

## P R E F A C E

This twenty-third volume of the "Thermophysics" series is devoted to the study of heat transfer and turbulent mixing in gas-cooled channels considering the effect of body forces (natural convection, or bouyancy, and centrifugal forces) conducted at the Institute for Physical Engineering and Problems of Energy Research of the Lithuanian Academy of Sciences (now Lithuanian Energy Institute). The current study continues the cycle of investigations into local heat transfer and turbulent transport in smooth and rough annular channels variable fluid properties which were considered in "Thermophysics 8, 21", viz. in "Heat Transfer in Gas-Cooled Annular Channels" (1977) by J. Vilemas, B. Čėsna and V. Survila and "Heat Transfer Enhancement in Gas-Cooled Channels" (1989) by J. Vilemas, V. Šimonis and J. E. Adomaitis.

The present monograph presents the results of detailed measurements of hydrodynamic and thermal characteristics in a vertical tube with coincident directions of forced and free convections (aiding flows) at various heat loads, and also in curved tubes and coils. Numerical results for the characteristics of turbulent transport in mixed turbulent convection are reported.

The comprehensive numerical and experimental investigation of heat transfer and turbulent transport provided an explanation of some specific features of heat transfer in such intricate flows. The information obtained is needed for improving the operational reliability of various state-of-the-art gas-cooled heat exchangers.

Experimental apparatus and investigation techniques are described. The results are correlated in the form of similarity relations suitable for practical application over a wide range of operating conditions and geo-

metric parameters. Some of the most characteristic experimental data are tabulated in the appendices.

The monograph includes the results of studies performed by the present authors in collaboration with V. Kaupas, V. Mikaila, V. Šimonis, V. Šukys, E. Ušpuras, J. E. Adomaitis, R. Pabarčius, B. Teškevičienė and other researchers.

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**Authors**

## NOMENCLATURE

$b$	– channel width, m;
$c_f = 2\tau_w/\rho u^2$	– skin friction coefficient;
$c_p$	– specific heat, J/(kg·K);
$D$	– curvature diameter, m;
$d$	– diameter, m;
$d_1, d_2$	– wetted surface diameters of the tubes of annular or helical channel, m;
$d_e$	– equivalent diameter of the channels, m;
$E = (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$	– turbulence energy, m <sup>2</sup> /s <sup>2</sup> ;
$F$	– flow section area, m;
$g$	– acceleration due to gravity, m/s <sup>2</sup> ;
$h$	– channel height, m; enthalpy, J/kg;
$h_* = q_w/\rho u_*$	– characteristic enthalpy, J/kg;
$K = (v/u^2)(du/dy)$	– acceleration parameter;
$K = Gr_A/Re^2$	– natural convection (bouyancy) parameter;
$l$	– channel length along the axial line; turbulence scale, m;
$l_+ = lu_*/\nu$	– dimensionless turbulence scale;
$l_t$	– thermal turbulence scale, m;
$P, p$	– pressure, Pa;
$q$	– heat flux density, W/m <sup>2</sup> ;
$q^+ = q_w/h\bar{u}\rho$	– heat flux parameter;
$R$	– curvature radius, m;
$r$	– current radius, m;
$r_o$	– tube radius, m;
$s$	– twist pitch of the channel, m;
$T$	– temperature, K;
$T_* = q_w/\rho c_p u_*$	– characteristic temperature, K;
$T^+ = (T_w - T)/T_*$	– dimensionless temperature;
$u$	– flow velocity, m/s;

$u_* = \sqrt{\tau_w/\rho}$	– friction velocity, m/s;
$u^+ = u/u_*$	– dimensionless velocity;
$u, v, w$	– velocity components, m/s;
$\overline{u'v'_+} = -\overline{u'v'}/u_*^2$	– dimensionless turbulent shearing stresses;
$\overline{u'T'_+} = \overline{u'T'}/u_*T_*$	– dimensionless turbulent axial heat transfer;
$\overline{v'^2_+} = \overline{v'^2}/u_*^2$	– dimensionless intensity of radial velocity oscillations;
$v'T'_+ = -v'T'/u_*T_*$	– dimensionless turbulent radial heat transfer;
$x$	– longitudinal coordinate, distance from the heating origin, m;
$x_+ = xu_*/\nu$	– dimensionless longitudinal coordinate;
$y$	– transverse coordinate, m;
$y^+ = yu_*/\nu$	– dimensionless transverse coordinate;
$\alpha$	– heat transfer coefficient, W/(m <sup>2</sup> ·K);
$\beta = -1/\rho(\partial\rho/\partial T)_p$	– volumetric expansion coefficient, 1/K;
$\Delta$	– maximal relative error;
$\delta$	– boundary layer thickness, wall thickness, m;
	– relative rms error;
$\delta_{i,\alpha}$	– Kronecker delta;
$\gamma$	– intermittence coefficient;
$\varepsilon = E^{3/2}/l$	– dissipation function;
$\varepsilon_t$	– thermal dissipation function;
$\varepsilon_\tau = -\overline{u'v'}/(du/dy)$	– turbulent viscosity, m <sup>2</sup> /s;
$\varepsilon_q = -\overline{v'T'}/(dT/dy)$	– eddy diffusivity, m <sup>2</sup> /s;
$\theta$	– angle of curvature, deg;
$\kappa$	– universal constant;
$\lambda$	– thermal conductivity, W/(m·K);
$\mu$	– dynamic viscosity, Pa·s;
$\nu$	– kinematic viscosity, m <sup>2</sup> /s;
$\xi$	– friction factor;
$\Pi$	– wetted perimeter of the channel, m;
$\rho$	– density, kg/m <sup>3</sup> ;
$\tau$	– shear stress, N/m <sup>2</sup> ;
$\varphi$	– angle, °, deg;

