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K.G. Terry Hollands Thermal Radiation Fundamentals

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THERMAL RADIATION FUNDAMENTALS

by

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Acknowledgements

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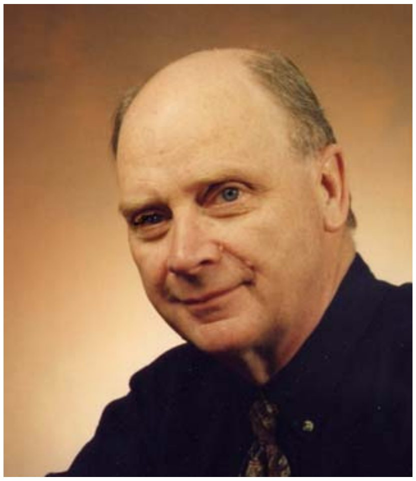
Finally, my heartfelt thanks are given to my wife Mara for her generous support and encouragement of this project, which mirrored how she has supported all my projects over the years. This book is dedicated to her.

About the Author

A graduate of the University of Toronto (BASc, 1959) and McGill University (PhD, 1967) Terry Hollands worked for several years in the solar energy labs of the Australian government scientific agency (CSIRO) before joining the University of Waterloo in 1969 where he stayed until his retirement in 2001. He is now a Distinguished Professor Emeritus at the University of Waterloo. He has held executive positions in a number of societies, including the International Solar Energy Society for which he has served as Board Member and Editor-in-Chief of its Journal, *Solar Energy* (1994-1999). A Fellow of both the Canadian Society for Mechanical Engineering (CSME) and the American Society of Mechanical Engineers (ASME), Dr Hollands has won several awards, including the Jules Stachiewicz Medal of the CSME (1997), the Best Paper Award of the Heat Transfer Division of the ASME (1989), the Special Service Award of the International Solar Energy Society, and the Chandrashekar life time service award of the Solar Energy Society of Canada. He has pioneered in several research areas, including the honeycomb transparent insulation, the micro-flow strategy for solar water-heating, the synthetic generation of solar radiation sequences, the conduction layer method for solving natural convection problems, the hybrid technique for natural convection heat transfer measurements, and the smoothed band model for the radiation properties of gases.



K.G.T. Holland

Contents

PREFACE

This book is intended as a textbook for a graduate-level course for students in engineering. It is hoped that it will also find use as a reference text for practicing thermal analysts and designers of heat transfer equipment. The book is an accumulation of my lecture notes from thirty years of regularly teaching a thermal radiation course at the University of Waterloo. It also draws on a somewhat longer experience of research in heat transfer and solar energy applications.

During the early part of that lecturing experience, I had followed the approach used in several popular texts. Despite the excellence of these texts, I was never fully satisfied with their approach, which I will call here the traditional treatment. The traditional treatment divides radiation into two separate parts: the first treats radiation as a surface phenomenon, and the second (dealing with participating media) treats radiation as a volumetric phenomenon, which it truly is. This meant that the fundamental equation of radiative transfer—the Radiative Transfer Equation or RTE—was not introduced until more than halfway through the course, when the volumetric treatment is undertaken.

In the latter part of my lecturing experience, I experimented with a more holistic approach, wherein the volumetric treatment and RTE are introduced at the beginning of the course, and the treatment of radiation as a surface phenomenon is developed later as a special case. I found this approach gave the students a more fundamental understanding, and also that it was a more efficient, in that more material could be covered in one course. Moreover, given that the students had been exposed to the traditional treatment at the undergraduate level, they found the holistic approach at the graduate level gave a refreshing contrast—an altered perspective that made the subject more interesting.

It is admitted that this approach may not be suitable for all. The students must spend longer on fundamentals before making engineering heat transfer calculations. Some students prefer the topics to be more concrete and directly relevant. Professors who are especially concerned for these students, or whose natural proclivities are also along these lines, will want to continue to use the traditional approach. But for those professors who prefer the advantages of the holistic approach, this book should provide a useful text.

There are several other novel aspects to the text, including the manner of handling radiant calculations. In times past, no textbook formula designed for routine problem-solving would ever contain a definite integral. But now every graduate student and serious thermal analyst has a personal computer, and moreover, software packages, such as Mathcad® and Mathematica®, are readily available—many universities have site licenses for one or both of these packages. So now textbook formulas can realistically contain definite integrals (as well as the inverse of high-order matrices) because they are easy to set up and can be quickly evaluated numerically. This new practice is adopted in this text, wherever applicable. More on this topic is given in the Introduction.

