NUMERICAL SIMULATION OF VISCOUS PERFECT GAS DYNAMICS

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Preface

The present monograph summarizes the results of many years of research on numerical simulation of transonic, supersonic, and hypersonic viscous perfect gas flows based on the continuum mechanics equations for the problems of external aerodynamics. These results were obtained by the authors and their colleagues and have been published in different Russian journals.

The monograph is split into two sections. Steady and unsteady two-dimensional problems are considered in the first section. Three-dimensional steady problems are the subject of the second section. Specifically, steady-state uniform flow over a body of relatively simple configuration, which surface is given analytically, is studied.

Each part begins with the mathematical problem statement and the description of its numerical solution method, followed by a detailed discussion of the numerical data for several aerodynamic problems. The results are obtained within a certain range of the key similarity parameters. The considered aerodynamic problems are divided into two groups.

The problems of the first group are aimed at theoretical study of the flow field near a body, of the body local and integral aerodynamic characteristics and of the effect of the key similarity parameters on these characteristics. This requires numerical data within a wide range of variation of the key similarity parameters. Such investigations are usually performed by the example of the flow over bodies of simple configuration, with much attention being paid to verification of the numerical method and to validation of the numerical data. Typical bodies are considered in the present monograph: circular and elliptical cylinders, a sphere, and sharp circular and elliptical cones.

The problems of the second group follow the aerodynamic experiment in different supersonic and hypersonic wind tunnels at TsAGI. In this case calculations are performed for a body of interest as applied to the experimental conditions. The calculations involve estimation of the flow field along the whole wind tunnel duct, i.e., the working fluid flow is computed in the nozzle and in the test section of the wind tunnel both with and without a model in it. Numerical data are compared with the experimental results. This monograph describes mostly blunt axisymmetric bodies like American and European prototypes of Martian probes.

This monograph is of interest for specialists in the area of computational and applied aerodynamics as well as for graduate students, whose research is related to applied aerodynamics.

Introduction

A complex flow-field pattern is observed in case of the body motion in incompressible and compressible fluid or in the case of incompressible and compressible fluid flow in different engineering devices. Furthermore, in most cases the continuum fluid flow is followed by the flow separation and reattachment, which have a significant effect on the aerodynamic characteristics of a moving body or an engineering device. The phenomena of flow separation and reattachment are of complex nature and depend on many factors. One of the most important fundamental tasks of current aero- and hydrodynamics is to study the principles of evolution of these phenomena.

There are two ways to study these complicated fundamental problems: experimental and theoretical.

The experimental approach for investigation of the fluid motion laws appeared at the origin of mankind and evolved significantly together with the human society. First it was a full-scale experiment; later, special aerohydrodynamic plants and the corresponding measuring equipment were created. With advances in technology and larger fluid velocities, experimental facilities become more complex and the cost of the experimental data increases; therefore each measurement becomes more expensive.

The theoretical science-based approach originated in the 17th century, when the main laws of mechanics were stated due to the work of Galileo, Newton, and other scientists.

In the 18th century Leonhard Euler (1755) derived the equations of ideal incompressible and compressible fluid dynamics within the framework of continuum mechanics, which are referred to as Euler equations. Equations of viscous incompressible and compressible fluid dynamics were derived in the 19th century in the works of Navier (1826), Poisson (1831), Saint-Venant (1843), and Stokes (1847). These equations are called Navier–Stokes equations. At the end of the 19th century Osborne Reynolds proved experimentally the existence of the turbulent flows (1883) and suggested an approach for studying these complex flows (1895) based on the decomposition of hydrodynamic variables into average and pulsation components. He derived the equations that describe the averaged flow referred to as Reynolds equations.

However, these equations turned out to be too complicated for the application problems and first of all for the fundamental problems of aerohydrodynamics, namely the problems of lift and drag.