$$\begin{aligned} \psi &= T_w/T_f & - & \text{temperature factor;} \\ \text{De} &= \text{Re}\sqrt{d/D} & - & \text{Dean number;} \\ \text{Gr}_q &= g\beta q_w d^4 \rho^2 / \lambda \mu^2 & - & \text{Grashof number defined by the heat flux} \\ & & & \text{specified on the surface;} \\ \text{Gr}_A &= g\beta d^4 \left( \frac{dT_f}{dx} \right) \frac{1}{16\nu^2} & - & \text{Grashof number defined by the longitudinal} \\ & & & \text{gradient of the bulk temperature of the} \\ \text{or } \text{Gr}_A &= \frac{\text{Gr}_q}{4\text{RePr}} & - & \text{liquid;} \\ \text{Nu} &= \alpha d_e / \lambda & - & \text{Nusselt number;} \\ \text{Pr} &= \mu c_p / \lambda & - & \text{Prandtl number;} \\ \text{Pr}_t &= \varepsilon_\tau / \varepsilon_q & - & \text{turbulent Prandtl number;} \\ \text{Re} &= \bar{u} d_e / \nu & - & \text{Reynolds number;} \\ \text{St} &= \text{Nu} / \text{RePr} & - & \text{Stanton number.} \end{aligned}$$

### Subscripts:

- o – refers to a straight channel;
- 1 – inner tube;
- 2 – outer tube;
- $\infty$  – in the external flow, in the stabilized flow region;
- cr – laminar-turbulent flow transition;
- cr1 – beginning of transition;
- cr2 – end of transition;
- f – in the flow;
- in – at the inlet;
- L – laminar;
- T – straight tube, turbulent;
- w – at the wall;
- $\psi = 1$  – at constant physical properties;
- ( $\bar{\phantom{x}}$ ) – averaging;
- ( $\prime$ ) – oscillating component.

The remaining symbols are defined in the text.

# Introduction

The development of modern engineering, the development of new technologies, and also the increase of the operation reliability of large power units necessitate an improvement of the accuracy of their thermal hydraulic calculations. This is especially important for energy-intensive systems utilizing gases since, unlike liquids, gases have low heat transfer coefficients, and small errors of their determination may cause significant deviations of the wall temperatures from the design values. To obtain a high heat flux density requires a setting up of a large temperature difference between the wall and the gas. This runs into the problem of the variability of physical properties of the gas and in some cases, especially with relatively small Reynolds numbers, into the problem of the action of natural convection (buoyancy) forces on forced turbulent transport. Apart from straight channels, heat exchangers frequently employ curved channels of various cross sections in which the heat transfer has specific features due to the action of centrifugal forces.

A good deal of attention has been afforded to studies of the effect of thermogravitational or natural convection (buoyancy) forces on turbulent transport and heat transfer (mixed convection) in channels. Results of these studies are generalized in monograph [1] and in some reviews [2, 3 and oth.].

With the joint influence of forced and free convection, the flow and heat transfer depend on the mutual direction of the forced flow and the thermogravitational forces. In the case of a heated or a cooled liquid flow in horizontal tubes, liquid particles move by the action of buoyancy in the plane normal to the tube axis, and under the effect of pressure drop, along the tube. The interaction of these flows along the axis

gives rise to an intricate vortex flow. The stability of a laminar vortex flow in horizontal heated tubes is disturbed at higher Re numbers than for an isothermal flow without vortices. Heat transfer in both laminar and turbulent flows varies noticeably around the tube perimeter. In the case of a laminar flow and of an appreciable effect of thermogravitational forces, the Nu number on the upper generatrix can 10 times as small as that on the lower generatrix. Here the heat transfer, averaged over the tube perimeter, is higher than in a forced convection. A relative reduction in heat transfer at the upper generatrix with turbulent mixed convection is more significant than with a laminar convection, whereas a relative augmentation of heat transfer at the lower generatrix is much greater with laminar mixed convection. Therefore heat transfer in turbulent flow is more uniform around the tube perimeter than in laminar flow. In turbulent mixed convection, the heat transfer, averaged over the tube perimeter is only slightly sensitive to the effect of thermogravitational forces. The flow patterns in such conditions have been analyzed thoroughly in study [1]. This study also presents correlations for heat transfer in laminar and turbulent flows.

Two typical flow cases may be discerned in vertical tubes: 1) when the directions of the forced flow and the vector of thermogravitational forces are opposite (an ascending flow of a cooled liquid or a descending flow of a heated liquid), 2) when the directions of the forced flow and of the vector of the thermogravitational forces coincide (an ascending flow of a heated liquid or a descending flow of a cooled liquid).

In the first case (opposite directions of forced and free convections), the liquid velocity decreases at the wall and increases in the flow core, in comparison with the case of forced flow. In laminar mixed convection, with an increase in the action of thermogravitational forces ( $Gr_q/Re$ ) the velocity gradient at the wall goes rapidly to zero, and afterwards a return flow originates near the wall. The flow with such a velocity profile is extremely unsteady and therefore quickly becomes turbulent. In laminar flow, with an increase in  $Gr_q/Re$ , the heat transfer reduces, which is associated with the velocity decrease at the wall.

