

PHYSICAL MECHANICS

LABORATORY MANUAL IN GAS DYNAMICS, HYDRODYNAMICS, AND PHYSICAL MECHANICS

EDITED BY
PROFESSOR E. E. SON
Moscow 2006

CONTRIBUTORS

A.S. Koroteev V.A. Volkov
M.A. Meshkov E.E. Son
A.T. Onufriev S.M. Starikovskaya
A.A. Paveliev A.Yu. Starikovskii
Yu.G. Rakogon B.K. Tkachenko
R.A. Safarov V.P. Vakатов
V.A. Sechenov M.V. Vasiliev
Yu.A. Shcherbina V. Yu. Velikodnyi
N.N. Shirokov E.N. Voznesenskii
A.P. Zuev

PHYSICAL MECHANICS

LABORATORY MANUAL IN GAS DYNAMICS, HYDRODYNAMICS, AND PHYSICAL MECHANICS
EDITED BY E.E. SON MOSCOW 2006

Copyright © 2012 by Begell House, Inc. All rights reserved. This book, or any parts thereof, may not be reproduced in any form or by any means, or stored in a data base retrieval system, without written consent from the publisher.

This book represents information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. A wide variety of references are listed. Every reasonable effort has been made to give reliable data and information, but the author and the publisher cannot assume responsibility for the validity of all materials for the consequences of their use.

ISBN: 978-1-56700-265-2

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

Library of Congress Cataloging-in-Publication Data

Physical mechanics : laboratory manual in gas dynamics, hydrodynamics, and physical mechanics / edited by professor E.E. SON, Moscow, 2006 ; contributors, A.S. Koroteev [and eighteen others].

pages cm

Includes bibliographical references and index.

ISBN 978-1-56700-265-2 (alk. paper)

1. Mechanics--Laboratory manuals. 2. Dynamics--Laboratory manuals. I. Son, E. E. (Elena E.), editor.

QC129.5.P48 2012

531.078--dc23

2012041807

Direct inquires to Begell House, Inc., 50 Cross Highway, Redding, CT 06896

PHYSICAL MECHANICS

LABORATORY MANUAL IN GAS DYNAMICS, HYDRODYNAMICS, AND PHYSICAL MECHANICS

Edited by Professor E. E. Son
Moscow 2006

Contributors: A. S. Koroteev, M. A. Meshkov, A. T. Onufriev, A. A. Pavlieliev, Yu. G. Rakogon, R. A. Safarov, V. A. Sechenov, Yu. A. Shcherbina, N. N. Shirokov, E. E. Son, S. M. Starikovskaya, A. Yu. Starikovskii, B. K. Tkachenko, V. P. Vakarov, M. V. Vasiliev, V. Yu. Velikodnyi, V. A. Volkov, E. N. Voznesenskii, & A. P. Zuev.

Reviewers: Department of Plasma Physics of the Moscow Engineering-Physics Institute (State University) A. I. Leont'ev, Full Member of the Russian Academy of Sciences.

Physical Mechanics–Laboratory Manual in Gas Dynamics, Hydrodynamics, and Physical Mechanics, Edited by E. E. Son, Moscow. Moscow Engineering-Physics Institute, 2006, pp. 383.

A series of experimental laboratory works on gas dynamics, hydrodynamics of a viscous fluid, thermo- and aerodynamics, turbulent flows, plasma physics at normal and high temperatures that constitute the subject matter of physical mechanics for students of the second, third, and fourth years of study at the Faculty of Aerophysics and Space Investigations (FASI) of the Moscow Engineering-Physics Institute (State University) in "Applied Mathematics and Physics." The educational cycle, "Physical Mechanics," at the Faculty incorporates a course of lectures, seminars, and laboratory work. Experimental works are intervened with numerical simulation on personal computers with visual observations.

The book is intended for engineering physics and applied physics specialties of higher educational establishments.

INTRODUCTION

The trends in the activities of the base departments of the Faculty of Aerophysics and Space Investigations (FASI) of the Moscow Engineering-Physics Institute are connected with theoretical and experimental investigations of the phenomena of external and internal gas dynamics and hydrodynamics. Among them are the problems of entry of space apparatuses into dense atmospheric layers, design and creation of rocket engines, gas-dynamic lasers, electrophysical systems, and other technical facilities that use a gas, liquid, or plasma as the working medium. Fundamental investigations carried out at the base departments of FASI are related to the study of the phenomena occurring in the atmosphere, ocean, and other geospheres with respect to the physics of combustion and explosion.

