THE PHYSICS OF COMPOSITE SUPERCONDUCTORS

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Introduction

The tempting idea of using superconductors to produce strong magnetic fields arose soon after the discovery of superconductivity, but could not be implemented for a long time. It was found that the critical magnetic field of the pure metals in which superconductivity was initially studied did not exceed 0.2 T.

The situation changed radically for the better in the late fifties and early sixties, following the discovery of *hard superconductors*, e.g., Nb–Ti and Nb–Zr alloys with different composition, Nb_3Sn compounds, and so on. Small solenoids manufactured from these materials were used to generate magnetic fields of up to 10 T. Very high critical current densities were reached (up to $10^9 - 10^{10}$ A/m^2) after special thermomechanical treatment, but this introduced defects into the hard superconductors.

These results evoked some optimism and it seemed at the time that the main obstacles to the practical utilization of superconductivity had been overcome. However, studies of hard superconductors with high critical-current densities soon revealed the phenomenon of thermomagnetic instability, which was seen as an abrupt change in the magnetic flux in a superconductor, with a characteristic time constant of $10^{-4} - 10^{-6}$ s. Naturally, the process was accompanied by an intensive heat release and, as a rule, returned the medium to its normal state. Even small perturbations of temperature and magnetic field were found to be capable of initiating the thermomagnetic instability. Moreover, this was practically independent of the method of cooling because hard superconductors have a relatively low thermal conductivity, i.e., heat transfer is relatively ineffective. Subsequent studies showed that the thermomagnetic instability could be prevented by ensuring that the transverse size of the conductor (e.g., the radius of a wire or the thickness of a layer) was less than a critical value (usually of the order of 10^{-4} m).

However, it became clear later that this was still not the complete solution of the problem. Actually, these superconductors are not 'good metals' above the critical temperature, e.g., their thermal and electrical conductivities are lower by three or four orders of magnitude than those of copper. Hence, if a region of normal phase appears fortuitously in the conductor close to the critical current density, the Joule heat release in the normal region will be very high. Indeed, it will sometimes be so high that it will actually melt the conductor with a transverse size of $10^{-5} - 10^{-6}$ m. This can be avoided by coating the hard superconductor with a metal having high thermal and electrical conductivities. This coating shunts regions in which the transition to the normal state has taken place and at the same time promotes effective heat transfer. This ensures that, even for equal superconductor and metal thicknesses, the specific Joule heat release is reduced by between two and four orders of magnitude. This usually suffices to stabilize the superconducting state with respect to the effect of random heat sources that initiate normal-phase regions.

It follows that hard superconductors can be used with high current densities to produce,for example, strong magnetic fields, but only if they are in the form of very thin filaments or layers. They then lack the necessary strength, and the large number of crystal defects means that they do not have the required plasticity. The natural and, indeed, the only, way out of this dilemma is to use superconducting composites in which a hard superconductor and a normal metal are in thermal and electrical contact with each other and are combined in a single whole. The configuration of components in such materials depends on their function; it can be very complex and assume a variety of forms. Existing structures include multifilament composites in which a regular structure of superconducting filaments is imbedded in a normal-metal host; film composites consisting of alternate layers of normal metal and superconductor,; and *insitu* composites in which highly elongated superconducting 'needles' form a disordered grid in a normal-metal host.

Composite superconductors offer the solution to a wide range of problems, the principal of which are: thermomagnetic instability, instability of the superconducting state with respect to strong pulsed perturbations, heat release under varying external conditions, and inadequate strength and plasticity. It is important to note that the optimum composite superconductor must often satisfy conflicting demands. Thus, the superconducting state must be stabilized by increasing the relative concentration of the normal metal, in which case the current density averaged over the wire cross section may fall well below its critical value. A host with high electrical conductivity will then suppress the thermomagnetic instability, but will give rise to a higher specific heat release in variable magnetic field, and so on. For all these reasons, the structure of a composite is always a compromise that is achieved by a suitable choice of the hard superconductor, hosts with high and low resistivity components, thin filaments twisted around the wire axis, and so on. The electromagnetic, thermal, and mechanical processes in composite superconductors are intimately related because the current–voltage characteristic of a hard superconductor is very dependent on temperature, magnetic field, and – frequently – the deformation. Theoretical and experimental studies of the electrodynamics and low-temperature mechanics of a complex heterogeneous anisotropic nonlinear medium such as a superconducting composite are virtually impossible at the level of its individual structure elements. Moreover, we are usually interested only in the mean temperature, electric field, current density, magnetic induction, mechanical stress and strain, and so on. The situation is typical of heterogeneous media for which it is natural to pass from 'microscopic' to macroscopic description. The superconducting composite is then looked upon as an anisotropic homogeneous medium with effective parameters determined by averaging the parameters of the hard superconductor and the normal host over regions containing a large number of composite structure elements (filaments, layers, fibers, and so on).

