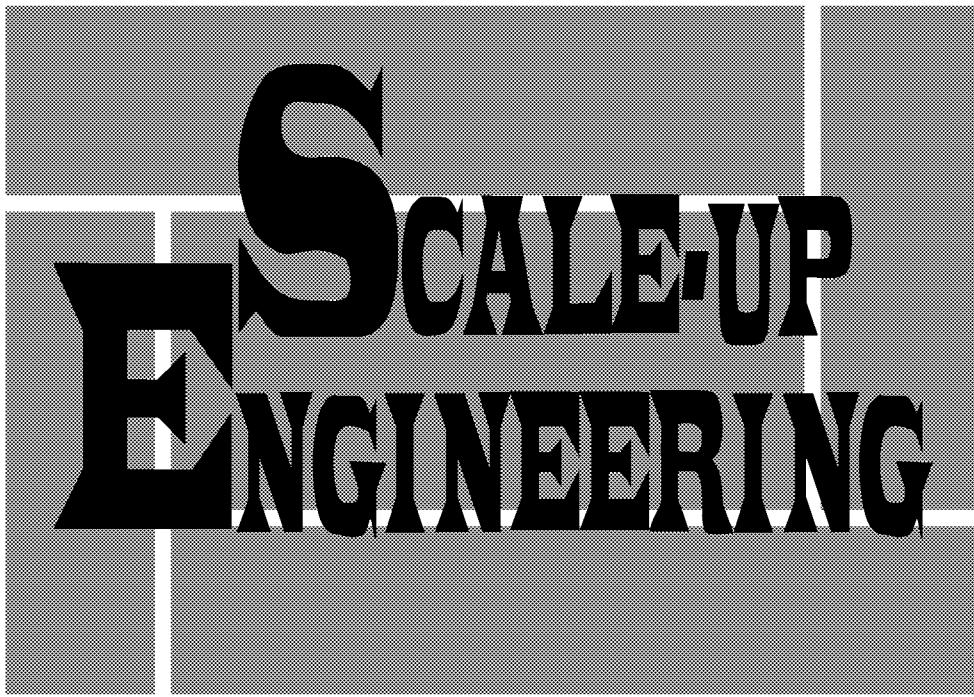


**E SCALE-UP  
ENGINEERING**





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## **Scale-up Engineering**

Johann G. Stichlmair

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## Foreword

Scale-up engineering is an extremely effective tool for engineers of all disciplines. It is based on the description of technical objects by complete sets of dimensionless numbers. The scale-up laws are very helpful in solving engineering problems by simple or small-scale experiments. Thus, dimensionless numbers and scale-up laws are used in many fields of natural and engineering sciences. Classic fields of applications are fluid mechanics, heat and mass transfer, mechanics (statics, dynamics, stress and strain), and aeronautics. Novel fields of applications are reaction engineering, acoustics, and space sciences.

The basis for the effectiveness of dimensionless numbers is the fact that fundamental physical laws are used in the definition of the units (dimensions) of physical quantities. For instance, the unit of force ( $N = kg \cdot m/s^2$ ) reflects the second law of Newton, force = mass times acceleration. Analogously, the unit of electrical resistance ( $R = V/A$ ) stems from Ohm's law, voltage = resistance times intensity of current. Thus, the units reflect elementary physical laws. The formulation of physical relationships by dimensionless numbers uses (mostly unconsciously) the relevant fundamental physical laws. This fact explains the enlarged range of validity of a single data point when presented with a complete set of dimensionless numbers. Such presentations include not only the experimental results but also the relevant physical laws in a rudimentary form.

The book is divided into five Chapters. Chapter 1 deals with the fundamentals of formulating physical relationships. In this Chapter, it is explained why and how physical laws are used for the definition of the units of physical quantities.

The Chapter 2 explains the structure of dimensionless numbers and presents methods for the systematic derivation of such numbers. A very elegant and systematic method developed by Pawłowski /0.12, 1.1/ for calculating dimensionless numbers using the knowledge of the relevant quantities is presented. In this Chapter, it is demonstrated that only complete sets of dimensionless numbers facilitate the solution of technical problems. Single dimensionless numbers and their interpretation are of little or no help. It is demonstrated that a large number of different sets of dimensionless numbers exists for each problem, which indeed allows a rigorous description. However, only a few of them are favorably applied to solving a given problem.

