READER'S NOTE

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics and the other is the turbulent motion of fluids. And about the former I am rather optimistic."

Sir H. Lamb, 1932 [99]

This monograph present a review of methods that exist for describing the structure of a turbulent flow and the behavior in the flow under various conditions of the average velocity, variance of the velocity pulsations, one-point and two-point correlation moments of various orders, integral scales and microscales, spectral distributions. The turbulent motion often demonstrates a lot of peculiarities because of the reason that the integral correlation scale is comparable to the characteristic size of the flow.

There is a problem of closure of equation sets that describe the turbulent flow. Methods of solving the closure problem in the continuum mechanics are considered: when the free path is short, and especially in transport process descriptions with a finite free path. Analogies are found for many approximations introduced to describe the turbulent flow; the fruitfulness of the kinetic approach is advocated.

Methods of statistical modeling of various level of complexity are considered as the more detailed investigation of the turbulent flow structure is being pursued. The book elaborates on models of the mixing path theory; those are models that use equations for energy and the turbulence dissipation, and equations of transport for the components of a second-order correlation tensor.

It is demonstrated that the nonlocal nature of the transport is caused by a relatively large ratio of the integral correlation scale to the characteristic size of the flow. Taking into account the diffusion of the two mentioned values permits to derive a more flexible relationship between the stress tensor and the strain rate tensor the principal axes of which may be of different directions. This makes the transport processes strongly anisotropic, where lateral gradients may cause diffusion flows to appear in the field homogeneity direction. The effects of the nonlocal transport are demonstrated by a number of problems:

- development of a planar wake after a cylinder, where an "apparent negative turbulent viscosity" effect appears;
- a diffusion stage of the development of a turbulent wake behind a body that moves in a stratified medium;
- peculiarities of turbulent transport in a vortex core;

- turbulent diffusion of momentum and heat in a planar flow for an atmosphere layer of a constant friction stress and heat flux.

The construction of a transport equation for determining the integral scale of turbulence is discussed. In this regard, a model of the spectrum for a homogeneous isotropic field is identified and rules of similarity for it are formulated.

A set of kinetic equations for the probability densities in one-point and two-point description is used to derive an equation set for one-point and two-point second-order correlation moments. A set of Fokker-Planck-type approximate equations is employed. A method for introduction of relaxation times is described. A connection is established between constants in semi-empiric equations for the moments with the Kolmogorov spectral constant. An example is discussed of attenuation of a homogeneous isotropic turbulence.

The book presents experimentation results on the behavior of a turbulent flow under the influence of a vorticity created by rotation of a tube with respect to its longitudinal axis; on the behavior of spectral distributions which correspond to the second, third, and fourth-order moments; on the behavior of one-dimension and joint probability densities. The applicability of the assumption of the fourth-order cumulants being equal to zero is discussed (a Millionschikov approximation). Peculiar features of the behavior of the joint probability densities at abnormal values of the pulsations are addressed. Results obtained in Moscow Physical and Technical Institute are used.

Although some problems receive only a cursory attention, this approach seems even useful for the reader in order to get a more thorough comprehension of the behavior of a highly dynamic turbulent flow under various changing conditions. Bibliographic references are given for further studies on problems of choice. The book provides a list of issues which are important for the improvement of the semi-empiric theory of turbulent transport but not yet investigated to a sufficient extent.

In short, the objective of this book is a propaganda of usefulness of the kinetic approach in the construction of methods for describing turbulent flow properties.

First chapter discusses general models of continua used in the turbulence theory. Analogies with a molecular transport in a continuous medium are developed. A stress tensor is introduced, balance equations for the mass, momentum and energy fluxes are presented. This set of equations is not yet closed. Its closure is based on models of the medium: a perfect fluid, a viscous fluid, a stress-relaxation medium.

Second chapter presents a description of the behavior of a gas medium based on the kinetic theory, a Boltzmann equation. This chapter is important from the standpoint of an analogy between a turbulent and a free molecular flow. Equations of transport for various order moments of the velocity field are given (Maxwell's equations): mass, momentum, energy, the components of the stress tensor. A kinetic equation model in the relaxation approximation is discussed. The short free path means a nearly equilibrium state. An appropximate solution of the kinetic equation is provided. Expressions for diffusion fluxes of a "gradient type" are presented.