An essential breakthrough in the solution of these problems was made in the beginning of the 20th century, when Nikolay Zhukovsky proved a theorem (1906) that relates lift to circulation, and Ludwig Prandtl showed (1904) that at large Reynolds numbers the viscous forces should be taken into account only in a thin near-wall layer, where the viscous fluid flow is described by the boundary layer equations (Prandtl equations). These equations being simpler as compared to the Navier–Stokes and Reynolds equations made it possible to study the principles of viscous flow over a body at large Reynolds numbers. As computational tools improved, a class of problems that could be solved with the boundary layer equations extended.

However, the boundary layer equations are effective only in the area of unseparated flow; in the vicinity of the flow separation they are invalid. Therefore, full Navier–Stokes and Reynolds equations should be used for computation of separated flows.

With recent advance in computational aerodynamics and computer technology, efficient software packages have been developed for numerical analysis of unsteady two-dimensional and three-dimensional Navier–Stokes and Reynolds equations at an acceptable cost. These software packages are used both for parametric computations of different problems of external and internal aerodynamics, and for solving the modeling problems as applied to the experimental conditions in the wind tunnels. The latter is an important division of computational aerodynamics.

Both experimental and theoretical approaches are the basis of our understanding of the principles of fluid flow and peculiarities of heat transfer. Used together they are a powerful tool for solving various application problems.

First, the aerodynamic experiment provides very important but limited data on the distribution of gasdynamic variables along the model surfaces (primarily pressure and heat transfer coefficients) and in some flow-field cross sections (for example, total pressure distribution in the exit section of a duct or a nozzle). Some information on the flow-field visualization (for example, Schlieren images, spectra of limiting streamlines) is also obtained in the aerodynamic experiment. This information gives general idea of the flow-field pattern. However, it is often not enough for the complete identification of a complex flow-field pattern associated with the fields of gasdynamic variables. Computational aerodynamics provides such information, thus completing experimental data analysis and understanding of the flow aerodynamics.

Second, the aerodynamic experiment reconstructs partially the full-scale conditions, primarily according to Mach and Reynolds numbers. Other free-stream parameters are not simulated. For example, such an important parameter as turbulence level, which affects significantly the position of laminar–turbulent transition, and therefore, the flow-field pattern and the body aerodynamic characteristics, is not simulated in the aerodynamic experiment. This inability of the aerodynamic experiment to faithfully reconstruct the full-scale free-stream parameters causes the problem when transforming the experimental wind tunnel data into the full-scale conditions. Computational aerodynamics in turn helps to analyze the influence of parameters that cannot be simulated in the experiment and to use efficiently the wind tunnel data under the full-scale conditions.

Third, in most of the cases the aerodynamic experiment does not capture the onset of a steady-state flow mode. Even with this moment being short, it is important for understanding of the start-up process associated with the reconfiguration of the flow field and redistribution of gasdynamic variables. Computational aerodynamics makes it possible to simulate this process and to examine all the aerodynamic characteristics of this unsteady flow.

Fourth, some mathematical model with a number of parameters is used for simulation of the flow of interest using the tools of computational aerodynamics. Sufficiency of the applied model is checked by comparing numerical and experimental data. A noticeable disagreement between numerical and experimental data (if observed) is analyzed to result in corrected parameters of the mathematical model and in an improved experimental setup.

Fifth, an estimate of the expected experimental data obtained by the methods of computational aerodynamics is helpful in choosing more appropriate geometric parameters of the model and experimental conditions. It reduces the possibility of failure of the experiment and contributes to reduction of its cost.

Therefore, the implementation of the aerodynamic experiment should be followed by the numerical solution of the corresponding modeling problem at all the stages—from setting up the experiment and up to the analysis of the results. This is a mutually complementary process, which validates the resulting data and enriches our knowledge in the area of external and internal aerodynamics.