In turbulent flow, conversely, with an increase in  $Gr_q/Re$ , the heat transfer is augmented, since a decisive influence is exerted by the increasing turbulent transport. Few studies were performed for a turbulent flow under such conditions but fairly dependable recommendations are available for calculating the heat transfer [1 - 3].

With coincident directions of the forced flow and of the vector of thermogravitational forces, the flow velocity increases at the wall and decreases in the tube center (an M-shaped velocity profile), in comparison with the case of forced flow. Therefore, the heat transfer in laminar mixed convection is greater than in forced convection. The presence of bending points in the velocity profiles and especially the origination of a return flow encourages a disturbance of the laminar flow stability and transition to turbulent flow occurs at  $Re = Re_{cr} < 2300$ . As  $Gr_q/Re$  increases, the critical Reynolds number decreases. The coexistence of laminar and turbulent flows can be observable in various tube sections at fairly large values of  $Gr_q/Re$ . In turbulent flow, under the increasing effect of thermogravitational forces, the heat transfer initially reduces, and at very large values of  $Gr_q/Re$  begins increasing as compared with the case of forced convection, since not only the average velocity profile changes but turbulent transport as well. The latter is especially noticeable at the wall, where velocity oscillations diminish sharply as a result of the flow acceleration (a flow laminarization). It is also characteristic that wall temperature nonuniformities may arise in definite modes. Notwithstanding certain advances in studying the mechanism of turbulent transport and heat transfer under these conditions, at present there are no adequate models of turbulent transport for numerical investigation of the occurring processes, and the available correlations for heat transfer [1 - 3] were obtained under the assumption of flow stabilization over the tube length, which is very rarely observed in real facilities. For this reason, a major part of the current investigations deals with experimental and numerical study of turbulent transport and heat transfer in a vertical gas-cooled tube with various heat loads and with coincident directions of the forced flow and of the vector of the thermogravitational forces, i.e. under the conditions of flow laminarization by the action of thermogravitational forces.

Specific features of the turbulent flow in channels under the effect of thermogravitational forces have an analogy with a flow in curvilinear channels in which the action of centrifugal forces manifests itself differently on convex and concave parts of the channel perimeter (suppression or stimulation of turbulence). In this connection, we consider it useful to set forth, in a single book, the results of our investigations into these, different, at first glance, flows. Generally, three flow types are differentiated according to their swirling mode:

- curved flows realized in the presence of a contiguous curved surface;



- rotating flows originating in the rotation of various sections confining the flow;
- flow with an initial swirl effected via various facilities, for example, with a tangential liquid feeding to the inlet section, in the presence of a swirler at the channel inlet or with a rotation of the inlet section, etc.

A swirling flow has some specific properties that distinguish it from rectilinear flows. In some cases, the flow swirl enables an enhancement of heat and mass transfer, therefore it is frequently employed in various engineering facilities. Therefore the questions as to the effect of flow swirl on hydrodynamics and heat transfer in channels have been treated in a number of monographs [4 - 7] and reviews [8, 9 and oth.]. Studies [5, 6] considered in detail the influence of the initial flow swirl on heat transfer and hydrodynamics for flow in tubes, annular channels and various technical devices. Study [7] placed primary emphasis on rotating flows. Study [4] allotted a significant place for the problems of heat transfer and hydraulic resistance in curved channels (a flow curving).

Studies [5, 6] mainly present an in-depth analysis of the flow hydrodynamics and average heat transfer in curved channels of circular cross section. It should be noted that fairly reliable relations for average heat transfer and hydraulic resistance in curved tubes (coils) are now available [4, 9], however the local heat transfer over the tube length and perimeter, the turbulent transport mechanism, and heat transfer and turbulent transport in noncircular curved channels need further investigations. For this reason, this monograph placed a focus on the experimental investigation of local heat transfer and local shear stress at the wall, and also on studying some turbulent characteristics of flow in circular (coils) and plane (helical channels) curved channels. Consideration is given to curved channels of rather large curvature in which, as the obtained data indicated, the transition of a laminar vortex flow to turbulent flow around the curved tube perimeter or at the concave and convex walls of a curved plane channel occurs at different Re numbers. Attention was centered on analyzing and unifying the data for laminarized flow regions, where the heat transfer coefficient is minimal.

Flow laminarization occurs not only by the action of thermogravitational or centrifugal forces. A minute review of the studies on flow relaminarization under the effect of various factors (dissipation, steady stratification, acceleration, rotation, heating, injection, suction and magnetic fields in conducting liquids) is presented in ref. [10]. There-

fore we consider only briefly some cases having a certain bearing on the problems solved in the current study. But first we discuss the general parameters governing flow in field of mass (body) forces.

### Basic parameters governing flow in a field of mass forces

The liquid flow and heat transfer in a field of mass forces are defined by the standard system of differential equations that consist of equations of momentum continuity and energy. The terms that account for mass forces enter only in the momentum equation and by acting on the velocity distribution in the field of mass forces, affect the heat transfer.

In analytical or numerical solutions of the problem the differential equations are usually written in an inertial (immovable) coordinate system. Here, mass inertial forces do not enter in the right-hand side of the momentum equation in explicit form.