Third chapter is dedicated to ε-models of turbulence. It analyzes a dissipation rate equation and suggests new approximations of terms in the equation for ε. Further, equations for second and third moments are given where the effect of volume forces is taken into account directly, which has not been done in other publications. This makes it possible to derive new equations of the turbulent motion for rotating flows and employ them to describe experimental results. Third part of the chapter deals with applications of the developed semi-empiric theory to descriptions of rotating flows in channels. General equations in cylindrical coordinates are derived for the second-order and third-order moments. General formulations for differential and algebraic models are discussed.

Fourth chapter deals with a homogeneous isotropic turbulence based on the approach of a local-isotropy theory by Kolmogorov and Obukhov. The chapter presents contemporary results on a generalized model of spectral distribution for an isotropic field. There is a description of a spectral transport model, of a von Karman model approximation for the shape of the spectrum, an extended von Karman (EVK) model for the spectrum in a homogeneous isotropic field, its similarity properties, and an EVK model for a scalar field. The spectral models are a basis for the theory of spectral models of inhomogeneous turbulence, which is currently being developed by Harlow and Orzaga schools in USA.

Fifth chapter discusses semi-empiric turbulence models based on a set of equations for the pulsation moments of velocities. These equations are the basis for the method of statistical moments: SM-1 of first order, SM-2 of second order, and SM-3 of third order. Approximate models of closure based on an hierarchy of relaxation times are discussed. The natural method of closure is the Millionschikov hypothesis that fourth moments can be expressed via second moments. A local-balance hypothesis is used; under this assumption, algebraic equations can be used for second-order and third-order moments. Those form the basis for a differential model (DRSM) and an algebraic model (ARSM). A problem of a parallel flow with a constant shift of velocity is under consideration; the object of study is the behavior of the second-order moments depending on the magnitude of the shift. The ARSM model is used; Rodi's approximation is employed to take account of the rapid convection and diffusion processes. The Langevin equation for a one-point probability density function takes into consideration the effect of a lift in the shear flow that acts on a pulsating "mole". The solution is in good qualitative accordance with experimental data and Launder's solution. Further, similar approximations are used to consider turbulent diffusion. Approximations are used that have been introduced for the description of the diffusivity, based on its being proportional to the structural function and the two-point relaxation time. If the distance between two points is small, the diffusivity value is proportional to the squared distance, and if the distance is big, it is proportional to the product of the integral scale and the pulsation velocity. In the inertial interval of the distances, it follows the "4/3" law by Richardson.

In addition to the method of moments that follows from the Navier–Stokes equations, an equation for the probability density function (PDF) can be also derived. Therefore sixth chapter discusses the derivation and applications of a model equation for the probability density in the semi-empiric theory of turbulent transport. This equation plays the role of a kinetic equation in the turbulence theory for the probability density of pulsation fields (Monin, Lundgren, Ievlev). In addition to the one-point density, a model equation for the two-point pulsation probability density is considered, which has the form of a Fokker–Planck equation. Methods for closure of the moment equation set are discussed, together with the example of attenuation of a homogeneous isotropic turbulence in a flow behind a grate. The PDF equations contain the relaxation times for which approximations and relationships between the model's constants are discussed and a link is established to Kolmogorov's spectral constant. Results are presented on the behavior of the joint probability density of the pulsation field in a turbulent flow. An isotropic flow behind a grate is investigated; a comparison is made to the Gaussian distribution; the behavior of the pulsation's joint probability density is analyzed. Discussed examples include flows in a jet and in a tube; an analysis of intermittency effects is given by the example of a jet flow. An example is presented of the behavior of spatial correlation functions; a comparison is made with Millionschikov's hypothesis. There is a discussion on the importance of a nonlocal description of transport processes and some unresolved problems in both theory and experiments.

The next chapter deals with cases of anisotropy of the turbulent transport under external actions on the turbulent flow; the discussed example is a turbulent diffusion wake in a stratified medium. This case is a wake limited vertically; similarity rules are under consideration. Another example is a turbulent motion in the bottom layer of the atmosphere, in a layer of constant friction stress and constant heat flux. The problem is based on a set of equations for the second-order moments. Asymptotic laws for an unstable, neutral, and stable stratification are investigated; the role played by the transport's anisotropy is analyzed.