An effective numerical technique for two-dimensional and three-dimensional aerodynamic problems based on unsteady Navier–Stokes and Reynolds equations has been developed at TsAGI and presented in the following works: Bashkin et al., 1993, 2002, 2001, 2003. This technique has been used successfully and is still being applied for parametric computations of various problems of external and internal aerodynamics. This approach has also been applied as a numerical supplement of a number of wind tunnel tests.

The goal of the present monograph is to describe the problem statement for supersonic viscous gas flow over two-dimensional and three-dimensional bodies, to explain the numerical simulation approach based on unsteady Navier–Stokes and Reynolds equations, to illustrate the approach efficiency by the example of several problems of external aerodynamics associated with transonic, supersonic, and hypersonic viscous perfect gas flow in the presence of closed separation zones, and to analyze evolution of the separated flow and heat transfer on the streamlines surfaces.

The monograph is organized as follows. It is divided into two sections according to the dimension of the considered aerodynamic problems.

The first section of the book consists of seven chapters. It reviews two-dimensional problems. The problem statement and the numerical method are described in the first chapter. The other chapters comprise discussion of the results of the following problems: transonic and supersonic cross flow over a circular cylinder (Chapter 2 and Chapter 3); supersonic cross flow over an elliptic cylinder (Chapter 4); a sphere in supersonic flow (Chapter 5); a flat plate and models of Martian probes with a thin groove on the frontal surface in supersonic and hypersonic flow (Chapter 6); models of Martian probes at zero angle of attack in supersonic and hypersonic flow (Chapter 7).

The second section consists of five chapters. It considers three-dimensional aerodynamic problems. Chapter 8 outlines the problem statement and the numerical simulation method. Verification of the numerical scheme follows in Chapter 9. The other chapters deal with the results for the following problems: sharp thin circular and elliptic cones in supersonic and hypersonic flow (Chapter 10 and Chapter 11); models of Martian probes at small, moderate, and large angles of attack in supersonic and hypersonic flow (Chapter 12).

SECTION 1. NUMERICAL SIMULATION OF TWO-DIMENSIONAL PROBLEMS OF EXTERNAL AERODYNAMICS

The computational aerodynamics methods are being developed actively nowadays and are being used successfully to solve various problems of external and internal aerodynamics. Many different approaches for numerical simulation of the viscous gas dynamics equations have been created within the framework of this direction. Among others is a method based on the implicit Beam-Warming finite-difference scheme (Beam and Warming, 1978), and its further modification (Steger, 1978; Hollanders and Devezeaux de Lavergne, 1987).

Newton approach to implicit finite difference schemes with subsequent linearization and solution of a system of algebraic equations is considered to be the most complete mathematically (Egorov and Zaitsev, 1991). This approach has been developed for numerical integration of unsteady two-dimensional Navier-Stokes equations (Bashkin, et al., 1993) and Reynolds equations (Bashkin, et al., 2000a) under Boussinesq assumption about Reynolds stresses using two-parameter turbulence model (Huang and Coakley, 1993). This approach has been implemented numerically in a code that can be run on a personal computer. It has been used successfully for solution of a number of supersonic problems of external and internal aerodynamics: a circular cylinder, a sphere, flat and axisymmetric channels (Bashkin, et al., 1998), a basic flat hypersonic air inlet (Bashkin, et al., 1996, 1997ab, 1999ab, 2001).

This approach is described in the first chapter as applied to two-dimensional perfect gas flow and some results of parametric computations for supersonic separated flow are discussed. The results of numerical simulation of a number of two-dimensional problems of external aerodynamics concerning transonic, supersonic, and hypersonic perfect gas flow over flat and axisymmetric bodies are analyzed in the next chapters.