Principal among these numerical integrations is spectral integration, whether over surface properties or over the complex gaseous radiation properties. But just as important is the area integration associated with the form factor. In this book, the well-known formulas for form factors are supplemented by numerical methods of double or quadruple integration, using the parametric representation of surfaces. The principle of parametric surface representation is explained in an Appendix (Appendix A), which also gives a catalog of parametric representations suitable for quick entry into Mathcad. With this data, form factors between more than 100 combinations of pairs of surface-types can be readily calculated, regardless of the orientation and placement of the surfaces with respect to each other. Although the actual calculations take a few minutes for the computer to complete, the computational time is still fast enough for course-work problem-solving.

A set of problems is included at the end of each chapter. Some of the problems require the use of Mathcad, and the wording of the problems has been designed to make it easy to spot the ones that will require Mathcad or a similar software package to complete.

The recent "smoothed-band" model of gaseous radiation and the availability of Mathcad and similar packages has enabled the isothermal gas enclosure to be handled in a relatively simple way—without the band-energy approximation and the mean beam length approximation. This is expounded on in Chapters 20 and 22.

It is not possible to do a proper coverage of radiation without getting into some aspects of quantum mechanical theory and electromagnetic wave theory. Whereas the other modes of heat transfer draw only on classical (prequantum) physics, the same is not so true of radiation.[1] I have strived to cover enough quantum mechanical and electromagnetic wave theory to meet the needs of the student.

Experience has shown that the material covered in this book can be completely covered in about 39 lectures, which is the length of an academic term at the University of Waterloo. Nevertheless, if a shorter time is available, some sections can be cut without losing the grand sweep of the material. Supplementary material, say on current research topics, such as the incorporation of radiative effects into the current CFD codes, can be added by the instructor

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[1] Thermal radiation is very much bound up with the history of modern physics. It had no adequate explanation in classical physics. Planck's derivation of his Radiation Law was the first break from classical physics and the start of quantum mechanics. Electomagnetic wave theory, while older and not so revolutionary as quantum physics, was a necessary prerequisite to a fundamental understanding of thermal radiation.

when additional time is available or when an enriched treatment is sought. No material has been included on the important area of radiation measurements. At Waterloo, a separate laboratory exercise accompanied the lectures.

My own experience has been that the study of thermal radiation not only provides the wherewithal to solve engineering heat transfer problems—important as that is. It also provides a new and exciting way of looking at the world around us. For example,it exposes us to some revolutionary branches of physics, and it explains how the light from objects around us actually come to our eyes. I have attempted to capture some of that excitement in this book.

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INTRODUCTION

In learning thermal radiation, we need to master a few differential equations expressing the appropriate conservation laws together with the appropriate boundary conditions. In this way, learning thermal radiation is similar to learning heat conduction and convection. But there are several important variations that make radiation decidedly different. Understanding the general nature of these differences should help in the total learning process, and so we will highlight them in what follows.

The fundamental differential equation describing the conservation of the radiant energy in a specified direction, say $\hat{\mathbf{d}}$, is the Radiative Transfer Equation, which for a gray nonscattering medium can be written as

$$\vec{\nabla} \cdot \mathbf{i} = -a \left(i - \sigma T^4/\pi \right) \tag{1}$$

where $\mathbf{i}$ is the intensity vector for radiation in direction $\hat{\mathbf{d}}$, which is a measure of the strength of that radiation, i is the magnitude of $\mathbf{i}$, T is the local temperature, and a is a property for the medium and σ is a constant[2]. If we were to compare this with another conservation equation, say that for the conservation of heat in a stationary medium at steady-state

$$\vec{\nabla} \cdot \mathbf{q} = q''' \tag{2}$$

or the conservation of mass for a flowing fluid:

$$\vec{\nabla} \cdot (\rho \mathbf{V}) = -\frac{\partial \rho}{\partial t} \tag{3}$$

we immediately see some similarities, particularly in the structure of the left-hand side. The important difference is not so much the details of the equations, but the fact that Eq. (1) must be written an infinite number of times, once for each possible direction $\hat{\mathbf{d}}$ in space, whereas Eqs. (3) and (2) need be written only once.[3] Moreover, because very few media are gray, in practice, an equation

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[2] We use a slightly different nomenclature for intensity in this Introduction from that used in the main body.