When analyzing the effect of inertial mass forces on a liquid flow and heat transfer using a similarity method it is convenient to write the differential equations in a noninertial coordinate system [4]. Then, the inertial mass forces enter in the right-hand side, of the momentum equation in explicit, form, i.e. become external relative to the in question. For this end, a coordinate system in which a liquid flow is analyzed should shift so that its direction and velocity coincide with the direction and velocity of the motion that gives rise to inertial mass forces.

In the case of an incompressible liquid with constant physical properties in a field of gravitational and inertial forces, the momentum written in a noninertial coordinate system is of the form

$$\rho \frac{D\mathbf{u}}{d\tau} = \mathbf{F} - \text{grad } p + \mu \nabla^2 \mathbf{u}, \quad (1.1)$$

where  $\mathbf{u}$  is the velocity vector,  $\mathbf{F}$  is the vector of mass force per unit volume,  $\tau$  is the time,  $p$  is the pressure and  $\mu$  is the dynamic viscosity.

In the general case, the vector  $\mathbf{F}$  in Eq. (1.1) incorporates gravitational, centrifugal and Coriolis forces, and also an inertial force originating in an accelerated rectilinear motion, and  $\text{grad } p$  reflects the effect of external pressure forces and mass forces on pressure.

The mass force can be written in the form

$$F = j\rho, \quad (1.2)$$

where  $j$  is the acceleration by gravity or inertia.

The liquid flow in a uniform field of mass forces is not affected by the mass forces. Thus, gravitational forces in a uniform isothermal flow have no effect on its characteristics. With the axis  $x$  taken to be parallel to the vector  $g$ , it is possible to write

$$\rho g - \left( \frac{\partial p}{\partial x} \right)_{mf} = 0, \quad (1.3)$$

where  $(\partial p / \partial x)_{mf}$  is the pressure gradient due to the gravitational mass force.

Thus, with a uniform field of mass forces, the terms that display the effect of mass forces on flow characteristics disappear from the momentum equations (1.1). Correspondingly, for a nonisothermal liquid flow in a field of gravitational forces we have

$$\rho g - \left( \frac{\partial p}{\partial x} \right)_{mf} = g\rho - g\rho_o = F_1 - F_2 = \Delta F, \quad (1.4)$$

where  $\rho$  and  $\rho_o$  are the liquid densities at two characteristic points of the system,  $F_1$ , and  $F_2$  are the mass forces at these points, and  $\Delta F$  is the excess mass force.

Hence, with a nonuniform field of mass forces the excess mass force  $\Delta F$  enters in the right-hand side of momentum equation (1.1).

As ensues from Eq. (1.2), the inertial mass force can vary not only with density but also with the acceleration  $j$ . By analogy with the gravitational field, its effect can also be expressed with the aid of the excess mass force equal to the difference of inertial mass forces at two characteristic points of the systems.

As was shown in ref. [4], introducing the excess mass force in Eq. (1.1) results in an additional dimensionless number that reflects the effect of mass forces on the flow

$$K = \frac{l \Delta F}{\rho u^2}. \quad (1.5)$$

In the absence of forced motion, the flow velocity does not enter in the unambiguity condition. Therefore, by multiplying the  $K$  number into

$Re^2$  it is possible to obtain the most convenient parameter for practical applications

$$P = K Re^2 = \frac{l^3 \Delta F}{\rho \nu^2}, \quad (1.6)$$

where  $l$  is the characteristic dimension,  $\Delta F$  is the excess mass force,  $\rho$  is the flow density and  $\nu$  is the kinematic viscosity.

At a constant flow acceleration ( $j = \text{const}$ ), the  $P$  criterion reduces to the generalized Archimedes criterion ( $Ar$ ), and at a constant density ( $\rho = \text{const}$ ), to the  $S$  criterion [4]:

$$Ar = \frac{j l^3 \Delta \rho}{\nu^2 \rho}, \quad S = \frac{l^3 \Delta j}{\nu^2}, \quad (1.7)$$

where  $\Delta j = j_{\max} - j_{\min}$ .

With no phase transitions,  $\Delta \rho / \rho = \beta \Delta T$  and the  $Ar$  number transforms into the Grashof number ( $Gr$ )

$$Gr = \frac{g l^3}{\nu^2} \beta \Delta t, \quad (1.8)$$

defining the magnitude of the effect of thermogravitational forces resulting from the temperature difference  $\Delta T$  characteristic of the conditions considered. For each specific problem,  $\Delta T$  is related to an appropriate thermal boundary condition. When the wall temperature  $T_w$  is specified,  $\Delta T = (T_w - T_{in})$  and, accordingly,

$$Gr_{\Delta T} = g \beta d_e^3 (T_w - T_{in}) / \nu^2. \quad (1.9)$$

When the heat flux at the wall  $q_w$  is specified the temperature drop is  $\Delta T = q_w d_e / \lambda$ . In this case

$$Gr_{\Delta T} = g \beta d_e^4 q_w / \nu^2 \lambda. \quad (1.10)$$

Sometimes  $\Delta T$  is defined in terms of the longitudinal gradient of the average liquid temperature  $A = dT_f/dx$ , i.e.  $\Delta T = d_e (dT_f/dx)$ . In this case,

$$\text{Gr}_A' = g\beta d_e^4 \left( \frac{dT_f}{dx} \right) / \nu^2 = \frac{4\text{Gr}_q}{\text{Re Pr}}. \quad (1.11)$$

If  $\Delta T = r_o (dT_f/dx)$  is applied, then

$$\text{Gr}_A = g\beta d_e^4 \left( \frac{dT_f}{dx} \right) / 16\nu^2 = \frac{\text{Gr}_q}{4\text{Re Pr}}. \quad (1.12)$$

As was noted above, in studying isothermal incompressible flows, the  $P$  number, Eq. (1.6), is reduced to the  $S$  number:

$$S = \frac{l^3 \Delta j}{\nu^2}. \quad (1.13)$$

Study [4] showed that for a curvilinear channel

$$S = B^2 \frac{l^2 \bar{u}^2}{R\nu^2}, \quad (1.14)$$

since the variation in the inertial acceleration within the confines of a cross section is  $\Delta j = u_{\max}^2/R$ . Here,  $B$  is the ratio of maximum and mean mass velocities of the liquid motion, and  $R$  is the curvature radius of the channel.