CHAPTER 1

Mathematical Problem Statement and Numerical Analysis

1.1 PROBLEM STATEMENT

1.1.1 Differential Navier-Stokes Equations

Viscous gas flow is described by a system of equations, which express the laws of conservation of mass, momentum, and energy. Hereinafter, these equations are referred to as Navier-Stokes equations. In case of the two-dimensional problem (plane flow and axisymmetric flow) solved in an arbitrary curvilinear reference frame ξ, η, where $x = x(\xi, \eta)$, $y = y(\xi, \eta)$ are Cartesian coordinates, Navier-Stokes equations are written in the divergent form as

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial \xi} + \frac{\partial \mathbf{G}}{\partial \eta} = \mathbf{B}. \tag{1.1}$$

Here $\mathbf{Q}$ is a vector of conservative dependent variables of the problem, $\mathbf{E}$ and $\mathbf{G}$ are flux vectors in curvilinear reference frame, and $\mathbf{B}$ is a source vector. Vectors $\mathbf{Q}$, $\mathbf{E}$, $\mathbf{G}$ are related to the corresponding vectors $\mathbf{Q}_c$, $\mathbf{E}_c$, and $\mathbf{G}_c$ in Cartesian reference frame by the formulae

$$\mathbf{Q} = J\mathbf{Q}_c, \quad \mathbf{E} = J\left(\mathbf{E}_c\frac{\partial \xi}{\partial x} + \mathbf{G}_c\frac{\partial \xi}{\partial y}\right), \quad \mathbf{G} = J\left(\mathbf{E}_c\frac{\partial \eta}{\partial x} + \mathbf{G}_c\frac{\partial \eta}{\partial y}\right),$$

where $J = \partial(x, y)/\partial(\xi, \eta)$ is transformation Jacobian.

Cartesian components of vectors $\mathbf{Q}_c$, $\mathbf{E}_c$, and $\mathbf{G}_c$ for two-dimensional Navier-Stokes equations are written as follows

$$\mathbf{Q}_c = \left\| \begin{array}{c} \rho \\ \rho u \\ \rho v \\ e \end{array} \right\|, \quad \mathbf{E}_c = \left\| \begin{array}{c} \rho u \\ \rho u^2 + p - \tau_{xx} \\ \rho u v - \tau_{xy} \\ \rho u H - u\tau_{xx} - v\tau_{xy} - \lambda\dfrac{\partial T}{\partial x} \end{array} \right\|,$$

$$\mathbf{G}_c = \left\| \begin{array}{c} \rho v \\ \rho u v - \tau_{xy} \\ \rho v^2 + p - \tau_{yy} \\ \rho v H - u\tau_{xy} - v\tau_{yy} - \lambda\dfrac{\partial T}{\partial y} \end{array} \right\|,$$

where ρ is density, u, v are Cartesian components of the velocity vector, p is pressure, $e = \rho\left(c_v T + \left(u^2 + v^2\right)/2\right)$ is total energy per unit volume, $H = c_p T + \left(u^2 + v^2\right)/2$

is total enthalpy, c_p and c_v are specific heats at constant pressure and volume, λ is heat conductivity coefficient, μ is dynamic viscosity coefficient, $\boldsymbol{\tau}$ is viscous stress tensor with components

$$\tau_{xx} = \mu\left(-\frac{2}{3}\mathrm{div}\mathbf{V} + 2\frac{\partial u}{\partial x}\right)$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{yy} = \mu\left(-\frac{2}{3}\mathrm{div}\mathbf{V} + 2\frac{\partial v}{\partial y}\right).$$

The source term **B** in Eq. (1.1) for plane ($\nu = 0$) and axisymmetric ($\nu = 1$) cases is written as

$$\mathbf{B} = J\left(0,\ 0, \nu\left(p + \mu\left(\frac{2}{3}\mathrm{div}\mathbf{V} - 2\frac{v}{r}\right)\right),\ 0\right)^T,$$

where r is the distance from the symmetry axis.