[3] Why do not conduction and convection also require conservation equations in every direction? After all, the medium for Eqs. 3 and 2 may be a gas, and gas molecules travel in all directions in space. The answer is that because a gaseous molecule travels very short paths before colliding with a neighbor, the overall effect of many, many molecules can be captured by a single vector equation like $\mathbf{q} = -k\vec{\nabla} T$.

like Eq. (1) will have to be written an infinite number of times, once for each wavelength, there being a different intensity and value of a at every wavelength. So in a sense there is a double infinity of governing equations, and this is the main thing that makes radiation different. The saving feature of radiation, at least for a nonscattering medium, is that for any one combination of wavelength and direction, the differential equation is relatively easy to solve: it readily transforms into a first-order, linear, ordinary differential equation, the general solution to which can be written down at once.

Another difference is in the boundary condition for Eq. (1), which, as in conduction, is prescribed at an interface between adjacent media. The boundary condition for the intensity in one direction $\hat{\mathbf{d}}$ depends on the intensity in all the other (incident) directions. This means that it also depends on the intensity at all other points on the interface, whereas that for Eqs. (3) or (2) (say zero velocity or a specified heat flux) is independent of what is going on elsewhere on the interface. The net result is that the complete statement of the radiative boundary conditions, written over the entire interface, reduces to an integral equation. Solving this integral equation is one of the more daunting tasks in radiant analysis. (One approximate scheme for solving these integral equations leads to the well-known "form factors.")

Once the differential equations of a conduction or convection problem have been solved, the entire solution is more or less complete—although sometimes one wants to integrate the solution over a bounding area to get the heat flow over a surface. But once a *radiation* problem has been solved for the intensity field, there is always the need for further integration because the intensity itself is not normally of engineering interest. Normally one wants to know the heat flow over a surface, and this requires integration over directions and wavelength, as well as over area. For this reason, multivariable integration plays a major role in radiant analysis.

All of the above assumes that the temperature field is given [notice the appearance of temperature in Eq. (1)], but often one does not know the temperature field beforehand. Rather radiation itself plays an important role in fixing the temperature in the medium. Clearly one needs an additional equation, and this is the familiar Energy Equation for the medium, which must now be generalized to include a new term, one representing the local rate, per unit volume, at which energy is taken out of the radiant field, a quantity that can be positive or negative. The Energy Equation and the Radiative Transfer Equation must be solved simultaneously to establish the temperature field and the intensity field.

Perhaps some comparisons with steady-state conduction will make the above clear. Finding the intensity corresponding to a given temperature distribution is like using Fourier's law: $\mathbf{q} = -k\vec{\nabla}T$ to find the conductive heat flux corresponding to a given a temperature distribution. We note, however, that in conduction, it is the conduction process itself that will fix the temperature distribution, so Fourier's Law is of little use unless it is combined with the Energy Equation, namely Eq. (2). These two equations must be solved simultaneously. It happens, however, that for pure conduction, one equation can be substituted directly into the other to reduce the pair to a single equation: thus $\mathbf{q} = -k\vec{\nabla}T$ substituted into $\vec{\nabla}\cdot\mathbf{q} = q'''$ gives $\nabla^2 T = -q'''/k$, the familiar partial differential equation that is solved routinely in conduction analysis. Such simple substitution and reduction are not normally possible for radiation; nevertheless, the underlying structure is the same.

Because of the complexity of the general case, it is in fact very rare to solve a radiant problem exactly, and there is a heavy reliance on models that simplify the problem and admit tractable solutions. One such model—one with which the student should be familiar, as it is treated in undergraduate texts—is the enclosure containing a transparent medium bounded by a diffuse opaque surface. In fact, there are three models at work here: the "diffuse surface," for which incident radiation is reflected uniformly in all directions; the "transparent medium," for which $a = 0$ in Eq. (1); and finally the "opaque surface," for which the radiant term in the Energy Equation is zero except in a very small volume so close to the surface that for all practical purposes, the radiant exchange happens at the surface itself.

This text may breaks into three principal parts. The first, Chapter 1 to 8, is about the Radiative Transfer Equation: its derivation, some solutions for presumed internal and boundary conditions, and some examples of integrating these solutions over direction and wavelength. This part also includes chapters characterizing scattering and radiation's role in the Energy Equation for the medium.

The second part, Chapter 9 to 14, is mainly about how we determine the boundary conditions to the Radiative Transfer Equation, so it deals with the radiant properties of interfaces and surfaces, and with surface models. When an interface is perfectly smooth and the material on each side is homogeneous, the properties of surfaces can be treated exactly by electromagnetic wave theory, and the second part presents a precis of that theory.