If the distance between the points of maximum and minimum mass forces is taken as the governing dimension, then the governing dimension for a circular tube is equal to a half-diameter. For a plane curved channel ( $b \gg h$ ), the governing dimension is equal to half of a distance between curvilinear surfaces ( $h/2$ ). Thus, for a circular curved tube we have [4]

$$S = \frac{B^2}{4} \frac{d}{D} \text{Re}^2, \quad (1.15)$$

where  $D$  is the mean curvature diameter of the channel.

For laminar flow in straight tubes  $B = 2$ . Therefore, in circular channels [4],

$$S = \frac{d}{D} \text{Re}^2. \quad (1.16)$$

In studying a laminar liquid flow in curvilinear channels, Dean [127] demonstrated theoretically that, with the same pressure gradient, the ratio of mass flow rates in two different-curvature tubes is defined by the dimensionless relation

$$\text{De} = \text{Re} \sqrt{d/D}, \quad (1.17)$$

later referred to as the Dean number (De).

Consequently, there is the following relationship between  $S$  and  $\text{De}$  [4]:

$$S = \text{De}^2. \quad (1.18)$$

Thus, in studying isothermal laminar flow in a curvilinear channel, the effect of mass forces can be accounted for by the  $S$  or  $\text{De}$  numbers definitively related to one another.

As ref. [4] demonstrated, before turbulence initiation, the  $\text{De}$  number unambiguously determines the change in the hydraulic resistance and heat transfer in curvilinear channels over a wide range of channel curvatures, in comparison with a straight channel. Only with a very large channel curvature these features are governed by the  $\text{Re}$  and  $\text{De}$  numbers (or  $d/D$ ). Because, in turbulent flow, the ratio of max and mean mass velocities in a straight channel is not a constant, the  $\text{De}$  number for turbulent flow does not unambiguously determine the effect of the field of mass forces in a curvilinear channel. Under these conditions, the effect of mass forces on the flow depends on  $\text{Re}$  and  $\text{De}$ , or on  $\text{Re}$  and  $d/D$  separately.

The effect of mass forces on the flow development near an infinite curvilinear plate is expressed by the Görtler (Gö) - number, which is analogous to the Dean number,

$$\text{Gö} = \frac{u_\infty \delta}{\nu} \sqrt{\delta/R} \quad (1.19)$$

or

$$Gö = \frac{u_{\infty} \Theta}{\nu} \sqrt{\Theta / R}, \quad (1.20)$$

where  $\delta$  is the boundary layer thickness,  $\Theta$  is the momentum thickness,  $u_{\infty}$  is the flow velocity outside the boundary layer and  $R$  is the surface curvature radius. This dimensionless relation was introduced by Görtler when studying the stability of a boundary layer on a curved surface.

### Laminarization due to dissipation

If Re number in a divergent channel decreases from  $Re_1$  at the section upstream of the divergence region to  $Re_2$  corresponding to the downstream zone, and  $Re_1 > Re_{cr}$  and  $Re_2 < Re_{cr}$  (where  $Re_{cr}$  is the pertinent critical value), the turbulent flow can be anticipated to pass into a laminar state. A flow of this type has some distinctive features. Experiments manifest [10] that the skin friction attains a value corresponding to that for laminar flow long before the velocity distribution in the channel center reaches a magnitude corresponding to the well-known parabolic profile. That is, the transition to laminar flow at the wall is relatively rapid, and the processes in the external layer proceed much more slowly. Also, with distance, the longitudinal ( $u'$ ) and transverse ( $v'$ ) velocity oscillations damp exponentially, with  $v'$  damping more quickly than  $u'$ . Thus, the damping turbulent flow proves to be strongly anisotropic. The Reynolds shear stresses decrease with respect to  $x/d$  nearly linearly. The correlation coefficient of the Reynolds stresses also diminishes, which is indicative of the attenuation of linear effects and of the breakdown of the coherent motion in the boundary layer.

### Flow laminarization due to acceleration

The problems of heat transfer and turbulent transport in a boundary layer under the conditions of a negative pressure gradient ( $dp/dx < 0$ ) are dealt with in many studies. Their review and systematization can be found in refs. [10, 11].

The pressure gradient appreciably alters the momentum and heat transfer in comparison with a nongradient flow. Under the action of the negative pressure gradient, initially the flow core gets strained, and thereafter the velocity profile experiences changes in the logarithmic region as well. Here the viscous sublayer becomes thicker. Subsequently, the

shape of the velocity profile approaches that in a laminar flow. According to refs. [12 - 14], a flow laminarization becomes distinct at  $K = (v/u_\infty^3) \cdot (du_\infty/dx) \approx 3.5 \cdot 10^{-6}$ ,  $\Delta p = (v/\rho u_*^3)(dp/dx) = -0.0275$  or  $\Delta \tau = (v/\rho u_*^3) \cdot (\partial \tau / \partial y) \approx -0.009$ .