System of equations (1.1) for perfect gas is closed by the state equation

$$p = \rho R_\mathrm{g} T/M. \tag{1.2}$$

Here R_g is universal gas constant, M is molar weight of the gas. The transfer coefficients are determined as follows: dynamic viscosity coefficient varies with respect to temperature according to the power law $\mu/\mu_\infty = (T/T_\infty)^{0.7}$ or Sutherland's law $\mu/\mu_\infty = (T_\infty + T_\mu)\,(T/T_\infty)^{1.5}/(T + T_\mu)$, $T_\mu = 110.4$, and Prandtl number is assumed to be constant $\mathrm{Pr} = \mu c_\mathrm{p}/\lambda = 0.7$.

1.1.2 Boundary and Initial Conditions

Solution of the problem described by Navier-Stokes equations (1.1) is subject to the boundary conditions. No-slip and flow tangency conditions ($u = v = 0$) are imposed on the boundary of the computational domain, which coincides with the body solid surface; the streamlined surface is also considered to be heat-insulated ($[\partial T/\partial n]_\mathrm{w} = 0$) or isothermal ($T_\mathrm{w} = \mathrm{const}$). In some cases local or integral heat balance condition may be imposed on the streamlined surface. Outer boundary of the computational domain is subject to radiation conditions, which correspond to the diverging wave and are written in the form of Riemann invariants:

$$\alpha_1 = \frac{2a}{\gamma - 1} - u\frac{\partial \xi}{\partial x} - v\frac{\partial \xi}{\partial y} = \mathrm{const},\ \ \alpha_2 = \frac{p}{\rho^\gamma} = \mathrm{const},$$

$$\alpha_3 = v\frac{\partial \xi}{\partial x} - u\frac{\partial \xi}{\partial y} = \mathrm{const},\ \ \alpha_4 = \frac{2a}{\gamma - 1} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} = \mathrm{const},$$

where a is sound speed. In addition, signs of eigenvalues are checked at every point of the inflow boundary

$$\lambda_1 = u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} - a\sqrt{\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2},\ \lambda_2 = u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y},$$
$$\lambda_3 = u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y},\ \lambda_4 = u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} + a\sqrt{\left(\frac{\partial \xi}{\partial x}\right)^2 + \left(\frac{\partial \xi}{\partial y}\right)^2},$$

The signs of eigenvalues specify the direction of perturbation propagation with respect to $\xi = \text{const}$. At $\lambda_i \geq 0$ ("inflow boundary") the corresponding invariant on the outer boundary is calculated by the values of gas dynamic variables in the free stream, and in case of $\lambda_i < 0$ ("outflow boundary") a smooth interpolation of the form $\mathbf{U}_k - 2\mathbf{U}_{k-1} + \mathbf{U}_{k-2} = 0$, where $\mathbf{U}$ is a vector of Riemann invariants, is applied. A periodic constraint is imposed on the inner boundary of the computational domain, which coincides with the positive x-axis.

Uniform free stream condition is taken as an initial approximation with further evolution of the flow field according to the unsteady problem solution. In addition to the above, the time step increases as the flow pattern is generated, which eventually makes it possible to solve steady-state problem. The numerical implementation is more efficient if the problem is solved initially on a coarse grid ($21 \times 21 \times 21$) following the approach described above, and then the resulting interpolated solution is used as an initial approximation for a finer grid. The data obtained earlier with the closest values of the variable parameters to the required ones is used as an initial approximation for parametric computations by Mach and Reynolds numbers.

1.1.3 Differential Reynolds Equations

Numerical simulation based on integration of Reynolds-averaged Navier-Stokes equations, referred to as Reynolds equations, is prominent in theoretical analysis of fluxes with different flow modes. This system of equations is not closed. Different turbulence models, both algebraic and differential, are used for its closure.