The third part, the remainder of the book, is about solving the Radiative Transfer Equation for a prescribed set of boundary conditions. The first chapters of this part treat transparent enclosures, a situation that leads to an integral equation, which is solved to various levels of approximation. Next the assumption that the gas is transparent (but not the assumption that the gas is nonscattering) is relaxed, and the associated chapters give a model for the radiant properties of the gas, as well as an exposition of the solution methodology for isothermal gases. The final chapter treats some simply defined situations where the Radiative Transfer Equation may be solved in concert with the Energy Equation for a stationary medium. Some of these solutions allow for a scattering medium.

As has been mentioned in the Preface, many problems are solved using the software package Mathcad, and where that is done, a hardcopy of the Mathcad worksheet is included as a figure. Also, the CD in the back of the book includes an Evaluation Version of Mathcad® 11, Single User Edition, which is reproduced with permission[4]. This software is a fully functional trial of Math-

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[4] Mathcad and Mathsoft are registered trademarks of Mathsoft Engineering and Education,

cad which will expire 120 days from installation. For technical support, more information about purchasing Mathcad, or upgrading from previous editions, see http://www.mathcad.com. The CD also contains the computer files of the Mathcad worksheets that were included in hardcopy in the figures.

The book uses a slightly unorthodox referencing system that needs to be explained. The section "References, Bibliography, and Further Reading" at the back of the book lists many important literature sources. Some of these (but not all) are cited in the main section of the book, with the author, title, and year being mentioned at the point of citation if the citation is to a book and the author, journal, and year if it is to a paper. A more complete reference for these brief citations can be quickly found by searching through the References, Bibliography, and Further Reading Section.

Students are encouraged to review the radiation chapters in their undergraduate heat transfer textbook before starting on this graduate text. While the present text does not require any previous exposure to thermal radiation, the different perspective of the undergraduate text will help to set the framework for the present treatment, which takes a more fundamental approach. It is hoped that it will answer any fundamental questions encountered on reading the undergraduate text, although it will probably raise a few more, as there is much to learn about radiation.

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SYMBOLS

A	surface area
$A_{n\rightarrow m}$	Einstein Coefficient of spontaneous emission
$\bar{A}_b(s)$	effective bandwidth
a	mean absorption coefficient; gray-medium absorption coefficient
a_λ, a_η	absorption coefficient
a^*_η	dimensionless absorption coefficient
B	total number of V-R bands for a given gas or gas mixture
$B_{m\rightarrow n}$	Einstein coefficient of absorption
$B_{n\rightarrow m}$	Einstein coefficient of stimulated emission
$B_k\left(\mathbf{u}_k\right)$	quantity defined by Eqs. (19.8) - (19.9)
$B_{k-j}(n)$	quantity defined by Eq. (22.33)
b	integer indicating to which V-R bands reference is being made
C	constant; also, symbol for a curve in the plane
C_s	scattering cross section of a particle
C_a	absorption cross section of a particle
c	speed of electromagnetic wave
c_o	speed of electromagnetic waves in free space
c_f	factor correcting the geometric mean beam length
D	diameter, molecular diameter, plane layer thickness
$\hat{\mathbf{d}}$	unit vector indicating a particular direction in space
d	number of diffuse surfaces in a specular enclosure
$\mathbf{E}$	electric field intensity
E_x, E_y, E_z	components of $\mathbf{E}$
E_0	amplitude of a sinusoidal electric field
E	energy of a photon
E_{nm}	$e_n - e_m$
$E_\perp, E_\parallel$	components of $\mathbf{E}$ perpendicular and parallel to plane of incidence
E_g	semiconductor's energy gap
E_l	photon energy associated with the l^{th} line of a band structure
$E_n(x)$	exponential integral function
e	energy of a fundamental particle
e_m	energy of the m^{th} discrete energy level available to a particle
e_i	energy level associated with the i^{th} quantum state of a system
e_λ	emissive power
$e_{\lambda b}\ e_{\eta b}$	blackbody emissive power
e	total emissive power