The macroscopic properties of composite superconductors, and the processes that occur in them, are studied in the rapidly developing subject of the physics of composite superconductors. This monograph is an attempt to present a unified account of the subject.

Chapter 1 gives a brief review of information on hard superconductors, which is exploited in subsequent chapters. Particular attention is devoted to the critical state, the viscous flow of magnetic flux, and the nonlinearity of the current-voltage characteristic in weak electric fields.

Chapter 2 describes the most commonly used composites and their typical characteristics. The word 'typical' must be treated with caution because advances in fabrication technology and in the properties of hard superconductors, which have a direct bearing on the properties of composites, are constantly being reported. Chapter 2 surveys the thermal, electrical, and mechanical properties of the most frequently employed hard superconductors (Nb–Ti and Nb_3Sn), and of copper and aluminum.

Chapter 3 is devoted to dissipation processes in composite superconductors in varying magnetic fields. Hysteretic losses in hard superconductors and in twisted multifilament superconducting composites are considered. Dissipation processes in filamentary composite superconductors are discussed.

Chapter 4 deals with the stability of the superconducting state in hard and composite superconductors. The thermomechanical instability of low-temperature plastic flow of metals, and the related problem of training of superconducting composites are also discussed. Considerable attention is given to calculations of the current-carrying capacity of composite superconductors.

Chapter 5 examines nonlinear thermal phenomena in superconducting composites carrying a transport current. The propagation and localization of the normal zone in superconductors are reviewed. The current-voltage characteristics of composites and hysteretic effects due to Joule heating are reviewed. The superconducting to normal transition initiated by thermal disturbances is analyzed together with normal-zone propagation in composites with high contact resistance between the superconductor proper and the normal host metal.

Chapter 6 gives a brief summary of high-temperature superconductivity, including the basic properties of high–T_c superconductors that will be needed for the evaluation of advanced superconducting materials.

CHAPTER

ONE

HARD SUPERCONDUCTORS

In a magnetic field B, a type II superconductor is in a mixed state (or Shubnikov phase) if $B_{c1} < B < B_{c2}$. The properties of type II superconductors are described in detail in Refs. [1–6]. They were first studied experimentally by Shubnikov *et al.*[7] The quantity B_{c1} is referred to as the first (or lower) and B_{c2} as the second (or upper) critical magnetic field. Modern views on the microscopic structure of the mixed state were first formulated in Abrikosov's theoretical paper.[8] The essential idea is that the magnetic flux enters a type II superconductor in the form of quantized vortices (Abrikosov vortices) stretched out along the magnetic field. The set of these vortices, each carrying a strictly defined magnetic flux, i.e., the magnetic flux quantum Φ_0, forms a lattice of vortices permeating the entire sample. Microscopic theory shows that $\Phi_0 = \pi\hbar/e = 2 \times 10^{-15}$ Wb, where $\hbar$ is the reduced Planck's constant and e is the electron charge. Abrikosov's theory, developed from the Ginzburg–Landau equations, is supported by a great number of direct and indirect experiments (*cf.* [1–6], and [8–10]).

An isolated vortex can be visualized as follows. The size of the central part of a vortex (its core) is of the order of the characteristic scale ξ of the variation of the density of the superconducting electrons (the coherence length). This part is almost wholly in the normal state. Closed superconducting currents flow on the periphery of the vortex. The size of this region is of the order of the depth of penetration λ of the magnetic field into the superconductor. Figure 1.1 shows the current lines, the magnetic field B_z, and the superconducting current density j_φ at different distances r from the vortex axis. For type II superconductors, $\lambda > \xi$ or even $\lambda \gg \xi$. Typical values are $\xi \sim 10^{-8} - 10^{-7}$ m and $\lambda \sim 10^{-7} - 10^{-6}$ m.

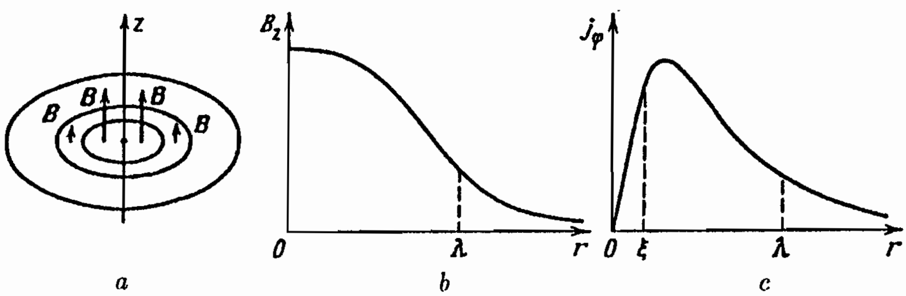

Figure 1.1 Vortex structure: magnetic lines of force and current lines (a) and the distributions of $B_z(r)$ (b) and $j_\varphi(r)$ (c)

In thermodynamic equilibrium, the vortices form an ordered structure, i.e., a two-dimensional lattice with vortex density $n = B/\varphi_0$ (Refs. 2 and 8–13). Its properties determine the properties of the mixed state. In particular, calculations show that $B_{c2} = \Phi_0/2\pi\xi^2$, whilst for $B_{c1} \ll B \leq B_{c2}$ the magnetic permeability of type II superconductor is practically equal to unity, i.e., $B = \mu_0 H$ (Ref. 2).