Reynolds-averaged Navier-Stokes equations can be written in divergent form in an arbitrary curvilinear reference frame (ξ, η), where $x = x(\xi, \eta)$, $y = y(\xi, \eta)$ are Cartesian coordinates, as follows

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial \xi} + \frac{\partial \mathbf{G}}{\partial \eta} = \mathbf{B}. \tag{1.3}$$

Here $\mathbf{Q}$ is a vector of conservative dependent variables, $\mathbf{E}$ and $\mathbf{G}$ are flux vectors in curvilinear reference frame, $\mathbf{B}$ is a source vector. Vectors $\mathbf{Q}$, $\mathbf{E}$, $\mathbf{G}$, and $\mathbf{B}$ are related to the corresponding vectors $\mathbf{Q}_\text{c}$, $\mathbf{E}_\text{c}$, $\mathbf{G}_\text{c}$, and $\mathbf{B}_\text{c}$ in Cartesian reference frame by the formulae

$$\mathbf{Q} = J\mathbf{Q}_\text{c},\ \mathbf{E} = J\left(\mathbf{E}_\text{c}\frac{\partial \xi}{\partial x} + \mathbf{G}_\text{c}\frac{\partial \xi}{\partial y}\right),\ \mathbf{G} = J\left(\mathbf{E}_\text{c}\frac{\partial \eta}{\partial x} + \mathbf{G}_\text{c}\frac{\partial \eta}{\partial y}\right),\ \mathbf{B} = J\mathbf{B}_\text{c}$$

where $J = \partial(x, y)/\partial(\xi, \eta)$ is transformation Jacobian.

Cartesian components of vectors $\mathbf{Q}_\text{c}$, $\mathbf{E}_\text{c}$, $\mathbf{G}_\text{c}$ and $\mathbf{B}_\text{c}$ for two-dimensional Reynolds (Favre)-averaged Navier-Stokes equations are written as follows

$$\mathbf{Q}_\mathrm{c} = \left\| \begin{array}{c} \rho \\ \rho u \\ \rho v \\ \rho(e+q^2) \\ \rho q \\ \rho\omega \end{array} \right\|, \quad \mathbf{E}_\mathrm{c} = \left\| \begin{array}{c} \rho u \\ \rho u^2 + p + \frac{2}{3}\rho q^2 + \tau_{xx} \\ \rho uv + \tau_{xy} \\ \rho u H + \frac{5}{3}\rho u q^2 + I_x \\ \rho u q + I_x^q \\ \rho u \omega + I_x^\omega \end{array} \right\|,$$

$$\mathbf{G}_\mathrm{c} = \left\| \begin{array}{c} \rho v \\ \rho uv + \tau_{xy} \\ \rho v^2 + p + \frac{2}{3}\rho q^2 + \tau_{yy} \\ \rho v H + \frac{5}{3}\rho v q^2 + I_y \\ \rho v q + I_y^q \\ \rho v \omega + I_y^\omega \end{array} \right\|, \mathbf{B}_\mathrm{c} = \left\| \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ h_1 \rho\omega q \\ h_2 \rho \omega^2 \end{array} \right\|,$$

where ρ is gas density; u, v are Cartesian components of the velocity vector $\mathbf{V}$; p is pressure; $e = h - p/\rho + \left(u^2 + v^2\right)/2$ is total energy per unit mass; $H = h + \left(u^2 + v^2\right)/2$ is total enthalpy, $h = c_\mathrm{p} T$ is static enthalpy; c_p is specific heat at constant pressure, $\boldsymbol{\tau}$ is viscous stress tensor with components

$$\tau_{xx} = (\mu + \mu_T)\left(\frac{2}{3}\mathrm{div}\mathbf{V} - 2\frac{\partial u}{\partial x}\right)$$

$$\tau_{xy} = \tau_{yx} = -(\mu + \mu_T)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{yy} = (\mu + \mu_T)\left(\frac{2}{3}\mathrm{div}\mathbf{V} - 2\frac{\partial v}{\partial y}\right),$$

$\mathbf{I}$ is a heat flux vector

$$\mathbf{I} = -(\lambda + \lambda_T)\,\mathrm{grad}\,(T) + \boldsymbol{\tau} V,$$