The above trends constitute the subject matter of the faculty cycle, "Physical Mechanics." The faculty cycle for training students includes courses of lectures, seminars, and laboratory work on aerophysics, applied gas dynamics, hydrodynamics, and physical mechanics for the training of bachelor of science candidates on "applied mathematics and physics" in the second, third, and fourth years of education.

The fundamental education in the field of continuum mechanics, hydrodynamics of media with different rheologies, and nonequilibrium systems is unthinkable if the skills of an experimental work have not been acquired, if students have not become familiar with the methods of creating dynamic media and their diagnostics. Despite the long history of the development of experimental methods, recent years have witnessed qualitative changes in this area due to the appearance of new types of probes based on a new elementary base and wide introduction of computation engineering in the methods of the processing of measurements.

The laboratory work at FASI on continuum mechanics and physical mechanics is intended for both getting acquainted with the methods of

measurements and carrying out laboratory works closely approximating up-to-date scientific experiments.

The general orientation of the cycle is associated with the investigation of the thermophysical properties of gases and plasma that include the thermodynamic properties of ideal and nonideal gases and plasma, chemical reactions, including dissociation and ionization, elementary processes in gases and plasma, and the optical properties of gases. Studied in the cycle are the hydrodynamics of motion of high-temperature gases and plasma, as well as radiative transfer. Considered in the cycle are the most general principles of hydrodynamic description with allowance for chemical reactions, self-consistent electromagnetic fields, and of translational, rotational, and vibrational nonequilibrium states; hydrodynamic, thermal, and plasma instabilities at linear and nonlinear stages; and transition from laminar modes of flow to turbulent ones. Most hydrodynamic flows are turbulent; therefore, attention is specially paid to practical methods of visualization and calculation of turbulent flows. It should be noted that the up-to-date methods of describing turbulent flows with combustion are based on the method of probability density functions that have been studied first in the world experimentally at the department of physical mechanics.

The laboratory work at the second year of education acquaints students with gas-dynamic flows; that at the third year deals with more complex types of laminar and turbulent flows of air and gases with internal degrees of freedom; that at the fourth year deals with investigation of present-day complex flows, as well as electrophysical and thermophysical phenomena, often at the level of modern scientific investigations. Experimental works are accompanied by numerical simulation on personal computers with visualization of the processes.

The work on compilation of the laboratory manual was begun at FASI at the end of 1960 by V. M. Ievlev, T. V. Kondranin, A. S. Koroteev, A. T. Onufriev, A. A. Paveliev, Yu. G. Rakogon, I. N. Rei, R. A. Safarov, V. A. Sechenov, A. A. Serebrov, Yu. A. Shcherbina, E. S. Shchetinkov, E. A. Son, B. K. Tkachenko, V. P. Vakotov, M. N. Vasiliev, V. A. Volkov, E. N. Voznesenskii, and by many other teachers. At the present time, this laboratory manual is being modified in an effort to update it, make it adequate for the needs of the FASI Department, and to reflect promising trends in high technologies in the field of continuum mechanics and physics.ii

The book prepared for publication rests on the materials submitted by the teaching staff of the Department of Mechanical Physics and those who served on the staff earlier. The setting of the text, preparation of figures, and preparation of laboratory aids were accomplished on the basis of modern technologies by D. E. Belov and O. S. Galkevich, to whom the contributors express their gratitude.

The publication of the Laboratory Manual was made possible due to the support from the REC-011 project of the joint program "Fundamental Investigations and Higher Education" at the American Civil Research and Development Foundation (CRDF) and from the Ministry of Education and Science of the Russian Federation.

E. E. Son

Head of Department, Professor, Doctor of Sciences

(in Physics and Mathematics)

Merited Scientist of the Russian Federation

Contents

Introduction	xiii
1. Measurement of Flame Temperature by the Spectral-Line Reversal Method	1
1.1. Introduction	1
1.2. Some Information on Radiation	2
1.3. Emission and Absorption of Real Media	8
1.4. Optical Methods of Measuring the Temperature of Real Media	13
1.5. Description of the Laboratory Work	17
1.5.1. Assignment to carry out the experimental part of the work	17
1.5.2. Assignment to process experimental data	18
1.5.3. Control questions	19
1.6. Technical Characteristics of the LOP-72 Optical Pyrometer. Construction of the Pyrometer. Order of Performing the Work	20
1.7. Gas Dynamic Parameters	21
1.8. Radiative Properties of Metals	22
1.9. References	26
2. Probe Methods of Plasma Research	27
2.1. Diagnostics of the Electric Parameters of Plasma	27
2.2. The Purpose of the Work	30
2.3. Current-Voltage Characteristic of a Single Probe	30
2.4. The Range of Validity of the Probe Methods	31
2.5. The Theory of a Single Electric Probe	32

