AN INTRODUCTION TO MODERN ANISOTROPIC ELASTICITY

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The author, a leading Russian authority in the field of elasticity, has obtained some fundamental results in the investigation of a number of general problems in the mechanics of solids, the theory of elasticity, the theory of shells, biomechanics, and the mechanics of elastomers. Fundamental theoretical investigations were combined with practical applications to real construction problems. He is the author of nine well-known monographs.

The first chapter of this book deals briefly with symmetry considerations. The second chapter is devoted to the author's original treatment of linear anisotropic elasticity. The succeeding chapters are concerned with some of the most interesting modern problems of the nonlinear theory of elasticity (constitutive equations, plane problems, anisotropic and reinforced shells, brittle fracture, Volterra's dislocations, etc.). The account is based mostly on the author's original investigations and contains a lot of new results.

This important book is intended for students, postgraduate students, engineers, and scientists specializing in the various fields of structural analysis and is an attempt to acquaint them with current problems of the modern theory of anisotropic mechanics of solids.

CONTENTS

TO THE READER

One must surely have a lot of courage to undertake the work of presenting the anisotropic theory of elasticity in its most general (nonlinear) form. After all, it is obvious that one must deal with cumbersome computations and unwieldy formulas that do not come easily across to the reader who must also be shown their practical significance. But the advantage of K. F. Chernykh's treatment is in that he knows how to overcome difficulties of that sort. In this book a successful (and in many ways original) classification of formulas belonging to the linear theory of elasticity is given as well as a detailed treatment of the nonlinear theory, even including specific strain-energy functions for the most important types of anisotropy. The problems of large deformations of thin anisotropic shells are dealt with in great detail. Since anisotropic and constructively anisotropic (i.e., composite) materials are used more and more extensively, this book containing a wealth of information about their elastic properties will find, undoubtedly, its reader.

V. V. Novozhilov, Member of the Russian Academy of Sciences

PREFACE

From a formal point of view, this monograph may be regarded as the second edition of the author's book "An Introduction to Anisotropic Elasticity" (Nauka-Fizmatlit, Moscow, 1988). However this is, essentially, a new book updated and considerably extended. Since it has preserved its character, I think it to be worthwhile to retain the preface written by my teacher and friend Valentin Valentinovich Novozhilov (see preceding page).

It is impossible even to list all the numerous publications in which various problems in the analysis and the design of structures of anisotropic materials are discussed. In this connection, the fundamental work of N. G. Chentsov and S. G. Lekhnitsky must be mentioned above all. At the same time the number of publications on the anisotropy of elastic properties is strikingly small. Among them the following ones [1.1, 1.2, 2.4–2.6, 2.8, 3.4, 3.5, 4.1, 7.1–7.3, 11.6] should be noted.

The present volume was designed as a brief systematic account of the class of problems that are connected with the laws of elasticity of anisotropic materials. Emphasis is placed on either new problems or those that are seldomly treated in the literature—problems that are of great significance (generalized plane strain and plane state of stress, incompressible and reinforced materials, geometrical and physical nonlinearity, the nonlinear theory of shells, plane problem, brittle fracture, nonlinear Volterra's dislocations).

A few words on the contents of the book.

In Chapter I a brief but systematic account is given of the use of symmetry considerations in the mechanics of solids (deformable bodies). The concept of a symmetry group for a finite body is made clear. Restrictions on possible types of symmetry due to the existence of a space crystal lattice are revealed. The existing crystal classes and textures are listed. The Neumann principle is formulated.

In Chapter II the structure of Hooke's law for anisotropic materials is discussed. A nontraditional approach makes it possible to introduce symmetric Poisson coefficients of different orders. This enables solution of P. Bekhterev's problem [2.2] of finding narrowest (unimprovable) bounds on the variation of elastic constants [3.2]. The ideas of V. V. Novozhilov concerning the principal axes of anisotropy and the reduction in the number of essentially different elastic constants are further developed. Attention is given to the problem of unification of elastic constant matrices within each crystal system.

In Chapter III nonlinearly elastic anisotropic materials are dealt with. The structures of strain-energy density functions corresponding to various anisotropic materials are investigated. For small deformations, the conditions for the appropriate law of elasticity to acquire the form of the corresponding Hooke's law are given.

In Chapter IV we deal with deformation anisotropy which occurs for large deformations of an elastic material. An incompressible isotropic material reinforced by families of cords (filaments) is considered.