Chapter 3 demonstrates procedures for the application of dimensionless numbers to the investigation of physical phenomena. Of primary interest is which of the different sets of dimensionless numbers offers the greatest advantages. In this context, the often used physical interpretation of single dimensionless numbers is questioned.

Chapters 4 and 5 deal with the development of the principles of similarity (scale-up models) from a knowledge of complete sets of dimensionless numbers. The scale-up laws allow the investigation of an unknown phenomenon by simple models instead of by a complex and expensive prototype. Because of the degree of freedom inherent to the laws of similarity, the model can be much simpler than the prototype. The constraints for developing simple model experiments are presented in detail. They refer to both the number and the type of free quantities. Since these constraints often prevent simple model experiments with total similarity, strategies for dealing with partial similarity are presented in Chapter 5.

In all Chapters of the book, the text is accompanied by a large number of thoroughly elaborated examples. These examples are arranged in boxes in order to not hinder the easy readability of the text. Only the principal procedures and the most essential results are explained in the text. All the details are contained in the boxes. In total, 65 examples from all disciplines of engineering sciences are presented in the book.

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## 0 Nomenclature

$A$	area	$m^2$
$a$	amplitude	$m$
$a$	sonic velocity	$m/s$
$\alpha$	thermal diffusivity, $\alpha = \lambda / (\rho \cdot c)$	$m^2/s$
$B, b$	breadth, width	$m$
$b$	acceleration	$m/s^2$
$c$	heat capacity	$J/(kg \cdot K) = m^2/(K \cdot s^2)$
$c$	concentration	$kmol/m^3, kg/m^3$
$D, d$	diameter	$m$
$D$	diffusion coefficient	$m^2/s$
$D_{ax}$	axial dispersion coefficient	$m^2/s$
$E$	elastic modulus	$kg/(m \cdot s^2)$
$E$	energy	$J = kg \cdot m^2/s^2$
$e$	unit	-
$F$	force	$N = kg \cdot m/s^2$
$F$	gas load, $F \equiv (v_G \cdot \sqrt{\rho_G})$	$Pd^{1/2}$
$f$	scale factor, $f \equiv (l/l_M)$	-
$\dot{G}$	volumetric gas flow	$m^3/s$
$g$	acceleration of gravity	$m/s^2$
$H, h$	height	$m$
$I$	area momentum	$m^4$
$J$	momentum of inertia	$kg \cdot m^2$
$k$	rate constant of reaction	$s^{-1}$
$\dot{L}$	volumetric liquid flow	$m^3/s$
$L, l$	length	$m$
$M, m$	mass	$kg$
$m$	number of $\pi$ groups in a set	-
$m$	fainting exponent	$m^{-1}$
$\dot{m}$	mass flow rate	$kg/s$
$n$	number of relevant quantities	-
$n$	rotation speed	$s^{-1}$
$P$	power	$W = kg \cdot m^2/s^3$

$p$	degree of homogeneity	-
$p$	pressure	$Pa = kg/(m \cdot s^2)$
$\Delta p$	pressure loss	$Pa$
$q$	exponent	-
$q$	degree of homogeneity	-
$\dot{Q}$	heat flux	$W = kg \cdot m^2/s^3$
$r$	rank of dimensional matrix	-
$r$	degree of homogeneity	-
$T$	Kelvin temperature	$K$
$\Delta T$	temperature difference	$K, {}^\circ C$
$\Delta T$	reverberation	$s$
$t$	time	$s$
$t$	Celsius temperature	${}^\circ C$
$\dot{V}$	volumetric flow	$m^3/s$
$v$	velocity	$m/s$
$W$	weir height	$m$
$\bar{x}$	quantity	-
$x$	number	-
$x$	coordinate	$m$
$\bar{y}$	quantity	-
$y$	number	-
$y$	coordinate	$m$
$\bar{z}$	quantity	-
$z$	number	-
$z$	coordinate	$m$
$z_\pi$	number of $\pi$ sets	-