μ and λ are molecular viscosity and heat conductivity coefficients, μ_T and λ_T are turbulent viscosity and heat conductivity coefficients;

$$\mathbf{I}^q = -\left(\mu + \frac{\mu_T}{\mathrm{Pr}_1}\right)\mathrm{grad}(q), \quad \mathbf{I}^\omega = -\left(\mu + \frac{\mu_T}{\mathrm{Pr}_2}\right)\mathrm{grad}(\omega).$$

The source vector in Reynolds equations for flat ($\nu = 0$) and axisymmetric ($\nu = 1$) flow takes on the form

$$\mathbf{B} = J\left(0,\ 0,\ \nu\left(p + \mu\left(\frac{2}{3}\mathrm{div}\mathbf{V} - 2\frac{v}{r}\right)\right),\ 0,\ h_1\rho\omega q,\ h_2\rho\omega^2\right)^T.$$

Two-parameter differential $q - \omega$ turbulence model (Huang and Coakley, 1993) is used in this work. The turbulent viscosity is expressed as follows:

$$\mu_T = C_\mu f \frac{\rho q^2}{\omega},\ f = 1 - \exp\left(-\alpha \frac{\rho r_w q}{\mu}\right),\ \alpha = 0.02,\ C_\mu = 0.09,$$

$$h_1 = C_{11}\left(C_\mu f \frac{S}{\omega^2} - \frac{2}{3}\frac{\mathrm{div}\mathbf{V}}{\omega}\right) - C_{12},\ h_2 = C_{21}\left(C_\mu \frac{S}{\omega^2} - C_{23}\frac{\mathrm{div}\mathbf{V}}{\omega}\right) - C_{22},$$

$$S = \frac{4}{3}\left[\left(\frac{\partial u}{\partial x}\right)^2 - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2,\ \mathrm{div}\mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y},$$

where $C_{11} = C_{12} = 0.5$, $C_{21} = 0.055 - 0.5f(q, r_w, \rho, \mu)$, $C_{22} = 0.833$, $C_{23} = 2.4$, $\mathrm{Pr}_1 = 2$, $\mathrm{Pr}_2 = 2$, r_w is a distance from the wall, q and ω are turbulent velocity and frequency accordingly.

Perfect gas state Eq. (1.2) is used to close the resulting system of equations; molecular viscosity coefficient depends on temperature according to a power law $\mu/\mu_\infty = (T/T_\infty)^{0.7}$ and Prandtl numbers are assumed to be constant $\mathrm{Pr} = \mu c_p/\lambda = 0.7$, $\mathrm{Pr}_T = \mu_T c_p/\lambda_T = 0.9$.

1.1.4 Boundary and Initial Conditions

Solution of the problem determined by Reynolds equations (1.3) is subject to the same boundary conditions as the Navier-Stokes problem solution, since the type of equations remains the same. In addition, the values of the turbulence parameters in the free stream should be set as $q = q_\infty$, $\omega = \omega_\infty$.

No-slip and flow tangency conditions ($u = v = 0$) are imposed on the boundary of the computational domain, which coincides with the body solid surface; the streamlined surface is considered to be heat insulated ($[\partial T/\partial n]_w = 0$) or isothermal ($T_w = \mathrm{const}$). In addition, the solid surface boundary conditions are used for the equations that determine behavior of turbulence parameters: the condition of turbulent pulsation damping ($q_w = 0$) and the frequency impermeability condition ($[\partial\omega/\partial n]_w = 0$).