1.1. Viscous Flow of Magnetic Flux; Pinning

Let us now consider a transport current flowing in a type II superconductor in the critical state. We begin by analyzing this case qualitatively, using a simple hydrodynamic analogy.

The motion of conduction electrons in metals can be looked upon as the motion of a quasi-neutral incompressible liquid if the characteristic scales of the resulting flow are appreciably greater than the interatomic separations. In superconductors, this is also the motion of a superfluid, i.e., there is no viscosity. The Abrikosov vortex is then literally a vortex in the fluid of conduction electrons.

The distribution of current lines in an isolated vortex in a superconductor carrying a transport current of density j is shown diagrammatically in Fig. 1.2. It is clear that the resultant velocity of the electron fluid on the right of the vortex core is greater than on the left. According to Bernoulli's theorem, this means that the pressure on a vortex from the left is higher than from the right. This gives rise to the the Lorentz force f_L per unit vortex length in the direction perpendicular to the magnetic field and the transport current.

The Lorentz force f_L produces the motion of the vortices when the transport current density reaches a certain value determined by the interaction between the vortices and the crystal structure. In ideal type II superconductors, the vortices are practically not bound to the crystal lattice. The motion of the vortices begins in this case for low values of j. In nonideal type II superconductors, the vortices are pinned to crystal structure defects. This is known as *pinning*, and superconductors with strong pinning of the vortex structure are referred to as *hard superconductors*. It is clear that the motion of vortices in hard superconductors begins for a finite value of j.

A detailed derivation of the expression for f_L can be found, for example, in Refs. 2, 3, and 6. The final result of this derivation is

$$\mathbf{f}_L = \mathbf{j} \times \mathbf{\Phi}_0 \tag{1.1}$$

where

$$\mathbf{\Phi}_0 = \Phi_0 \frac{\mathbf{B}}{B} \tag{1.2}$$

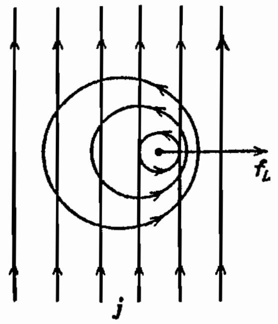

Figure 1.2 Lines of current associated with a vortex and lines of transport current

The force produced by the transport current per unit volume of the vortex lattice is then readily shown to be

$$\mathbf{F}_L = n\mathbf{f}_L = \mathbf{j} \times B \tag{1.3}$$

It will be shown later that the motion of vortices in type II superconductors, occurring under the influence of the Lorentz force effect, is accompanied by the dissipation of energy[14]. This means that a potential difference must be applied to the sample to maintain a given transport current. In ideal type II superconductors, in which the vortex lattice is not pinned to crystal-structure inhomogeneities, electric resistance will therefore be present even for arbitrarily low values of transport current density. We emphasize that superconductivity persists under these conditions provided the temperature of the sample does not exceed the critical temperature $T_c(B)$.

Let us now examine qualitatively the basic physical mechanisms of energy dissipation during the motion of the vortices. The first of these mechanisms is almost obvious. During their motion in the superconductor, the vortices cross the current lines (by virtue of their continuity), so that part of the transport current flows through vortex cores that are practically in the normal state. Clearly, this is only possible if an electric field is present in the vortex core, i.e., heat is released [14] in the core at a rate $\mathbf{j}.\mathbf{E} > 0$. The second mechanism of energy dissipation is less obvious [15]. It can be understood on the basis of the following physical considerations. When a vortex passes through a point in the sample, a phase transition occurs at this point from the superconducting state (peripheral part of the vortex) to the normal state (vortex core) and *vice versa*. We know from thermodynamics that this is a reversible process (i.e., it is not accompanied by the dissipation of energy) provided it proceeds at an infinitely slow rate [16]. Since the vortex velocity is finite, part of the difference between the free energies of the normal and superconducting phases is released as heat.