### Greek Symbols

$\alpha$	heat transfer coefficient	$W/(m^2 \cdot K) = kg/(K \cdot s^3)$
$\alpha$	sound absorption coefficient	-
$\beta$	angle	-
$\dot{\beta}$	angle velocity	$s^{-1}$
$\ddot{\beta}$	angle acceleration	$s^{-2}$

$\delta$	distance	$m$
$\dot{\delta}$	velocity	$m/s$
$\ddot{\delta}$	acceleration	$m/s^2$
$\gamma$	temperature sensitivity coefficient	$K^{-1}$
$\varepsilon$	volume fraction	-
$\eta$	dynamic viscosity	$Pa \cdot s = kg/(m \cdot s)$
$\vartheta$	Celsius temperature	$^{\circ}C$
$\kappa$	heat capacity ratio	-
$\lambda$	conductivity of heat	$W/(m \cdot K) = kg \cdot m/(K \cdot s^3)$
$\lambda$	wave length	$m$
$\mu$	constant in viscosity function, friction coefficient	-
$m$	friction factor	-
$\xi$	drag coefficient	-
$\pi$	dimensionless number	-
$\pi_c$	circular constant	-
$\rho$	density	$kg/m^3$
$\Delta\rho$	density difference	$kg/m^3$
$\sigma$	surface tension	$N/m = kg/s^2$
$\sigma$	breaking stress coefficient	$kg/(m \cdot s^2)$
$\varphi$	angle	-
$\omega$	frequency	$s^{-1}$

### Subscripts

$c$	continuous phase
$d$	dispersed phase
$G$	gas
$g$	glass
$i$	variable
$j$	variable
$L$	liquid
$m$	mold
$M$	model
$o$	end, reference
$p$	particle

<i>s</i>	swarm
<i>S</i>	solid
<i>x</i>	quantities of type x
<i>y</i>	quantities of type y
<i>z</i>	quantities of type z

### Mathematical Operations

=	equal
$\neq$	not equal
$\equiv$	definition
$\approx$	approximately equal
$\sim$	proportional
<i>Det</i>	determinante
<i>f</i>	function
$\Delta$	difference
$\Pi$	product

### Comments

- The exact meaning of the symbols is explained at the end of the tables.
- In Chapter 1, quantities are marked by a bar ( $\bar{x}$ ) to avoid any confusion with numbers (*x*).

## 1 Description of Physical Phenomena

Every engineer is well trained in the use of physical equations, that is, mathematical relationships between physical quantities. Because of this training, many aspects of such equations have never been questioned and, in turn, have never been fully understood. Often the right procedure is applied just by simple intuition. The "unit check", for instance, is often used for proving a physical relationship. In its simplest form, the "unit check" states that physical quantities of different dimensions must not be added or subtracted. Even in elementary schools, students are taught - without any reasoning - one cannot add apples and oranges. The addition of length and time is nonsense. Why? Is there a physical law behind it?

This question seems well qualified, especially when considering the fact that dividing length by time is a sensible operation; the result is velocity. What reasoning exists for one mathematical operation (addition) of length and time to be forbidden and the other one (division) to be legal?

The square of a length is a legal mathematical operation; the result is an area. However, why is the logarithm of a length nonsense? Or is it not nonsense at all? Such doubts may arise, since the logarithm of temperature,  $\ln T$ , and especially its differential,  $d\ln T$ , is often found in scientific literature.

Such considerations lead, finally, to the question of whether or not physical relationships can be formulated by mathematical equations. Mathematics is, in essence, a science of numbers. In physical relationships, the elements are not numbers but physical quantities, for instance, length, time, temperature, mass, etc. Can mathematical operations be applied to physical quantities? If yes, can they be applied without any restrictions? What is, in essence, a physical quantity?

### 1.1 Physical Quantities

The object of natural science is not the nature of natural phenomena, but rather the relationships between some characteristics (features, entities) of these phenomena. If, for instance, scientists consider the Earth, they are not interested in the nature of the Earth, nor in why it exists or where it comes from. They concentrate, in wise self-restriction, on single characteristics of the Earth, for example, shape, size, mass, density, rotation speed, etc. Furthermore, they try to formulate relationships between some features of the Earth.