When  $dp/dx < 0$  the temperature profile deviates upward from standard relationships, indicates a growth of the heat transfer resistance along the entire acceleration region. As the results of study [11] demonstrate, the dynamics of the magnitudes of the longitudinal velocity oscillations are strongly dependent on  $K$ . When  $K < 3.5 \cdot 10^{-6}$ , i.e. prior to the onset of a pronounced laminarization, these oscillations monotonically increase with flow acceleration in the wall region of the boundary layer and remain nearly unchangeable away from it. With higher acceleration rates, the absolute values of the longitudinal velocity oscillations increase insignificantly in the wall region, and decrease markedly in the external region, i.e. signs of flow laminarization appear. The correlation coefficient of the Reynolds stresses on the greater part of the boundary layer remains with a constant value of the order of 0.5 [10], i.e. the fact that there is any considerable decorrelation between the velocity oscillation components  $u'$  and  $v'$  in accelerating flows is not confirmed.

Study [15] indicated that, with the flow acceleration, the complex  $[\overline{u'v'}(\partial u / \partial y - (\overline{u'^2} - \overline{v'^2})(\partial u / \partial x))] / \overline{u'v'}(\partial u / \partial y)$ , characterizing turbulence generation in the boundary layer decreases noticeably and even becomes negative when  $y/\delta > 0.2$ , which results in flow laminarization. This is manifested most clearly in the external region of the boundary layer, where a vortex stretching occurs under the effect of  $dp/dx < 0$  [11], which adds to the flow anisotropy and enhances the role of normal stresses. In the wall region, the principal role in turbulence generation is retained by the term  $\overline{u'v'}(du/dy)$  which transfers the turbulence energy to the longitudinal component  $u$ , and turbulence is redistributed between the velocity components  $v$  and  $w$  only via the "mixing" action of pressure fluctuations.

In studying, the intermittence of the velocity oscillations, ref. [16] advanced a hypothesis that a new external region is formed in the boundary layer when  $dp/dx < 0$ . Study [11] notes that on the interface of the wall and external regions of the boundary layer there are structures of small longitudinal vortices which ensure a transfer of large temperature pulses to the wall. This is verified by measurements of the excess coefficient of temperature fluctuations which evidently increase in the re-



gion  $40 < y^+ < 200$ , attesting to an increased likelihood of a small-scale vortex structure. The formation of such a structure is responsible for a reduction in the heat transfer efficiency and a displacement of the maximum of temperature fluctuations through an ever larger distance from the wall, whereas the location of the zone of velocity oscillation generation remains almost unchanged.

Study [11] demonstrated that, when  $dp/dx < 0$ , the value of  $Pr_t$  varies dramatically across the boundary layer thickness, with two pronounced peaks occurring and generally  $Pr_t$  is much larger than in a nongradient flow.

It should be pointed out that in highly accelerated flows, a reverse transition occurs gradually and asymptotically rather than stepwise, but markedly more rapidly than does a dissipative transition.

At a high heating intensity, the flow also becomes highly accelerated, which may result in its laminarization. This trend is discussed in detail in ref. [17]. The flow heating and acceleration are defined using two dimensionless parameters, viz. the heat flux parameter  $q^+$  and the acceleration parameter  $K'$ :

$$q_{in}^+ = \frac{q_w}{\rho u c_p T_{in}} = \frac{1}{4} \frac{1}{T_{in}} \frac{\partial T_f}{\partial (x/d)}, \quad (1.21)$$

$$K' = \frac{4q_f^+}{Re}, \quad (1.22)$$

$$q_f^+ = \frac{q_w}{\rho u c_p T_f} = \frac{1}{4} \frac{1}{T_f} \frac{\partial T_f}{\partial (x/d)}, \quad (1.23)$$

The parameters  $q_{in}^+$  and  $q_f^+$  are proportional to the relative rate of flow temperature variations over the tube length and, in the case of an ideal gas and a negligible acceleration due to the lengthwise pressure drop caused by friction, they are proportional to the relative flow acceleration, since

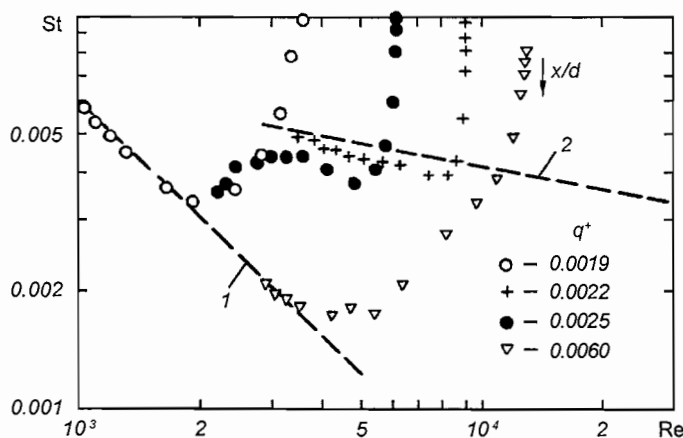
$$\frac{1}{T_f} \frac{\partial T_f}{\partial (x/d)} \approx \frac{1}{4} \frac{\partial u}{\partial (x/d)}. \quad (1.24)$$

Thus, under these conditions the parameter  $K'$  is identical to the acceleration parameter in external boundary layers  $K = \nu/u_\infty^2(du_\infty/dx)$ .