Outer boundary of the computational domain is subject to radiation conditions, which correspond to the diverging wave and are written in the form of Riemann invariants:

$$\alpha_1 = \frac{2a}{\gamma - 1} - u\frac{\partial \xi}{\partial x} - v\frac{\partial \xi}{\partial y} = \mathrm{const},\ \alpha_2 = \frac{p}{\rho^\gamma} = \mathrm{const},$$

$$\alpha_3 = v\frac{\partial \xi}{\partial x} - u\frac{\partial \xi}{\partial y} = \mathrm{const},$$

$$\alpha_4 = \frac{2a}{\gamma - 1} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} = \mathrm{const},$$

$$\alpha_5 = q = \mathrm{const},\ \alpha_6 = \omega = \mathrm{const},$$

where a is sound speed. Furthermore, signs of eigenvalues are checked at every point of the inflow boundary

$$\lambda_1 = u\frac{\partial\xi}{\partial x} + v\frac{\partial\xi}{\partial y} - a\sqrt{\left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2},\ \lambda_2 = u\frac{\partial\xi}{\partial x} + v\frac{\partial\xi}{\partial y},$$

$$\lambda_3 = \lambda_2,\ \lambda_4 = u\frac{\partial\xi}{\partial x} + v\frac{\partial\xi}{\partial y} + a\sqrt{\left(\frac{\partial\xi}{\partial x}\right)^2 + \left(\frac{\partial\xi}{\partial y}\right)^2},\ \lambda_5 = \lambda_6 = \lambda_2.$$

The signs of eigenvalues specify the direction of perturbation propagation with respect to $\xi = \text{const}$. In case of $\lambda_i \geq 0$ ("inflow boundary") the corresponding invariant on the outer boundary is calculated by the values of gasdynamic variables in the free stream, and a smooth interpolation of the form $\mathbf{U}_k - 2\mathbf{U}_{k-1} + \mathbf{U}_{k-2} = 0$, where $\mathbf{U}$ is a vector of Riemann invariants, is applied in case of $\lambda_i < 0$ ("outflow boundary").

Periodicity conditions are imposed on the inner boundary of the computational domain, which coincides with the positive x-axis.

Due to the similarity between Navier-Stokes and Reynolds equations, all that was said above in Section 1.1.2 concerning the choice of initial conditions for the system of Navier-Stokes equations holds true for the system of Reynolds equations.

1.2 APPROXIMATION OF EQUATIONS

For numerical analysis, systems of Eqs. (1.1) and (1.3) are nondimensionalized. This is done by dividing Cartesian coordinates by a characteristic length L (for example, in case of simulation of flow over a cylinder or a sphere, the characteristic length is taken to be $L = R$, where R is the body radius), velocity components by velocity V_∞, pressure by doubled impact air pressure $2q_\infty = \rho_\infty V_\infty^2$, time by a characteristic time, during which a fluid particles stays in the body proximity, $t_* = R/V_\infty$; other gasdynamic parameters are divided by their values in the free stream. The initial boundary value problem stated above is solved numerically using an integral-interpolation method (finite element method). Application of the finite element method for Navier-Stokes (1.1) and Reynolds (1.3) equations yields the difference analog of conservation laws

$$\frac{\mathbf{Q}_{i,j}^{n+1} - \mathbf{Q}_{i,j}^{n}}{\tau_{i,j}} + \frac{\mathbf{E}_{i+1/2,j}^{n+1} - \mathbf{E}_{i-1/2,j}^{n+1}}{h_\xi} + \frac{\mathbf{G}_{i,j+1/2}^{n+1} - \mathbf{G}_{i,j-1/2}^{n+1}}{h_\eta} = \mathbf{B}_{i,j}^{n+1},$$

where n is a time step number; $\tau_{i,j}$ is a time increment. The time increment is specified as follows

$$\tau_{i,j} = \tau_0\left(a_{\text{min}} + (a_{\text{max}} - a_{\text{min}})\frac{J_{i,j} - \min(J_{i,j})}{\max(J_{i,j}) - \min(J_{i,j})}\right),$$

where τ_0 is a value of the time step that corresponds to the largest computational cell at given values of parameters $a_{\min}$ and $a_{\max}$ (for example, $a_{\min} = 0.02$ and $a_{\max} = 1$); i, j, and h_ξ, h_η are numbers of the nodes and increments of ξ, η accordingly. Using the variable in space time increment proportional to the area of an elementary cell speeds up the computation process for the steady-state time relaxation solution.

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