There is no difficulty in evaluating the specific rate of energy dissipation, $\dot{Q}_1$, due to the Joule heat release in the vortex cores. Indeed, as already mentioned, the vortex core is practically in the normal state. The specific rate of heat release in the vortex core is of the order of $\rho_n j^2$ where ρ_n is the resistivity of the superconductor in the normal state. Multiplying this quantity by the relative volume of the cores of the vortices ($\sim n\xi^2$), we find that

$$\dot{Q}_1 \sim n\xi^2 \rho_n j^2 \tag{1.4}$$

Since $n = B/\Phi_0$ and $B_{c2} \sim \Phi_0/\xi^2$, we can rewrite (1.4) in the form

$$\dot{Q}_1 \sim \frac{B}{B_{c2}} \rho_n j^2 \tag{1.5}$$

Using (1.5), one can readily assess the contribution of the Joule heat release in the vortex cores to the resistivity of a type II superconductor, ρ_f. The specific rate of heat release is given by

$$\dot{Q} = \rho_f j^2 \tag{1.6}$$

and if we compare (1.5) with (1.6), we find that

$$\rho_f^{(1)} \sim \rho_n \frac{B}{B_{c2}} \tag{1.7}$$

Physically, this is an almost obvious result. Indeed, for $B = 0$, the superconductor has no nonsuperconducting regions, and $\rho_f^{(1)} = 0$; for $B = B_{c2}$ the entire sample is in the normal state, and $\rho_f^{(1)} = \rho_n$. For $B_{c1} < B < B_{c2}$, the relative volume occupied by the normal phase is of the order of $n\xi^2 \sim B/B_{c2}$, so that $\rho_f^{(1)} \sim \rho_n \xi^2 n \sim \rho_n B/B_{c2}$.

Let us now determine the specific rate of energy dissipation, $\dot{Q}_2$, due to the irreversibility of phase transition from the superconducting state to normal (and *vice versa*), which occurs in the neighborhood of the cores of moving vortices. Let τ_r be the characteristic relaxation time of the electronic subsystem of the superconductor, and let τ_0 be the time taken by a vortex to traverse a distance ξ (the phase transition of interest to us occurs within this very space scale). The time τ_0 is given by $\tau_0 = \xi/v$, where v is the velocity of the vortex.

The relaxation time τ_r determines the characteristic time of phase transition from the superconducting to the normal state (and *vice versa*). Consequently, if $\tau_r/\tau_0 \to 0$, the phase transition is reversible and unaccompanied by energy dissipation. For real values of the velocity of the vortex structure, the 'macroscopic' time τ_0 is always much greater than the 'microscopic' time τ_r, i.e., $\tau_r/\tau_0 \ll 1$. For a finite but small ratio τ_r/τ_0, a small fraction of the difference between the free energies of the normal and superconducting phases, $\Delta F = F_n - F_s$, which is proportional to τ_r/τ_0, will be converted to heat in a time of the order of τ_0. (ΔF is related [2] to the critical magnetic field B_c by $\Delta F = B_c^2/2\mu_0$ and $B_c = B_{c2}\xi/\lambda\sqrt{2}$). Since the relative volume of the vortex core in whose neighborhood the phase transition of interest to us occurs is of the order of $n\xi^2$, the specific rate heat of release is

$$\dot{Q}_2 \sim n\xi^2 \frac{\Delta F}{\tau_0}\frac{\tau_r}{\tau_0} \sim \frac{B}{B_{c2}}\frac{B_c^2}{\mu_0\tau_0}\frac{\tau_r}{\tau_0} \tag{1.8}$$

The motion of the magnetic flux (vortex structure of type II superconductor) with velocity v produces an electric field whose mean strength is given by [17] the well-known relation $\mathbf{E} = -\mathbf{v}\times\mathbf{B}$, from which

$$v = E/B \tag{1.9}$$

and therefore $\tau_0 = \xi B/E$. Substituting the expression for τ_0 and $B_c = B_{c2}\xi/\lambda\sqrt{2}$, in (1.8) we obtain

$$\dot{Q}_2 \sim \frac{\tau_r}{\mu_0\lambda^2}\frac{B_{c2}}{B}E^2 \tag{1.10}$$

It follows from the microscopic theory that, in type II superconductors, [2, 4]

$$\lambda^2 = \frac{m}{\mu_0 n_e e^2} \tag{1.11}$$

where n_e is the density of conduction electrons and m is the electron mass. Using the well-known Drude formula [18] for the resistivity of a normal metal,

$$\rho_n = \frac{m}{n_e e^2 \tau_r} \tag{1.12}$$

we find that

$$\dot{Q}_2 \sim \frac{B_{c2}}{\rho_n B} E^2 \tag{1.13}$$

On the other hand, the specific rate of heat release can be written as $\dot{Q} = E^2/\rho_f$. Comparing the latter expression with the result for $\dot{Q}_2$, we find the contribution to ρ_f due to the irreversibility of the phase transition during vortex motion:

$$\rho_j^{(2)} \sim \rho_n \frac{B}{B_{c2}} \tag{1.14}$$

We conclude that the two mechanisms of energy dissipation provide contributions to the resistivity ρ_f of type II superconductor that are of the same order of magnitude. Exact calculation shows that

$$\rho_f = \gamma \rho_n B/B_{c2} \tag{1.15}$$

where γ is a number of the order of unity (*cf.* Refs. 9 and 14).