In Chapter V the results obtained are used for shells subjected to large deformations. A brief outline of the simplest "working" nonlinear theory of elastic shells (without loss of generality) is given. The problem of the choice and design of strain-energy density functions is treated.

In Chapter VI we are concerned with shells reinforced by two families of inextensible or slightly extensible filaments. Two types of reinforcement are considered: in the middle surface and continuous through the shell thickness. Solutions to a number of applied problems are given.

In Chapter VII the linear plane problem for an orthotropic material is discussed. Extensive use is made of the complex variable method developed by the author and also of his algebraic method for solving boundary-value problems.

In Chapter VIII we deal with the nonlinear plane problem. An original approach makes it possible to obtain in finite form solutions to geometrically nonlinear problems.

In Chapter IX a brief outline of the results obtained by the author on the nonlinear theory of cracks in an isotropic material is given. The ways to extend the theory to anisotropic materials are discussed.

In Chapter X we deal with the essentials of the nonlinear theory of Volterra's dislocations. The wedge disclination is considered in detail.

In Chapter XI we are concerned with those aspects of theory that are closely connected with the subject matter of the preceding chapters. The theoretical material was published at different times by the author [11.2–11.6] and is, essentially, an extension of the fundamental work of V. V. Novozhilov and L. I. Sedov. The relevant published theoretical results bear witness to the undiminished attention paid to the problems of the structure and properties of the constitutive equations of the mechanics of deformable bodies. Instead of going into the contents of this chapter, we shall only mention that in §12 the general relations for large deformations are introduced, and in §13 the reasons are given for the substantial advantages of the conventional stresses (the Biot stresses) over the true stresses (the Cauchy stresses) in geometrically nonlinear mechanics.

Appendices A and B are given for the convenience of the reader. They contain the fundamentals of curvilinear coordinates, curves, and surfaces, which are needed for reading the book.

At the end of the book a list of references and a subject index are given. The bibliography mostly contains references to Russian authors that are not widely known outside the former Soviet Union.

Many of the chapters in the book may be read independently of the other chapters, yet there are many cross references. Equations within the same chapter are referred to by the section number followed by the number of the equation, and equations in the other chapters are indicated using the triple-number notation, the first number being that of the chapter in which the equation is given. A similar triple-number notation is used for both figures and tables throughout the book. In the book the repetition of a *Greek* index implies summation with respect to that index.

When writing this book, the author sometimes felt as if he were performing a sort of a balancing act on a nebulous boundary between the fundamental theoretical results and their applications. While having preference for the former, the author, nevertheless, also wanted to show how the theory works (naturally, trying not to get bogged down in cumbersome details). The book contains both the standard material and some of the new results obtained by the author and his colleagues. Certain "nonhomogeneity" of the material may be justified by the attempt to stimulate the interest of the reader in doing research work in this difficult but promising area of the mechanics of solids.

CHAPTER I

SYMMETRY CONSIDERATIONS

It is difficult to find an area of knowledge where symmetry considerations are not taken into account to a greater or lesser degree. They are widely used in theoretical elasticity when dealing with natural (crystal) or artificial (composite) materials.

The mechanical properties of media depend largely on whether the material structures in question have some symmetry elements of finite bodies. Therefore, at the beginning of this chapter we provide a short, elementary, though systematical, treatment of all possible symmetry groups for a finite body. Further the restrictions on the form of symmetry imposed by the presence of a space (crystal) lattice are discussed. Crystal classes and textures are listed.

The reader who wishes to gain a more thorough knowledge of the classical topics of symmetry presented below and also of some of the extended versions of the symmetry notion are recommended the monograph [1.2].

§1. Symmetry of finite bodies

Consider an arbitrary three-dimensional body. We may talk of its symmetry only if the body can be split up into several identical parts forming some regular configuration. The laws of regularity are defined by the *symmetry transformations* that carry the identical parts of the body into one another. If no distinction is made between the identical parts of the body, we can say that symmetry transformations carry the original body into itself.

Simple geometric constructions show [1.2] that symmetry transformations can be reflections in planes, rotations about the axes of symmetry and rotoflections (rotary reflections).

Figure 1.1.1 shows a body consisting of two tetrahedrons with bases in the plane of the figure. A reflection in the *symmetry plane* passing through the common edge of the tetrahedrons at right angles to the plane of the figure carries the two parts of the body into each other.

Figure 1.1.2 shows bodies having axes of symmetry passing through the points of contact of the tetrahedrons at right angles to the plane of the figure. An axis of revolution is called an *axis of n-fold symmetry* if by a complete revolution it is carried n times into itself. Axes of 2-, 3-, and 4-fold symmetry are shown in Figure 1.1.2.