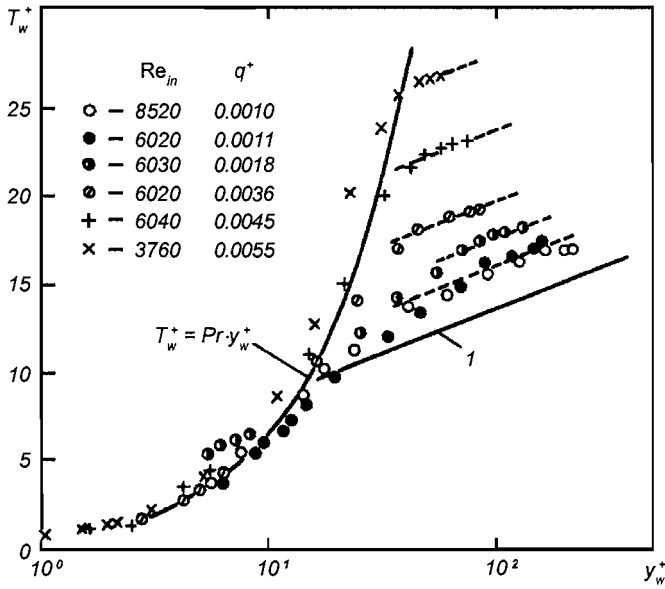
Therefore, the numerical value of  $K'$  at which a flow laminarization can be anticipated is also equal to about  $3 \cdot 10^{-6}$ .

The data on local heat transfer for various heat loads (Fig. 1.1), presented in ref. [18], indicate its obvious reduction at  $q_{in}^+ = 0.006$ . In this case, heat transfer at the tube end corresponds to that in laminar flow, although here  $Re \approx 3000$ . However, a deviation of heat transfer from the relationship characteristic of a turbulent flow is observable even for  $Re \approx 10^4$ .

Since experimental studies of turbulent transfer at large heat loads are complicated, no detailed data on turbulent flow under these conditions are available. However, temperature profiles measured in ref. [18] for a quasi-steady tube flow with various heat loads (Fig. 1.2) indicate that in this case, as in external boundary layers with a flow acceleration, an increase in the heat load causes the thickness of a viscous sublayer to increase monotonically and the zone with a logarithmic temperature profile to narrow. Also, the velocity profile deviates monotonically upward from the relationship characteristic of constant physical properties of the fluid [19].



**FIGURE 1.1.** Heat transfer in a tube at various heat loads [17]: 1-2 - stabilized laminar and turbulent heat transfer at constant physical properties of the fluid respectively



**FIGURE 1.2.** Temperature profiles for quasi-steady flow in a tube [18]; 1 - at constant physical properties of air [19]

Based on the experimental data obtained, study [96] proposed the following relations for predicting the onset

$$q_{in}^+ = 4.94 \cdot 10^{-3} Re_{in}^{0.05} [1 - (Re_{cr2}/Re_{in})^{0.75}] \quad (1.25)$$

and the termination

$$q_{in}^+ = 5.87 \cdot 10^{-3} Re_{in}^{0.05} [1 - (Re_{cr1}/Re_{in})^{0.75}] \quad (1.26)$$

of flow laminarization at large values of  $x/d$ . The onset of the flow laminarization is associated with the origination of an intermittent flow, and the termination, with a total disappearance of the turbulent pulsations. Quite logically, the reverse transition is also linked to the direct transition in isothermal conditions ( $Re_{cr1}$  and  $Re_{cr2}$ ).

### The effects of stable and unstable density stratifications

A stable stratification can be observed in the atmosphere or modeled in a plane channel with a heated upper side. The presence of a more lightweight liquid in the upper part of the volume implies that the ascending liquid should do work against gravitational forces, and thus the turbulent energy can be converted to a potential energy of gravity. Stratification effects were analyzed thoroughly in refs. [10, 20]. Recent studies in horizontal channels with stable (heating from above) and unstable (heating from below) systematized density stratifications in refs. [1, 21]. Study [1] shows that with an insignificant effect of thermogravitational forces, the velocity and temperature profiles in the universal coordinates  $u^+ = f(y^+)$ ,  $T^+ = f(y^+)$  are strained only at the flow axis, with the profiles  $u^+(y^+)$ ,  $T^+(y^+)$  for a stable stratification departing upward, and for an unstable stratification, downward from the laws of velocity and temperature variations with a neutral stratification (forced convection). As the influence of thermogravitational forces increases, the velocity and temperature profiles are disturbed even more, and the influence region is located closer to the wall. The distance from the wall, over which the buoyancy effect on turbulent characteristics can be neglected, was referred to as "a thickness of the dynamic turbulence sublayer" ( $\delta_{MO} = -u_*^3 \kappa g \beta (q_w / \rho c_p)$ ). Here, the minus sign corresponds to the heat flux direction at which the magnitude of  $\delta_{MO}$  is positive (in the case of a stable density stratification). However, this magnitude is established to be accurate up to a constant determined experimentally, i.e.  $\delta_{MO}^q = K \delta_{MO}$ . The results of measurements in the atmosphere produce the value of  $K \approx 0.01$  and channel,  $K \approx 0.03$  [1].

A stable density stratification results in the damping of the turbulent transport processes, and an unstable density stratification, in their enhancement.