An example for such a relationship between several features of the Earth is

$$\text{mass} = \text{volume times density}$$

Dealing with mechanics, scientists are not interested in the "nature of mass", the "nature of acceleration", or the "nature of force". Their interest lies in the formulation of a relationship between mass, acceleration, and force, that is, the second law of Newton

$$\text{force} = \text{mass times acceleration}$$

This relationship is worthless to philosophers but very important to engineers. It allows them, for instance, to calculate the motion of stars and rockets or the braking distances of cars.

The object of natural science is, in essence, the formulation of relationships between the characteristics of natural phenomena. Hence, the natural phenomenon itself is not included in such relationships. There is no relationship that contains the term "water". The water is represented by its features, that is, density, viscosity, and thermal conductivity. Analogously, no physical relationship contains the term "pump". The pump is replaced by its features: diameter, speed of rotation, suction height, etc.

The features of natural phenomena are called physical quantities. Examples of important physical quantities are

$$\text{length, mass, time, temperature, force, velocity, acceleration, etc.}$$

However, the time shown on a clock (e.g., 11:22) or a date (e.g., 1/6/2000) are not physical quantities. Rather the difference between two dates is a physical quantity.

Physical quantities can be distinguished between "quantities of the same dimension" and "quantities of different dimensions". Examples of quantities of the same dimensions are

$$\text{distance, length, diameter, height}$$

$$\text{or} \quad \text{kinetic energy, heat, work, electric energy}$$

The distinction between "quantities of the same and of different dimensions" is, however, not absolute. For instance, kinetic energy and thermal energy have been considered as quantities of different dimensions until the detection of the principle of heat equivalence by Robert Mayer in 1842. Today, both types of energy have the same dimension. Nevertheless, in special cases, they can even today be treated as quantities of different dimensions.

**Table 1.1 Euclid, 4th century B.C., writes in ΘEON /1.2/**

A ratio is a particular relation between two homogeneous quantities, as, for instance, double, triple, etc. But the relation between inhomogeneous quantities is, after Andrastos, impossible to know. For instance, the Cubit can not be compared with the Choinix (i.e., ancient unit for grain) or the Kotyle (i.e., ancient unit for liquids) nor the white with the sweet or the hot. With homogeneous quantities, however, such a comparison is possible, as length with length, area with area, volume with volume, liquid with liquid, grain with grain, dry with dry, number with number, time with time, motion with motion, sound with sound, taste with taste, color with color, etc.

The characterization of the features of natural phenomena by physical quantities is an abstraction of the reality that is the result of the long-term development of the natural sciences. The difficulties that have had to be overcome can be understood from the historic text in Table 1.1.

## 1.2 Formulation of Physical Relationships

At present, physical relationships are quantitatively formulated by mathematical equations. Being a science of numbers, mathematics can be applied to numbers only and not to quantities. Thus, the quantities have to be transformed into numbers first. This is, in principle, established by forming the ratio of the quantity at hand and a reference quantity of the same dimension:

$$\begin{array}{lll} x_i & = & \bar{x}_i \quad / \quad e_x \\ \text{number} & & \text{quantity} \quad \text{unit} \end{array} \quad (1.1)$$

Consequently, a reference quantity  $e_x$  must be defined for each type of quantity, for example, length, time, mass, etc. These reference quantities are called units  $e$ .

For the clear distinction between numbers and quantities, the quantities are marked by a bar in the first Sections of this book. Thus,  $x$  stands for a number and  $\bar{x}$  for a quantity. Quantities with different dimensions, that is, different units, are denoted by different symbols  $\bar{x}_i, \bar{y}_i, \bar{z}_i, \dots$ . The corresponding units are denoted by  $e_x, e_y, e_z, \dots$ . The subscript  $i$  of the quantities expresses the fact that several quantities  $\bar{x}_i$  of the same dimension may be contained in a relationship.

If the quantity  $\bar{x}$ , for instance, denotes the length of a rope, the number  $x$  of the length is determined by referring the quantity length to the reference quantity of

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