The relation given by (1.15) has often been subjected to verification and has been found to be in good agreement with experiment [3, 6, 9]. It describes well the data of relevant measurements. Figure 1.3 shows the current–voltage characteristics of niobium in the mixed state. It is clear that, when the potential difference U is not too small, the differential resistance of the sample, dU/dI, which is proportional to the resistivity ρ_f, is independent of U. According to (1.15), the differential resistance increases linearly with the magnetic field. Moreover, when U is small, there is always a relatively small (along the current axis) nonlinear portion on the $U(I)$

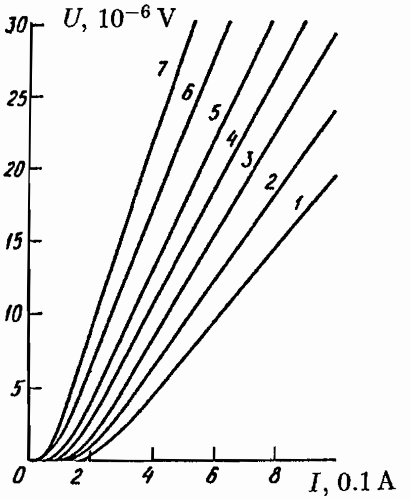

Figure 1.3 Current–voltage characteristics [9] of niobium foil in an external magnetic field B_a (B_a = 0.1000 T (*1*), 0.1125 T (*2*), 0.1250 T (*3*), 0.1375 T (*4*), 0.1500 T (*5*), 0.1750 T (*6*), 0.2000 T (*7*)

curve (we shall discuss this in greater detail in Sec. 1.3, devoted to the resistive state of type II superconductors).

If a transport current is present, the Lorentz force causes the motion of the vortices, which is accompanied by energy dissipation and is therefore viscous. The frictional force per unit volume of the vortex lattice can be conveniently written, as usual, in the form $F_f = -\eta v$, where η is the viscosity. The work done by the frictional force per unit time is the specific rate of heat release, $\dot{Q}$:

$$\dot{Q} = \eta v^2 = \eta \frac{E^2}{B^2} \tag{1.16}$$

This enables us to relate the viscosity to the resistivity of the superconductor in the normal state. Indeed, since $\dot{Q} = E^2/\rho_f$, we can use (1.15) to show that

$$\eta = \frac{B^2}{\rho_f} = \frac{BB_{c2}}{\gamma \rho_n} \tag{1.17}$$

Consequently, the steady motion of the vortex lattice in ideal superconductors is described by the dynamic equation

$$\mathbf{F}_L + \mathbf{F}_f = \mathbf{j} \times \mathbf{B} - \eta \mathbf{v} = 0 \tag{1.18}$$

where η is given by (1.17).

In nonideal type II superconductors, the motion of the vortex structure depends significantly on the interaction between this structure and the crystal defects. The interaction force, F_p, acting on a unit volume of the vortex lattice, is referred to as the *pinning force*. Clearly, in nonideal superconductors, the dynamic equation (1.18) is replaced with

$$\mathbf{F}_L + \mathbf{F}_f + \mathbf{F}_p = \mathbf{j} \times \mathbf{B} - \eta \mathbf{v} + \mathbf{F}_p = 0 \qquad (1.19)$$

A large number of original and review papers (*cf.* Refs. 6, 9, and 10 and the references cited therein) has been devoted to the evaluation of the pinning force. They deal mainly with different models. Without going into details, we shall examine the physics of the pinning force F_p.

From the standpoint of thermodynamics, the pinning of the vortex structure in nonideal type II superconductors means that the Gibbs free energy of the vortices depends on their position in the sample. The vortices are pinned to different crystal-structure defects, which are then referred to as *pinning centers*. The latter are associated, for instance, with the presence of grain boundaries in polycrystalline media, with dislocations, with precipitations of another phase (superconducting or normal), and so on[6, 10].

At this time, we must emphasize a point that is important for our understanding of the physics of nonideal type II superconductors. The pinning of the vortex structure as a whole is possible only in the absence of long-range order from the structure, i.e., when it is not strictly periodic [6]. Indeed, in an absolutely rigid vortex lattice, all the distances between vortices are fixed and depend exclusively on the interaction between vortices. This ensures that, when some vortices are in thermodynamically favorable positions, other vortices are in thermodynamically unfavorable positions. Since crystal structure defects are in general disordered, the result is that the total Gibbs free energy of the entire vortex lattice does not depend on its position relative to the superconductor lattice. This means that the absolutely rigid vortex lattice is not pinned despite the presence of pinning centers (the gain in the Gibbs free energy of some vortices is compensated by a loss in the corresponding energy of other vortices).