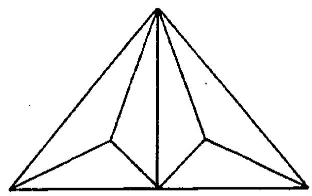

FIGURE 1.1.1

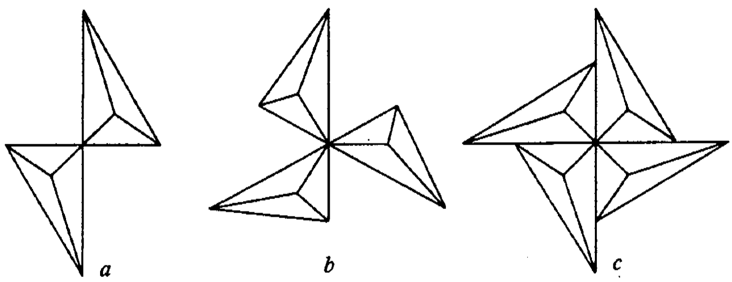

FIGURE 1.1.2

If under a rotation through $2\pi/n$ about an axis placed at right angles to a plane and a subsequent reflection in the plane, the body is carried into itself, then the axis is called an *axis of n-fold rotoflection*. Axes of 2-, 4-, and 6-fold rotoflection are shown in Figure 1.1.3.

Simple geometrical considerations imply [1.2] that in a finite body all symmetry transformations must leave at least one point immovable. Thus, if a body possesses several symmetry elements (axes and planes), they all have to pass through one immovable point called a *singular point*. This point may be outside the body itself (for example, the center of a pinion with an orifice).

Besides the symmetry transformations listed, the theory of elasticity also deals with *inversion*, under which the points of the body are carried into the points that are symmetric with respect to a particular point called the *center of symmetry*, and also with *turning-over*, in which the points of the body are carried into the points that are symmetric with respect to a given line (see Figure 1.1.4).

Symmetry transformations are said to be *equivalent* if each carries the body into itself (i.e., each part of the body will occupy exactly the same position as before). It is seen from Figure 1.1.4 that inversion is equivalent to 2-fold rotoflection, and turning-over to 2-fold rotation. Below we will show that an odd-fold rotoflection can be obtained by combining a suitable rotation with a reflection in the plane placed at right angles to the axis of revolution.

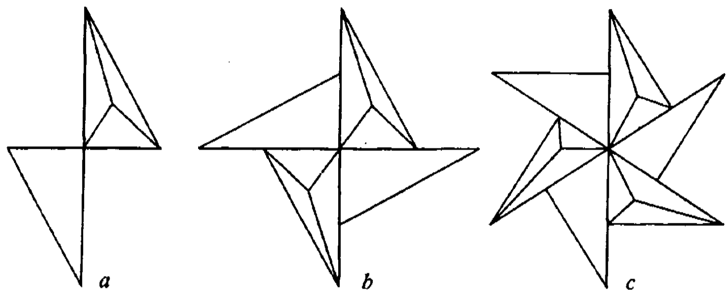

FIGURE 1.1.3

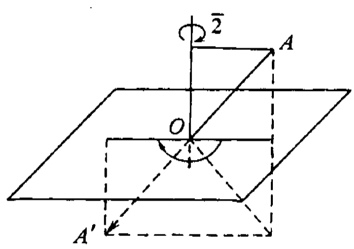

FIGURE 1.1.4

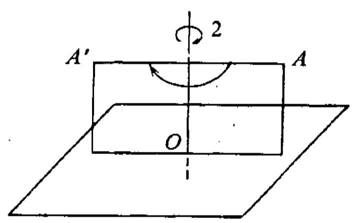

FIGURE 1.1.5

The description of all nonequivalent symmetry transformations is of particular interest. Therefore, we shall no longer be concerned with the turning-over transformation. The inversion transformation, though not of major importance by itself, is convenient to use. Finally, observe that a reflection in a plane is equivalent to a rotation about an axis perpendicular to the plane followed by an inversion with respect to the point of intersection of the plane and the axis.

§2. Notion of a symmetry group

The set of all nonequivalent transformations that carry a symmetric body into itself is called a *symmetry group*. Thus, the symmetry group of a body having one symmetry plane (see Figure 1.1.1) contains two transformations: the *identity transformation* (under which all identical parts continue to occupy their respective positions) and a *reflection in*

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