2.6. The Theory of Double Electric Probe	36
2.7. Experimental Part of the Work	38
2.7.1. Assignment to carry out the laboratory work	38
2.7.2. Description of the setup	39
2.7.3. Carrying out measurements with the aid of a single probe	40
2.7.4. Carrying out measurements with the aid of a double probe	41
2.7.5. Processing of the data of the laboratory work	41
2.8. A Criterion of Space Charge Layer Formation at Great Negative Probe Potentials	41
2.9. Determination of the Plasma Potential from Measurements of the Floating Potential	44
2.10. Measurement of the Energy Distribution Function	44
2.11. The Theory of Electron Current in the Case of a Thick Space Layer	46
2.12. Determination of the Electron Temperature	47
2.13. Conclusions	48
2.14. Control Questions	48
2.15. References	49
3. Study of a Glow Discharge in Helium	50
3.1. Introduction	50
3.2. Qualitative Description of a Glow Discharge	50
3.3. Assignment to Carry out Laboratory Work	54
3.3.1. Processing of data	55
3.3.2. Comparison between theory and experiment	55
3.4. Elementary Theory of the Cathode Part of the Glow Discharge	55
3.5. Positive Column of a Glow Discharge	61
3.6. Nonequilibrium State of a Weakly Ionized Plasma	66
3.7. References	66

4. Study of the Oscillations of a Fluid in a Channel	67
4.1. Purpose of the Work	67
4.2. Experimental Technique	67
4.2.1. Experimental setup	67
4.2.2. Probe of the fluid level displacement	68
4.2.3. Specifications	70
4.2.4. Determination of the spectral components of oscillations	71
4.3. Measurement Technique	71
4.4. Assignment	71
4.5. Elements of the Theory of Oscillations and Waves in a Fluid	72
4.5.1. General relations	72
4.5.2. Linearization of equations	74
4.5.3. Natural oscillations of a liquid	75
4.6. Reference	77
5. Determination of the Reynolds Number in Transition to Turbulence in a Boundary Layer	78
5.1. Introduction	78
5.2. Justification of the Experimental Technique	80
5.3. Description of the Experimental Setup	84
5.4. Guidelines for Carrying out the Work	86
5.5. Elements of the Boundary Layer Theory	87
5.6. Boundary Layer Thickness on a Flat Plate	88
5.7. Inferences of the Linear Theory of Hydrodynamic Stability for a Boundary Layer on a Flat Plate	90
5.8. Control Questions	91
5.9. References	92

6. Laminar Liquid Flow Over the Starting Length of a Plane Channel	93
6.1. Purpose of the Work	93
6.2. The Procedure of Measuring Flow Velocity	94
6.3. Principle of Doppler Signal Processing by a $55\alpha 90$ Counter	97
6.4. Experimental Setup	99
6.5. Operational Procedure for the Setup	101
6.6. Operating Controls of the $55\alpha 90$ Block	101
6.7. Operational Procedure for a $55\alpha 90$ Counter	103
6.8. Assignment	104
6.9. Laminar Flow Development in a Plane Channel	104
6.10. References	107
7. Investigation of a Laminar Boundary Layer on a Plate by a Doppler Laser Velocimeter	108
7.1. Elements of the Theory of the Doppler Laser Method of Velocity Measuring	108
7.2. Assignment	113
7.3. Elements of the Theory of a Boundary Layer on a Flat Plate	114
7.4. References	118
8. Methods of Generation and Registration of Shock Waves	119
8.1. The Purpose of the Work	119
8.2. Briefly on the Processes Occurring in Starting of Shock Tubes	119
8.3. Operation of a Shock Tube with the Cross Section Constant along the Length	120
8.4. ST-2 Shock Tube	128
8.4.1. General description	128
8.4.2. Pneumatic diagram of the setup	129