The non-local nature of the turbulent transport is studied by the example of the development of turbulence in a planar wake where the phenomenon of "negative" viscosity manifests itself. The numerical modeling is based on transport equations for the third-order moments.

Tenth chapter is dedicated to processes in a viscous sublayer and intermittency in the wall area. Approaches are discussed where a Loytsiansky influence function is introduced to express the dependence of the turbulent viscosity coefficient: a relationship is introduced where the intermittency coefficient depends on the value of the local Reynolds number. The possibility of introducing an "hierarchical" model is discussed, which could produce approximate estimations for the anisotropy coefficients, the spectrum and the scales based on a solution for velocity, energy of pulsation motion K, and dissipation rate ε. Ways to develop and improve a kinetic model for the description of turbulent transport are also discussed. The standard k-ε model has a singularity on the wall in the boundary layer. To remove the singularity, it is suggested to take into account the important parameters of the viscous sublayer: dynamic velocity v_* and viscosity ν. This approach means essentially the derivation of a new form for the wall influence functions. Further from the wall the equations become standard k-ε equations. An attempt is made to identify the wall influence functions with a function for the intermittency coefficient. Next, the development of an "hierarchical" model of turbulent transport is discussed. We try to use an expression suggested by Loytsiansky for the turbulent viscosity coefficient, which takes into account the molecular and molar interaction in the flow, as a relationship for the intermittency coefficient that depends on the local Reynolds number (the value ν_T/ν). A comparison is made to data available for the external part of the boundary layer, the external part of a round free jet, and a wake behind a cylinder. There seems to be a qualitative conformance. Equations derived by using the one-point intermittency PDF are in accordance with those given in publications.

Eleventh chapter elaborates on models to describe the turbulent motion in cylindrical tubes and vortex flows. The object of study includes mechanisms of turbulent transport in the vortex core and the role played by the anisotropy of the transport that causes a reduction of radial diffusion flows; in particular, the development of a thermal wind in atmosphere is investigated. Unique experimental results are presented: the behavior of characteristic parameters of a turbulent flow in the conditions of vorticity appearing in a rectilinear round tube that rotates with respect to its longitudinal axis; and a reduction of the pulsation energy and the friction stress along the flow. Results of hot-wire experimental studies are presented; they concern second, third, and fourth moments of spectral distributions. Anm approximate similarity in the energy-containing interval is demonstrated. The theoretical description is based on a semi-empiric ARSM model for a turbulent flow where the effect of rotation is taken into account. Using the one-point PDF equations, equations of the ARSM model were derived in the boundary (thin) layer approximation taking into account the effect of rotation on the coefficients of "turbulent viscosity". The effect of rotation is also taken into consideration on the values of fluxes of the pulsation motion energy and the dissipation rate. Simplifying assumptions are made in order to make the model feasible. The wave motion in some turbulent flows is also under study. Results of experimentations on the equilibration of fields in a turbulent flow inside a tube with strongly asymmetric data at the inlet are presented. A comparison is made to the concept of wave motion of perturbations in a turbulent flow.

Twelfth chapter discusses the numerical modeling of turbulent flows with rotation in channels. The derived ARSM model in the boundary layer approximation is used to analyze the behavior of a rotating flow of two mixing jets in a cylindrical mixing chamber, and so on. In order to stabilize the calculation process, a re-calculation of the longitudinal velocity field is performed on the basis of the flow rate conservation condition; a boundary condition that corresponds to the "law of wall" is used. The initial conditions (at the inlet) are set on the basis of experimental data for the longitudinal and azimuth components of the velocity and the pulsation motion energy; Prandtl's formula is used to set the conditions for the dissipation rate. An approximate solution has been obtained by setting the longitudinal pressure gradient to zero; all functions are in good qualitative correspondence with the relationships obtained experimentally. By using the dissipation rate as a parameter and varying the constant factors at the rotation influence functions we can achieve a stable calculation process for a given length of the flow and a better correspondence between the experimental and calculated results. In addition, rotating flows in a conical diffuser and in a tube with an elbow are considered.