The pinning of the vortex structure as a whole is therefore due to the interaction between the pinning centers and the individual, relatively weakly interacting, fragments of the structure (bundles of vortices). Experiments show that such bundles of vortices can contain up to $10^2 - 10^5$ vortices[9, 19].

A strong temperature dependence of the physical parameters of the superconducting state leads to a temperature dependence of F_p. The magnetic field in the superconductor determines the distance between the vortices. Repulsion between the latter is one of the main reasons for the dependence of F_p on the magnetic field[6]. We cannot consider here problems associated with the pinning of vortices in nonideal type II superconductors. We merely note that no adequately universal scheme is currently available for the determination of $F_p(T, B)$, even when the mechanism of the action of pinning centers is known.

In nonideal type II superconductors, the vortex structure is brought into motion if the Lorentz force F_L is strong enough to overcome the pinning force F_p. By analogy with the expression for $\mathbf{F}_L$, the pinning force $\mathbf{F}_p$ can be conveniently written in the form

$$\mathbf{F}_p = -\mathbf{j}_c \times \mathbf{B} \tag{1.20}$$

The quantity j_c in this expression is referred to as the *critical current density.* Since $j_c = F_p(T, B)/B$, the critical current density j_c is a function of temperature and magnetic field.

In type II superconductors with high enough j_c, the vortex structure is strongly bound to the crystal lattice. As already mentioned, such superconductors are referred to as *hard.* It will be convenient to conduct the remainder of our discussion of hard and composite superconductors in terms of the transport current density j and the critical current density j_c. In particular, energy dissipation in hard superconductors occurs only for $j > j_c$.

The critical current density j_c of modern superconducting materials may reach up to $10^9 - 10^{10}\,\mathrm{A/m^2}$, but this is still much less than the density of superconducting currents circulating in the peripheral part of a vortex.

The dependence of j_c on temperature T and magnetic field B has been investigated by many researchers, who have proposed a number of approximate formulas for the critical current density as a function of temperature and magnetic field. Some of them are reproduced below.

The critical current density usually decreases monotonically with increasing T and B (Fig. 1.4). For many hard superconductors, the function $j_c(T, B)$ is linear in a wide range of values of T and B, and can be written in the form

$$j_c = j_0(B)\left[1 - \frac{T}{T_c(B)}\right] \tag{1.21}$$

When $B \ll B_{c2}$ a good approximation is provided by the expression proposed by Anderson and Kim[19]

$$j_c = \frac{\alpha_0(T)}{B + B_0(T)} \tag{1.22}$$

where $B_0(T) \ll B_{c2}$.

Figure 1.5 shows the form of $j_c(B)$ at $T = 4.2$ K for a sample of the superconducting alloy Nb–50% Ti. The curves represent (1.22) for $\alpha_0 = 7.7 \times 10^9 \,\mathrm{A\,T/m^2}, B_0 = 1.2$ T (curve *1*) and $\alpha_0 = 8.1 \times 10^9 \,\mathrm{A\,T/m^2}, B_0 = 1.5$ T (curve *2*). It is clear that the Kim–Anderson model describes well the experimental data in the entire range of magnetic fields in which the measurements were performed. Curve 1 is in good agreement with experiment for $0 < B < 0.5$ T, whilst curve 2 is practically identical with the experimental results for $1\text{ T} < B < 5\text{ T}$.

We note that $B_0 \sim 1\,\mathrm{T}$ is typical for superconducting Nb–Ti alloys of different composition. If in (1.22) we assume that

$$\alpha_0(T) = \alpha_0(0)\left[1 - \frac{T}{T_c(B)}\right] \tag{1.23}$$

we find that it provides a qualitatively correct description of the temperature and magnetic-field dependence of the critical current density for both $T \approx T_c$ and $B \approx B_{c2}$ because $j_c(B_{c2}) = 0$. In practice, however, it is more convenient in the neighborhood of $B \approx B_{c2}$ to write