e	total blackbody emissive power
$e_{\lambda bk}$, $e_{\eta bk}$	$e_{\lambda b}$, $e_{\eta b}$ evaluated at temperature T_k
$\hat{\mathbf{e}}$	unit vector along a line joining two points on an enclosure surface
$\mathbf{e}_{\lambda b}$	blackbody vector; its elements are the set of $e_{\lambda bk}$'s
$F_{d\mathrm{i}-j}$	point form factor from elemental area di to surface j
F_{i-j}	form factor from surface i to surface j
$F_{0-x}(x)$	(universal) fractional blackbody energy function
$\mathbf{F}$	form factor matrix with elements F_{i-j}
F^s_{k-j}	specular form factor from surface k to surface j
$\mathbf{F}^s$	specular form factor matrix with elements F^s_{k-j}
$\Im_{\lambda k-j}$	exchange factor from surface k to surface j
$\overline{\Im}_{k-j}\,(T)$	total exchange factor from k to j, for temperature T
$\overrightarrow{\Im}_{\lambda}$	exchange factor matrix
$\Im^s_{k-j}$	specular enclosure exchange factor from k to j
$\overrightarrow{\Im}^{s}$	specular enclosure exchange factor matrix
$\overrightarrow{\Im}(a_\eta)$	gaseous exchange factor matrix
$f_{0-\lambda}\,(T)$	fraction of $e_{\lambda b}$ at temperature T with wavelength $\leq \lambda$
$f_{\lambda_1-\lambda_2}\,(T)$	fraction of $e_{\lambda b}$ at temperature T with wavelength between λ_1 and λ_2
f_v	volume fraction of a sooty gas occupied by soot particles
$G_{d1-j}(x)$	gaseous point form factor (function) from area di to surface j
$G_{k-j}(x)$	gaseous form factor (function) from surface k to surface j
G_{sc}	solar constant
$\mathbf{G}(x)$	matrix of gaseous form factor functions
$\mathbf{H}$	magnetic field intensity
H_x, H_y, H_z	components of $\mathbf{H}$
H_0	amplitude of a sinusoidal variation in $\mathbf{H}$
H_n, H_k	hemispherical solid angle bisected by $\hat{\mathbf{n}}$ or $\hat{\mathbf{k}}$
h_P	Planck's constant
h_c	convective heat transfer coefficient
h_{rk-j}	radiative heat transfer coefficient between surfaces k and j
$\mathbf{I}$	identity matrix
I	number of image surfaces in a specular enclosure
i	quantum state number
i'_λ, i'_η	intensity
$i'_{\lambda b}$, $i'_{\eta b}$	blackbody intensity
$i'_{\lambda bn}$	blackbody intensity inside medium of index of refraction n
i', i'_b	total intensity, total blackbody intensity
$\widehat{\mathbf{i}}$	unit vector along x-axis
$J\,(u,v)$	surface factor for a parametric surface, $= \lvert\mathbf{J}(u,v)\rvert$
$\mathbf{J}(u,v)$	surface normal for a parametric surface

j	integer representing a particular enclosure surface
$\widehat{\mathbf{j}}$	unit vector along y-axis
$K_{x\lambda}$	extinction coefficient
$K(...)$	kernel of an integral equation
K, K_D	optical depths for gray medium: $K = ax$, $K_D = aD$
k_B	Boltzmann constant
$\hat{\mathbf{k}}$	unit vector along the z-axis
k	thermal conductivity of medium
k	integer representing a particular enclosure surface
$k(...)$	kernel of a single-variable integral equation
L	distance or dimension
l	integer representing a particular line in a band
M	molecular mass
N	number of particles per unit volume
N	total number of surfaces in an enclosure
N_p	number of scattering particles per unit volume
N_c	number of FCM surfaces in an enclosure
$\mathbf{N}$	normal to a surface or curve
N_f	number of terms in a truncated Fourier series
N_1, N_2	conduction/radiation parameters
n	index of refraction
$n_{P,E}$ $(n'_{P,E})$	spectral (directional) photon density
$\hat{\mathbf{n}}$	unit vector normal to a surface
n_r	rotational quantum number
n_v	vibrational quantum number
$P(e)$	probability that system is in quantum state of energy e
P	pressure
$\mathbf{P}$	vector of power carried by an electromagnetic wave
P_x, P_y, P_z	components of $\mathbf{P}$
P_E	equivalent-broadening pressure
P_A	partial pressure of active component of a gas mixture
P_0	reference pressure equal to one atmosphere
$P_{\mathrm{H_2O}}$	partial pressure of $\mathrm{H_2O}$
$P_{\mathrm{CO_2}}$	partial pressure of $\mathrm{CO_2}$
$Q_{r\lambda}$	radiant heat flow over a finite surface
Q_k	total rate at which radiative heat leaves surface k
Q_g	total rate at which radiative heat leaves the gas
$Q_{\lambda k}$, $Q_{\eta k}$	spectral rate at which radiative heat leaves surface k
$q_{r\lambda}$, q_{rE}, $q_{r\eta}$	radiant heat flux
$q_{r\lambda,bn}$	radiant heat flux in a medium of index of refraction n, at photonic equilibrium

