# INTRODUCTION

Turbulent flows are a fundamental phenomenon as they can be widely encountered in both the nature and engineering: nebulae of stars, motion of water in the rivers, atmospheric fronts, motion of air as the aircraft fly or vehicles move on the roads, motion of gas and oil in pipelines, motion of the air-fuel mix in internal combustion engines of vehicles and aircraft, compressors, fans, vacuum cleaners – all demonstrate turbulent motion. In spite of the many centuries of the usage and studies of the phenomena, we are still far from an adequate description and understanding. In some cases the turbulent motion is a desired result that facilitates the process; the examples comprise a range of phenomena from stirring tea in a cup to flow control systems and devices that make the flow laminar when the turbulent motion is undesirable.

The latter cases include craft that move at a great speed but with a small resistance of the medium. The nature demonstrates examples where fish have developed devices not yet reproducible by human technology. Some fish swim at a great speed which corresponds to a hypercritical Reynolds number where turbulence ought to take place but the resistance is nearly as low as laminar. The reason why the resistance of the fish is low seems to be their integument that reduces the resistance due to an effect similar to Toms' and to "feedback" effects where the integument of a moving fish adjusts its shape dynamically so as to suppress vortices which agitate the water flowing around. This example shows how much is yet to understand and apply in the control of the turbulent flow.

The goal of the turbulent flow studies can be formulated as **turbulence flow control**. The first step is to understand the nature of turbulence. In spite of a long history of research in fluid and gas mechanics and its seeming simplicity in comparison with microphysics, first cornerstones in the foundation of understanding of the turbulence phenomena were laid nearly at the same time when quantum mechanics, special relativity theory, quantum electrodynamics and laser physics were created, all the latter being acknowledged achievements of XX century. The science of turbulent motion in continua is based on a combination of stochastic and determinate phenomena. On one hand, at the first glance a turbulent flow seems to be totally stochastic: no two equal snapshots of the flow can ever be made. On the other hand, some integral properties remain the same with a high degree of accuracy: the flow rate, the logarithmic velocity profile etc. Sophisticated mathematical descriptions of turbulent flows make some researchers think the problem is mathematical in the first place, therefore more complicated equations, closure methods, computational schemes must be involved, more computing power needs to be employed etc. To the great surprise of the followers of this opinion, there are results beyond understanding when a more complicated model has a worse correspondence with the experiment than a simpler one. Speaking generally of the correspondence between experiments and theories, it should be noted that the experimentation usually involves geometries and conditions other than numerical modeling. To remove this contradiction, the conference of 1999 at Karlsruhe decided to define five classic cases of turbulent motion (a flow with a sudden expansion, a flow in an elbow etc.) so that further experiments and theories of turbulence were first to be tested on these models and only then, after proper conclusions are made, to be applied to other cases.

We would like to emphasize a few ideas which actually formed a basis for this book. First: the understanding of turbulence should be based on ideas of physics. Second: most conservative values are moments of stochastic functions. For example, first moments define average values such as the average flow velocity and the respective flow rate, second moment defines the friction in the turbulent flow which is responsible for the integral balance of forces, say, in a pipe or in a channel. Those are actually observable physical quantities. Higher moments, not accidentally, have less significant physical meanings. The similar roles are played by the moments in the kinetic gas theory. However, in the latter the presence of a small parameter, Knudsen number, helps identify two characteristic times: hydrodynamic and kinetic one. This allows, in most cases, to use a hydrodynamical description of phenomena via equations for first moments of the velocities. Turbulence has no explicit parameter like this; however, analysis of experimental works and a "direct computational modeling" of turbulence show that

1. moments of the same order have similar transient and relaxation times;

2. there exists an hierarchy of times: the greater the moment's order, the shorter the relaxation time;

3. in order to study the processes on characteristic hydrodynamical time intervals, we can assume the higher moments are in local equilibrium with the flow;

4. due to the equilibrium, of special significance are expressions of the higher moments via the lower ones, or their derivatives with respect to the coordinates; such relations include differential expressions of second moments via the derivatives of the first ones (a Boussinesq assumption) and algebraic cumulant expressions of fourth moments via second ones (a Millionschikov approximation);

5. the turbulent flow is closer to the motion of a dilute gas in the kinetic theory of gases when the free path is not much shorter than the characteristic size of the area; an anisotropy in the turbulent transport appears due to this – the gradient of the thermodynamic force in one direction causes a flux in the other, and a "negative viscosity" arises – the signs of the Reynolds stresses do not correspond to those of the local vorticities of the flow, i.e. the relations between the gradients and the forces become nonlocal.

The mathematical description of the turbulent motion of a medium has much in common with the kinetic description of dilute and solid gases. Just as the kinetic theory of gases, the turbulence theory has two general approaches to the description of motion. First approach is based on the Liouville equation for the particle ensemble distribution function. Its counterpart in the statistical theory of turbulence is based on an equation for the probability density function. Second approach (being the first historically) is based on equations for the moments of pulsation values. In this approach, the PDF (Probability Distribution Function) equation is a counterpart of Boltzmann's equation in the kinetic theory of gases. Navier–Stokes equations can be used to derive equations for the moments of second, third and higher orders. They are a chain of connected equations. The procedure of closure, i.e. getting a closed equation set, is based on an hierarchy of the characteristic times, similarly to the closure in the kinetic theory of gases. It is known that the Boltzmann kinetic equation describes a kinetic phase of processes with the characteristic times of the order of the mean free time. When averaged, the Boltzmann equation produces a motion equation (Euler or Navier–Stokes) that describes the hydrodynamical phase of the process, i.e. the phase that has the characteristic time of the order of the hydrodynamical time. It means first-order moments have a slow characteristic time. Higher-order moments become steady-state in a significantly shorter time. This idea, based on the hierarchy of times for higher-order moments, manifests itself also in turbulence: higher-order moments come to their steady state faster than lower-order moments do, and after a fast time there appears a parametric dependence of them on some integral quantities such as the local vorticity of the flow, average values of external forces etc. Further below we will deal with both approaches to the description of turbulence.

This book presents methods for describing average one-point and two-point, correlation and spectral characteristic functions of a developed turbulent flow in an incompressible medium. There is a discussion on the analogy with the molecular transport at a finite mean free path, on methods of closure and descriptions of flows based on the kinetic approach. Statistical models are presented for moments of various orders. A semi-empiric set of kinetic equations for one-point and two-point probability densities is employed; the equations are similar to those of Fokker–Planck. A discussion is provided on a nonlocal nature of transport in the turbulent diffusion process and on the anisotropy of transport caused by a difference in the directions of the principal axes of the stress tensor and of the strain rate tensor. Methods of solution are demonstrated by the example problems of the development of a turbulent wake in a homogeneous or stratified medium, and of peculiarities of the diffusion in a vortex core or in a rotating flow.

The following monographs are recommended for reading together with this book: Monin A.S. and Yaglom A.M. [136]; Hinze J.O. [70]; Batchelor G. [11]; Landau L.D., Lifshitz E.M. [100]; all those contain detailed derivations for many of the relationships found in this book.