$$j_c = j_0(T)\left[1 - \frac{B}{B_{c2}(T)}\right] \tag{1.24}$$

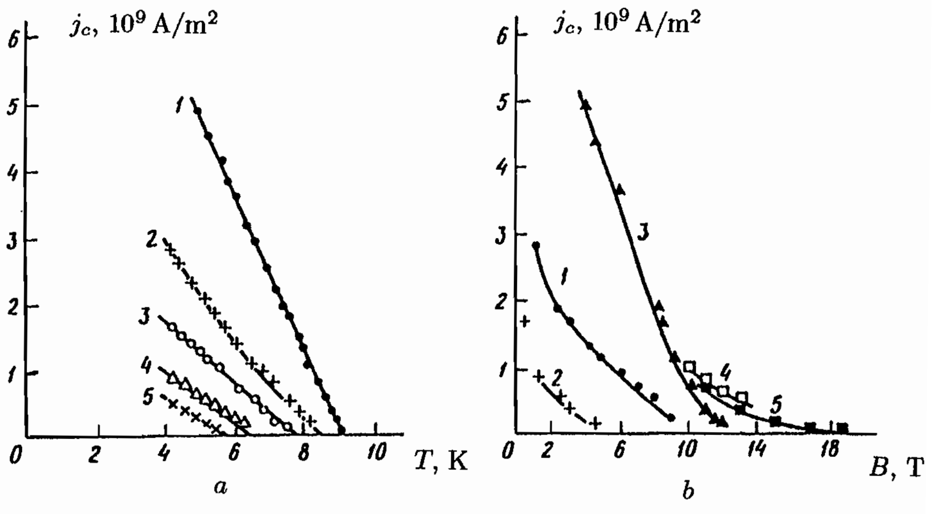

Figure 1.4 The critical current density j_c as a function of temperature for the alloy Nb–Ti [$B_a = 0$ (*1*), 1.2 T (*2*), 3 T (*3*), 6 T (*4*), 9 T (*5*)] (Ref. 20) and as a function of the magnetic field for the alloy Nb–44% Ti [$T_0 = 4.2$ K (*1*), 7 K (*2*)] (Ref. 20); Nb–65% Ti alloy 3, $T_0 = 4.2$ K] (Ref.20) and for Nb_3Sn (*4*, *5*, different samples at $T_0 = 4.2$ K) (Ref. 21)

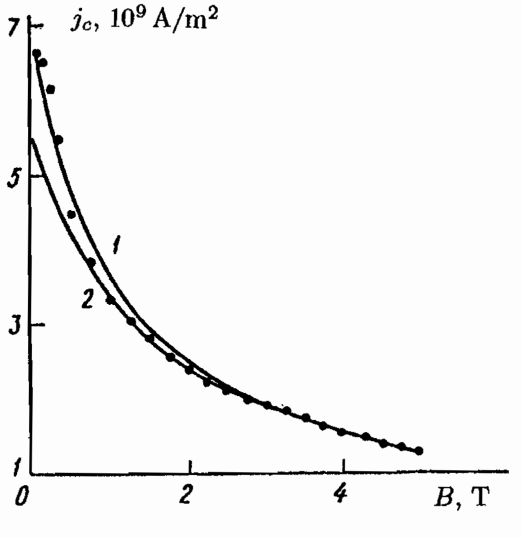

Figure 1.5 The function $j_c(B)$. Curves *1* and *2* represent (1.22) (Ref. 22)

We also note that, in a relatively narrow range of temperature and magnetic field, the critical current density may often rise as these quantities increase[6, 23]. This phenomenon has come to be known as the *peak effect*.

The dynamic equation (1.19) and the expression for F_p given by (1.20) can readily be used to obtain the current–voltage characteristic for the viscous flow of magnetic flux, i.e., for the situation in which the entire vortex structure is in motion. Indeed, if we substitute (1.20) and (1.9) in (1.19), we find that

$$j = [j_c(T, B) + \sigma_f E]\frac{\mathbf{E}}{E} \tag{1.25}$$

where $\sigma_f = \rho_f^{-1}$. Figure 1.6(a) shows the current density as a function of the electric field (1.25). The current-voltage characteristics of hard superconductors will be discussed in greater detail below. Here, we merely note that, for high critical current densities ($j_c \geq 10^8 - 10^{10}\ \mathrm{A/m^2}$), viscous flow of magnetic flux is observed if $E > E_f$, where $E_f \sim 10^{-4}\ \mathrm{V/m}$.

In the normal state, hard superconductors such as superconducting Nb–Ti of different composition and the compounds Nb_3Sn and V_3Ga, have ρ_n of the order of $10^{-6} - 10^{-5}\ \Omega\mathrm{m}$. Using (1.15), we find that $\rho_f \sim 10^{-7} - 10^{-6}\ \Omega\mathrm{m}$ if $B \sim 0.1 B_{c2}$.