8.4.3. Methods of measuring the shock wave parameters and instrumentation	130
8.4.4. System of optical recording of a shock wave	130
8.4.5. Interferometer	131
8.4.6. High-speed photographic recorder	131
8.4.7. Lighting source and synchronization block	135
8.5. Assignment	136
8.6. Calculation of the Shock Wave Intensity for a Shock Tube of Variable Cross Section	137
8.7. References	141
9. Study of the Characteristics of the Ballistic Installation	142
9.1. Introduction	142
9.2. Ballistic Installation	143
9.3. Estimation of the Shell Velocity	145
9.4. Assignment	150
9.5. Basic Relations for Unsteady Gas Flows	150
9.6. References	154
10. Investigation of the Supersonic Rarefied Gas Flow	155
10.1. Introduction	155
10.2. Generation of the Rarefied Gas Flow	157
10.3. Investigation of the Rarefied Gas Flow Parameters by a Pitot Tube	159
10.4. Measuring Instruments and Experimental Technique	161
10.5. Assignment	161
10.6. Elements of the Boundary-Layer Theory	163
10.7. Velocity and Temperature Jumps at a Wall in a Slip Gas Flow	168
10.8. References	170

11. Structure of a Shock Wave in a Low-Density Gas Flow Past a Cylinder	172
11.1. Introduction	172
11.2. Application of a Free-Molecular Thermal Probe to Investigate the Flow Field Near a Cylinder Immersed in a Transverse Flow	173
11.3. Application of a Glow Discharge to Visualize the Flow Pattern	175
11.4. Assignment	176
11.5. Structure of a Shock Wave	177
11.6. Shock Wave Withdrawal	183
11.7. References	186
12. Study of the Regimes of Gas Outflow from a Laval Nozzle	187
12.1. Introduction	187
12.2. One-Dimensional Theory of the Supersonic Nozzle	187
12.3. Gas Outflow with Overexpansion	192
12.4. Gas Outflow with Underexpansion	193
12.5. Experimental Setup	194
12.6. Experimental Procedure	195
12.7. References	195
13. Determination of the Time of the Vibrational Relaxation of CO₂	196
13.1. Introduction	196
13.2. Determination of the Time of the <i>VT</i> Relaxation of the Deformation Mode of CO ₂	200
13.3. Description of the Setup	203
13.4. The Order of Performing the Work	205
13.5. Processing of Results	205
13.6. Checking Questions	206

13.7. Vibrational Relaxation of Molecules (the Landau–Teller Theory)	206
13.8. Entropy Increment in the Process of Retardation and Vibrational Relaxation of a Flow	209
13.9. References	213
14. Atmospheric-Vacuum Supersonic Wind Tunnel	214
14.1. The Bernoulli–St. Venant Equation	215
14.2. Measurement of the Reduced Flow Velocity or of the M Number	219
14.3. Principles of Calculation of the Atmospheric-Vacuum Supersonic Wind Tunnel	221
14.4. Arrangement of the ST-4 Wind Tunnel	225
14.5. The Order of the Start-up and Shut-down of the Wind Tunnel	228
14.6. Study of Flow in the Laval Nozzle	229
14.6.1. Static pressure along the nozzle length	229
14.6.2. Nonuniformity of the profiles of gasdynamic quantities at the nozzle cut	230
14.7. Study of Some Characteristics of the Wind Tunnel	232
14.7.1. Flow field in the working chamber	232
14.7.2. Compression degree and time of wind tunnel operation	233
14.8. Assignment	234
14.8.1. Operational procedure	234
14.8.2. Students' report structure	235
14.9. Notation	235
14.10. References	236
15. Supersonic Flow Past a Plate	237
15.1. Supersonic Flow past a Plate	237
15.2. Assignment	239

15.3. Elements of Gas Dynamics	240
15.3.1. Oblique shocks	240
15.3.2. The Prandtl–Mayer flow	243
15.4. References	248
16. Measurement of Averaged And Pulsation Characteristics of a Turbulent Flow by a Constant Temperature Anemometer	249
16.1. Aim of the Work	249
16.2. Brief Information on Turbulent Motion	249
16.3. Thermoanemometric Method	253
16.4. Measurement of Average and Pulsation Velocities in a Flow	254
16.5. Instrumentation	257
16.6. Description of the Laboratory Setup	258
16.7. Order of the Fulfillment of the Work	260
16.8. Representation of Results	260
16.9. References	262
17. Investigation of a Free Turbulent Jet	263
17.1. Aim of the Work	263
17.2. Some Information on Turbulent Flows	263
17.3. Scheme and Basic Laws Governing the Development of a Free Turbulent Jet	264
17.4. Description of the Laboratory Setup and of the System of Measurements	269
17.5. The Order of the Fulfillment of the Work	272
17.6. Processing of the Results of Experiments and Drawing up of the Laboratory Work	272
17.7. Appendix	273
17.8. References	274