With a stable stratification, turbulent transport at the wall (when  $y^+ < 50$ ) becomes commensurate with molecular, transport and the viscous sublayer thickness. Under these conditions, the turbulent heat transfer attenuates more rapidly than the momentum transfer. A variation in the turbulent Prandtl number ( $Pr_t$ ) with the dimensionless coordinate  $y^+$  is largely dependent on density stratification conditions [1]. With neutral and unstable density stratifications, the  $Pr_t$  number varies slightly across the boundary layer thickness, from a value close to unity near the wall to 0.7 - 0.8 in the flow core. With a stable density stratification there is a pronounced maximum at  $y^+ \approx 60$ , where  $Pr_t \approx 3.5$ . With an

increase in the effect of an unstable density stratification,  $Pr_t$  decreases somewhat and in the case of a stable density stratification noticeably increases. In the opinion of the authors of study [1], this is related to the fact that, with a strong stable density stratification, turbulent transport has the form of random internal waves which transfer momentum via pressure forces but do not, actually, transfer heat.

### **The effect of the wall curvature on the boundary layer flow**

Study [22] reviewed in detail early investigations (conducted before 1973) of the flow curving effect on turbulent flow. The parameter  $\delta/R$  was taken as the measure of curvature, where  $\delta$  is the boundary layer thickness and  $R$  is the curvature radius assumed to be positive for convex surfaces and negative for concave surfaces. In such a flow there is a normal pressure gradient which strongly affects stability of the boundary layer flow. At a convex wall, the pressure increases in the direction of the boundary layer edge, therefore the liquid discharges traveling from the external part of the boundary layer to the wall with a higher velocity and, hence, a higher centrifugal acceleration overcomes additional obstacles while moving against this acceleration. Conversely, slow turbulent splashes at the wall are affected by a lower centrifugal acceleration than that for faster external layers, which hinders their movement from the wall. Therefore, there is a tendency toward turbulence suppression. For flow in concave boundary layers, the pattern is reversed the pressure increases toward the wall and therefore destabilizes the boundary layer flow. As a consequence, centrifugal forces in the boundary layers on convex walls increase the stability limit and on concave walls reduce it significantly. In the latter case, the disturbances have the form of vortices with axes parallel to the main flow. They are named Taylor-Görtler vortices. In the damping of turbulent transport on convex boundary layers the vortex scale is smaller than on a plane boundary layer, whereas on concave boundary layers it is larger.

Study [23] drew on the analogy between the layer curvature and buoyancy effects. It was demonstrated that in the case of an insignificant influence of buoyancy, the Monin-Obukhov equation for mixing length

$$l/l_o = 1 - BRi \quad (1.27)$$

(where  $B$  is a constant and  $Ri = (-g/\rho)(dp/dy)(du/dy)_w$  is the Richardson number) can be used successfully for modeling slight-curvature effects,

if  $Ri = (2u/R)(\partial u/\partial y)$ . In this case, predictions of curved flows can employ a numerical value of the constant  $B$  determined from meteorological experiments under conditions of stable and unstable density stratifications. Even the first studies [22, 23] revealed that the boundary layer on a concave wall grows much more rapidly than on a plane wall, and on a convex wall, vice versa. The friction stress on a concave wall increases in comparison with that on a plane wall, and decreases on a convex wall. The average velocity profile in the immediate vicinity of the wall obeys the standard wall law, whereas at a distance from it, in the case of a convex wall it diverges upward and in the case of a concave wall, downward from that for a plane wall. The location of a point at which the velocity profiles fall off from the wall law depends on the curvature parameter  $\delta/R$ .

The wall curvature also affects the characteristics of turbulent transport, its influence being significant in large-curvature boundary layers. Thus, according to the data of ref. [24], the velocity oscillations and the Reynolds stresses on a concave wall with  $\delta/R \approx 0.1$  are twice as great as those on a plane wall. On a large-curvature convex wall [25, 26], the Reynolds stresses in the external part of the boundary layer approach zero or even become negative. With distance from the wall, the ratio of the Reynolds stresses and the turbulent kinetic energy  $-u'v'/E^2$  sharply decrease across the boundary layer thickness. This indicates a decorrelation between the velocity oscillations, i.e. a breakdown of the coherent structures. In small-curvature boundary layers ( $\delta/R \approx 0.01$ ) these variations are insignificant and involve the external region of the boundary layer.

As was previously noted, instability of the laminar flow over a concave wall gives rise to Taylor-Görtler vortices. Many researchers remark that such longitudinal vortices exist also in turbulent flow [24, 27, 28 and oth.]. These vortices cause the boundary layer thickness and the friction stress to alter in the lateral direction. The boundary layer thickness is largest when the flow from the wall takes place between the vortices and the wall friction is minimal. These large longitudinal structures affect also the external part of the boundary layer. Following recent studies [29], an increase in the Gö number changes the longitudinal structures and hence, the entire boundary layer. When analyzing local mass transfer, study [30] established the existence of a border (along the wall length) behind which the longitudinal vortex structures disappear, even though they existed ahead of it. The border position is depen-

dent on characteristics of the outflow (of the agitating grid). Studies of heat transfer on curved surfaces [31-33] demonstrate that its variation resembles that of wall friction, i.e. it increases on a concave wall and decreases on a convex wall.

As this brief overview of investigations of flow laminarization under various conditions reveals, in all cases turbulent transport changes dramatically compared to a typical turbulent boundary layer. These changes in various flow conditions differ both in magnitude and in their effect on individual zones of the boundary layer.

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