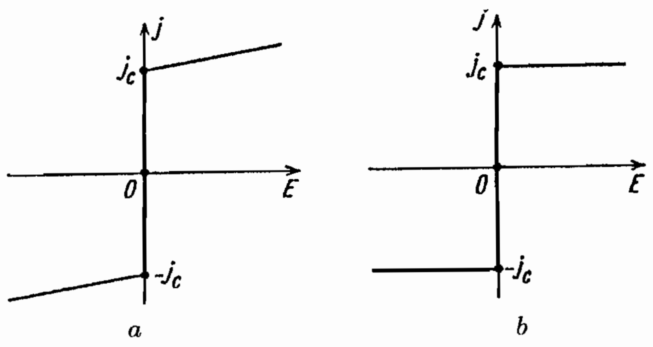

Figure 1.6 The function $j(E)$: a – viscous flow of magnetic flux, b – critical state model

It follows that the electric field appears in hard superconductors if the current density j is of the order of $j_c(T, B)$, and is accompanied by the specific Joule heat release $\mathbf{j}.\mathbf{E}$. Superconductivity can then survive only if the superconductor temperature T established by using a coolant is less than the critical temperature T_c. From these considerations, we can readily determine the maximum electric field E_m and the ratio of the critical current density j_c to the product $\sigma_f E$, often referred to in literature as the resistive current density.

Consider now the most typical case in which a sample is cooled by liquid helium at normal pressure and temperature ($T_0 = 4.2$ K). Under these conditions, heat is removed by the boiling liquid, which is possible in two steady-state regimes, namely, nucleate or film boiling. Figure 1.7 shows the heat flux density q from the sample surface to the coolant as a function of the temperature difference $\Delta T = T - T_0$ in a large pool of boiling liquid helium[24, 25]. The portion OA of the $q(\Delta T)$ curve corresponds to nucleate boiling whilst BC corresponds to film boiling. In a magnetic field $B \geq 0.1 B_{c2}$, the temperature difference $T_c - T_0$ does not exceed 10 or 20 K even for hard superconductors with record values of the critical temperature T_c. It is clear from Figure 1.7 that, for $\Delta T < 15$ K, the maximum heat flux density is $q_m \approx 7 \times 10^3$ W/m^2, which is attained for the nucleate boiling mode with $\Delta T \approx 0.65$ K.

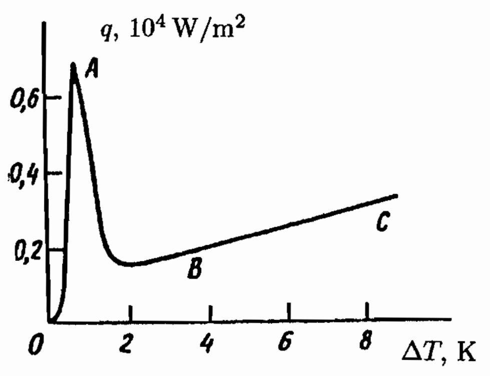

Figure 1.7 The function $q(\Delta T)$ at atmospheric pressure and $T_0 = 4.2$ K (Ref. 24)

In steady state, the rate of Joule heat release in a conductor is equal to the heat flux to the coolant, i.e., $jEA = Pq$ where A and P denote the cross-sectional area and perimeter of the superconductor, respectively. Since $q \leq q_m$ and $j > j_c$,

$$E = \frac{qP}{jA} \leq \frac{q_m P}{j_c A} = E_m \tag{1.26}$$

The quantity E_m is the required estimate of the maximum electric field in a hard superconductor. Suppose, for example, that $A/P = 10^{-4}$ m and $j_c = 10^9$ A/m^2, so that, using (1.26), we find that $E_m \approx 10^{-1}$ V/m.

Inequality (1.26) can now be used to determine the ratio of j_c to $\sigma_f E$. Simple substitution then yields

$$\frac{\sigma_f E}{j_c} \leq \frac{\sigma_f E_m}{j_c} = \frac{\sigma_f q_m P}{j_c^2 A} \approx \frac{B_{c2} q_m P}{B \rho_n j_c^2 A} \tag{1.27}$$

where we have used (1.15). For example, if $A/P = 10^{-4}$m, $j_c \simeq 10^{-9}$ A/m^2, $\rho_n \simeq 10^{-6}\,\Omega$m, and $B/B_{c2} \simeq 0.1$, then (1.27) gives $\sigma_f E/j_c \leq 10^{-3} \ll 1$. In practice, the resistive current density $j_n = \sigma_f E$ is therefore always much less than the critical current density.

1.2. Critical State of Hard Superconductors

The current–voltage characteristic of a hard superconductor can be written in the following general form for $E < E_m$:

$$\mathbf{j} = [j_c(T, B) + j_n(T, B, E)]\frac{\mathbf{E}}{E} \tag{1.28}$$

where the resistive current density $j_n(T, B, E)$ is, in general, a nonlinear function of E. We note that the subdivision of the current density j into j_c and j_n is largely terminological. (This is discussed in detail in Sec. 1.3.) The important point to note here is that the resistive current density j_n in hard superconductors is always much less than the critical current density j_c.

The fact that j_n/j_c was small enabled Bean[26, 27] and London[28] to formulate the concept of the *critical state.* According to this, a hard superconductor responds to any stimulation that

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