q_r	total radiant heat flux
$q_{r\lambda,\omega}$	partial radiant heat flux
$\hat{q}_{r\lambda}$	net radiant heat flux
$\bar{q}_{r\lambda k}$	average radiant heat flux over surface k
$\mathbf{q}_r$	vector of radiant heat fluxes
$\hat{q}_{ri}$, $\hat{q}_{rj}$, $\hat{q}_{rk}$	components of $\mathbf{q}$
q_r'''	rate per unit volume at which radiant energy leaves medium
$q_{s\lambda}$, $q_{s\eta}$	(spectral) surface heat flux
q_s	surface heat flux
$q_{\lambda ok}$, $q_{\eta ok}$	outgoing radiant heat flux at surface k
$\bar{q}_{\lambda ok}$, $\bar{q}_{\eta ok}$	average outgoing radiant heat flux at surface k
q_{ok}	total outgoing radiant heat flux at surface k
$\bar{q}_{ok}$	average total outgoing radiant heat flux over surface k
$\mathbf{q}_{\lambda o}$, $\mathbf{q}_{\eta o}$	vector of average outgoing radiant heat fluxes
$\mathbf{q}_o$	vector of total average outgoing radiant heat fluxes
$\mathbf{q}_\lambda$, $\mathbf{q}_\eta$	spectral heat flow vector
$\mathbf{q}$	total heat flow vector
$R_{sp,n\rightarrow m}$	rate of spontaneous emissions $n \rightarrow m$, per unit volume
$R_{st,n\rightarrow m}$	rate of stimulated emissions $n \rightarrow m$, per unit volume
$R_{ab,m\rightarrow n}$	rate of absorption transitions $m \rightarrow n$, per unit volume
R	radius
$R'_{sp,n\rightarrow m}$	directional rate of spontaneous emission transitions $n \rightarrow m$, per unit volume
$R'_{st,n\rightarrow m}$	directional rate of stimulated emission transitions $n \rightarrow m$, per unit volume
$R'_{ab,m\rightarrow n}$	directional rate of absorption transitions $m \rightarrow n$, per unit volume
R_{ki}	thermal resistance between kth surface and a nearby node at T_{ki}
R_k	$= \rho_k$ if k is T-specified and $= 1$ if it is q-specified
$\mathbf{r}$	position vector: $\mathbf{r} = (x, y, z)$
r_e, r_{eDC}	electrical resistivity, DC electrical resistivity
$\mathbf{r}_\lambda$	reflectivity matrix
$\mathbf{r}_k$	position vector of a point on surface k
S, S_j	surface, surface j
S'_E, S'_λ	source term in the RTE
$S'_{\lambda,i}$, $(S'_{\lambda,o})$	contribution to S'_λ due to inscattering (outscattering)
S_l, $\overline{S}$	line strength of l^{th} line, average line strength
$\overline{S}_0$	average line strength at the band center
s	distance measured along a ray
s, $s\,(\mathbf{u}, \mathbf{u}^*)$	distance between two points $\mathbf{u}$ and $\mathbf{u}^*$on an enclosure

s_{k-j}	distance between a point on k and a point on j
$\bar{s}_{k-j}$	mean beam length between surfaces j and k
$\bar{s}_{k-j,o}$	geometric mean beam length between j and k
T	temperature
T_g	gas temperature
T_s	surface temperature
T_j	temperature of surface j, $j = 1, 2..k....N$
T_k	temperature of surface k, $k = 1, 2..j....N$
T_{ki}	temperature of i^{th} node exchanging nonradiative heat with k
$\bar{T}_k$	mean temperature of surface k
t	time
$t_\lambda(s)$	optical thickness
t_f	film thickness of a composite surface
u, v	parameters relevant to a parametric surface representation
$\mathbf{u}$, (u, v)	vector with components u and v; $\mathbf{u}$ fixes a point on a surface
$\mathbf{u}_k$	$\mathbf{u}$ fixing a point on the kth surface
u	dimensionless path length, $= \overline{S}_0 s/\delta$
V	volume
V_p	particle volume
X	any extensive measure of the radiant field
x, y, z	Cartesian coordinates in space

