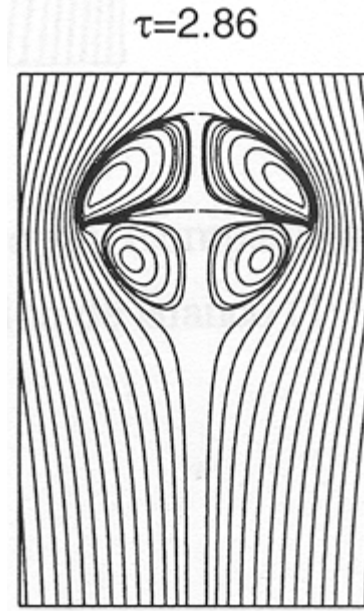
where  $g$  is the acceleration of the gravity. The time scale is therefore

$$t_c = \sqrt{d_e/g}. \quad (1.2)$$

Reduced parameters are  $\tau = t/t_c$  and  $u = U/U_c$ . According to these definitions, the non-dimensional reference calculation consist in the reduced time evolution of the speed of displacement of the center of mass:

$$u = U/U_c = f(\tau) = f(t/t_c). \quad (1.3)$$

The computation can be conducted either for an axisymmetrical domain or in a true three-dimensional domain. As the limited extend of the domain has an impact on the terminal velocity of the inclusion (see *e.g.* Harmathy (1960)), the size of the computational domain has to be increased as long as an effect on the results is noted. As a rough first estimate, we suggest that that the computational domain has a minimal extend equal to ten diameters in all directions. According to the work of Harmathy (1960), the shape of the bubble is not affected by the domain extension whereas the terminal rise velocity modification can be estimated through



**Figure 1.2:** Recirculation zone and stream lines at reduced time  $\tau = 2.86$ . After a figure of Benkenida (1999).

- As an example, we suggest the following physical properties:  $\rho_L = 1000 \text{ kg.m}^{-3}$ ,  $\rho_V = 10 \text{ kg.m}^{-3}$ ,  $\mu_L = 0.273556 \text{ Pa.s}^{-1}$ ,  $\mu_V = 0.00273556 \text{ Pa.s}^{-1}$ ,  $\sigma = 0.1 \text{ N.m}$ ,  $g = 10 \text{ kg.s}^{-2}$ ,  $d_e = 0.02 \text{ m}$ .
- Extract the position of the center of mass of the inclusion and then deduce its speed of displacement. The first point of comparison is the value of the reduced asymptotic velocity. This value can be obtained even with a peculiar point of the interface, such as the apex. Of course, in this later case, the temporal evolution around the overshoot (Figure 1.1) cannot be captured.
- In addition to the main result, additional features consist in comparisons of the non-dimensional values of the over-shoot in the build-up of velocity (Figure 1.1).
- Further comparisons are the shape of lines of current, the equilibrium shape of the inclusion and the size of the recirculation zone (Figure 1.2). This late characteristic requires that the inclusion has risen a length of more than ten diameters (Hnat & Buckmaster, 1976).

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