Greek Letters

α'_λ	absorptivity of a surface
α'	total absorptivity
$\alpha'_{\lambda n}$	normal absorptivity (applies when incident ray is normal)
$\alpha\,(T)$, $\alpha_b\,(T)$	tabulated function of T, see Tables 21.4 and 21.5
$\alpha_{g,j}(s)$	total gas absorptivity
β	exponential wide-band's line width to spacing parameter
β	angle measured from the x-axis
γ, γ_0	electrical permittivity, electrical permittivity of free space
γ	opening angle of a V-corrugated surface
$\gamma(T)$, $\gamma_b(T)$	tabulated function of T, see Tables 21.6, and 21.7
δ, δ_l	line spacing, line spacing of l^{th}line
$\overline{\delta}_l$	mean line spacing
$\delta_{i,j}$	Kronecker delta function: $= 1$ if $i = j$; $= 0$ otherwise
ϵ'_λ	emissivity
$\epsilon'_{\lambda n}$	normal emissivity
ϵ_λ $(\epsilon_{\lambda k})$	hemispherical emissivity (of kth surface)
ϵ	total hemispheric emissivity
ϵ_k	total hemispheric emissivity of surface k

ϵ'	total directional emissivity
ϵ'_n	total normal emissivity
ϵ	total hemispheric emissivity
$\boldsymbol{\epsilon}_\lambda$	emissivity matrix
$\epsilon_g(s)$	total gas emissivity
ϵ_{soot}	soot emissivity
$\boldsymbol{\epsilon}$	total emissivity matrix
$\boldsymbol{\epsilon}_s$, $\boldsymbol{\epsilon}_t$	emissivity matrices for enclosures with q-specified surfaces
$\boldsymbol{\epsilon}^s$	specular total emissivity matrix
η	wave number
η_l	wave number at center of l^{th} line
$\bar{\eta}_b$, η_c	wave number at center of vibration rotation band
η^*	dimensionless wave number distance from center of smoothed band
θ, θ_k	angle from surface normal, angle from normal to the kth surface
θ	colatitude angle; with φ, angle specifying a direction $\hat{\mathbf{d}}$; angle between $\hat{\mathbf{d}}$ and $\hat{\mathbf{k}}$ or between $\hat{\mathbf{d}}$ and $\hat{\mathbf{n}}$
θ	(in scattering) the angle between two directions, $\hat{\mathbf{d}}$ and $\hat{\mathbf{d}}'$
θ_{1i}, θ_i	angle of incidence
$\theta_{1,r}$	(for smooth surface) angle of reflection at interface 1-2
θ_{2t}	(for smooth surface) angle of refraction
θ_B	Brewster angle
$\theta_{\max}$	angle of total internal reflection
θ_r	(for rough surface) angle of reflected direction considered, from normal
$\theta(\eta^*)$	function used for characterizing the smoothed band
θ, θ_2	dimensionless absolute temperatures, $\theta .= T/T_1$; $\theta_2 = T_2/T_1$
κ	absorption index
λ_a, λ	wavelength, free-space wavelength
μ, μ_0	magnetic permeability, magnetic permeability of free space
μ	$\cos\beta$
ν	frequency of electromagnetic wave
ρ'_λ	surface reflectivity
$\rho'_{\lambda n}$	reflectivity for radiation incident normal to surface
ρ''_λ	bidirectional reflectivity
ρ'	total reflectivity
ρ_λ, $\rho_{\lambda k}$	hemispheric reflectivity, hemispheric reflectivity of kth surface
ρ, ρ_k	total hemispheric reflectivity, total hemispheric reflectivity of k
ρ	gas density
σ	Stefan-Boltzmann constant
σ_λ	scattering coefficient
φ	azimuth angle; with θ, angle specifying a direction $\hat{\mathbf{d}}$

φ_r	azimuth angle of reflected direction considered
φ_b, φ_g	dimensionless temperatures given by Eqs. (23.19) and (23.27)
χ	alternate symbol for θ_{2t}
ω, ω_j	solid angle, solid angle subtended by surface j
ω	bandwidth of an exponential wide band
ω_0	wide band property tabulated in Table 21.3
$\Phi\left(\hat{\mathbf{d}}, \hat{\mathbf{d}}'\right)$	phase function relevant to scattering

UNITS AND FUNDAMENTAL CONSTANTS

Units

This book endeavors to maintain strict adherence to the SI System of Units. Quantities must be in SI units when inserted into the equations, and when used properly, the equations will yield values in SI units. This means, for example, that wavelengths are to be expressed in m, wave numbers in m^{-1}, intensity i'_λ in $\mathrm{Wm}^{-3}\mathrm{sr}^{-1}$, intensity i'_η in $\mathrm{Wm}^{-1}\mathrm{sr}^{-1}$, angles in radians or steradians, photon energy in J, and temperatures in K. The only exception to this rule is in gas pressure, which is sometimes expressed in atmospheres.

Fundamental Constants

Name	Symbol	Value
Planck's constant	h_P	6.62618×10^{-34} Js
Boltzmann constant	k_B	1.38066×10^{-23} JK^{-1}
Speed of light in free space	c_o	2.997925×10^{8} ms^{-1}
Stefan-Boltzmann constant	σ	5.6702×10^{-8} $\mathrm{Wm}^{-2}\mathrm{K}^{-4}$

Chapter 1

NATURE OF THERMAL RADIATION

Thermal radiation is everywhere around us. Every material body in the universe emits thermal radiation, and we are bathed in these emanations. Thermal radiation radiates from our own human bodies and from the furniture and walls around us. Although we do not *directly* sense this low-temperature radiation, we would quickly feel very hot if it were to be suddenly stopped—a resting human body typically dispenses with two-thirds of its metabolic heat by thermal radiation. Thermal radiation from very hot bodies *is* directly sensed—by our eyes as visible light. Without the Sun's thermal radiation streaming through a nearby window or that from the filament in a nearby lamp, we would not be able to read this book. Indeed, without the thermal radiation emitted by the Sun, our very life would soon cease, as this radiation supports all life on Earth.

Ubiquitous and life-giving as it is, thermal radiation also represents an important mechanism for heat flow, and so we study it to understand heat transfer in its entirety. It plays a vital role as an engineered heat transfer mechanism in furnaces, boilers, and solar conversion systems. Engineers have to account for its presence when designing thermal control systems, like those for buildings and spacecraft, and when calculating the heat losses from objects like steam pipes, rocket plumes, electrical transmission lines, solar receivers, and humans. Radiation plays a role as important as conduction in cellular and fibrous insulations; indeed the thermal conductivity of such materials incorporates radiant effects. Thermal radiation also plays a decisive role in the propagation of fires, and engineers have to account for its presence when planning fire prevention programs and aids for firefighting.

To deal competently with these various facets, the thermal analyst needs a fundamentally based understanding of thermal radiation. This understanding begins with an appreciation of thermal radiation's fundamental nature.

1.1 Dual Nature of Thermal Radiation

Thermal radiation is one part of the electromagnetic spectrum. That is, it is one particular manifestation of electromagnetic waves. Electromagnetic (EM) waves are traveling fluctuations in coupled electric and magnetic fields. There are many different sources of EM waves, and one way in which they are categorized is by their source. Thus, we could say that thermal radiation consists of those electromagnetic waves given off by bodies *by virtue of their temperature*. Other sources of EM waves are radio and TV station towers, fluorescent light bulbs, lasers, radioactive materials, and indeed any wire carrying an AC current. It should be noted, however, that none of these other phenomena constitute thermal radiation because they do not fundamentally result from thermal phenomena. Nonetheless, once a wave of a particular frequency has been produced by a particular source, it is essentially indistinguishable from one produced by any other source.

There is another interpretation of thermal radiation: the photonic view. In this view, thermal radiation consists of a flow of fundamental particles called photons. Photons can be viewed as packets of electromagnetic energy. The energy carried by any one photon is very small, but a flow of many photons carries substantial energy. This photonic interpretation applies, in fact, to all EM radiation, not just to thermal radiation. Which view is the correct one to adopt depends on the context. If we want to explain interference, we need the wave interpretation; if we want to explain the photoelectric effect, we need the photonic interpretation. We will find both useful in the study of thermal radiation. We begin with the wave interpretation.

1.2 Thermal Radiation as EM Waves

Our understanding of EM waves originated with the work of James Clerk Maxwell in the mid-1800s. Maxwell expressed two important laws as field equations: Faraday's Law, which describes how a changing magnetic field induces a changing electrical field, and Ampere's Law, which describes how an electric current produces a magnetic field. He then introduced an intuitive correction to Ampere's Law to account for transient effects, thereby accounting for how a changing electrical field induces a changing magnetic field. He also showed how these laws (with a few additions) ultimately lead to the wave equation in each component of the fields: that is, in each component of the electric intensity $\mathbf{E}$ and the magnetic intensity $\mathbf{H}$. Thus, Maxwell was the first to predict electromagnetic waves. His predictions were later validated experimentally; for example by the experiments of Hertz. Now electromagnetic wave theory is a foundation stone of contemporary physics.

If the medium at hand has zero electrical conductivity (air approximates this condition closely), the wave equation obtained is the undamped wave equation; otherwise it is damped. We consider for the moment the undamped case. The speed c of the waves is found to depend on the material properties of the

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