18. Generation of a Low-Temperature Plasma by Electric-Arc Plasmatrons	275
18.1. Introduction	275
18.2. General Information on Arc Discharges and Plasmatrons	276
18.3. Construction of the DCP-2 Plasmatron	277
18.4. Methods of Measuring the Plasmatron Characteristics	281
18.4.1. Measurement of the arc current and voltage	282
18.4.2. Measurement of the working substance flow rate	282
18.4.3. Measurement of the cooling liquid flow rate	284
18.4.4. Measurement of the heating of water	285
18.4.5. Measurement of pressure in the working chamber	285
18.4.6. Measurement of heat flows	285
18.4.7. Measurement of the plasmatron characteristics	286
18.5. Instructions for Carrying out the Work and Assignment	286
18.5.1. Order of the fulfillment of the work	286
18.5.2. Program of the experimental part of the laboratory work	287
18.5.3. Assignment	287
18.6. The Thermophysical Properties of Helium	288
18.8. Checking Questions	290
18.9. References	292
19. Interaction of Concentrated Electron Beams with a Solid Body	293
19.1. Introduction	293
19.2. Production of Electron Beams	293
19.3. Experimental Setup and Procedures	296
19.4. Subject-Matter of the Work	299
19.5. Checking Questions	300
19.6. Interaction of Electron Beams with a Substance	301

19.6.1. Elastic collisions	301
19.6.2. Inelastic collisions	302
19.6.3. Thermal effect of electrons on solid bodies	304
19.7. Calculation of Electron Beam Trajectories	307
19.8. Calculation of the Concentration of Secondary Electrons in a Residual Gas	309
19.9. Neutralization of the Electron Beam Space Charge	310
19.10. Electron Beam Scattering	311
19.11. Inelastic Energy Losses by Electrons and Electron Beam Stopping Range	312
19.12. References	315
20. Hydrodynamic Stability of the Rotational Couette Flow	317
20.1. Aim of the Work	317
20.2. Brief Information on the Cylindrical Couette Flow	317
20.3. Laboratory Setup	319
20.4. Assignment	322
20.5. The Order of the Fulfillment of the Work	322
20.6. Processing of Results	323
20.7. Checking Questions	323
20.8. Stationary Distribution of Velocities in the Couette Flow	324
20.9. Stability of Motion of an Inviscid Fluid	326
20.10. Stability of Motion of a Viscous Fluid	331
20.11. Stability of the Couette Flow for a Narrow Gap	335
20.12. The stability of the Couette Flow for a Gap of Finite Thickness	340
20.13. References	342

21. Measurement of the Temperature of Heavy Particles in a Gas Discharge by the Radiation Spectrum of the Second Positive System of N₂	343
21.1. The Aim of the Work	343
21.2. Temperature and Temperature-Measurement Methods	343
21.2.1. Temperature measurement by contact methods	345
21.2.2. Optical methods of the measurement of temperature	346
21.2.3. Spectral methods of the measurement of temperature	348
21.3. Experimental Setup	350
21.3.1. Means of creation and sustainment of a glow discharge	350
21.3.2. Diagnostic part	351
21.4. Processing of Experimental Data	352
21.5. Checking Questions	353
21.6. Determination of Gas Temperature from the Radiation Intensity Distribution in the Electronic-Vibrational-Rotational Bands of Molecular Transitions	353
21.6.1. Introduction to the theory of molecular spectra	354
21.6.2. Designations of the electronic terms of diatomic molecules	355
21.6.3. Measurement of the rotational temperature from the relative intensity of the rotational structure of the electronic-vibrational spectrum	361
21.6.4. Relationship between rotational and translational temperatures	362
21.6.5. Structure and principal characteristics of a glow discharge	363
21.7. References	365

22. Propagation of Sound Waves in a Microbubble Medium	366
22.1. Aim of the Work	366
22.2. Experimental Procedure	368
22.2.1. Experimental setup	368
22.2.2. A set of devices for measuring the speed of sound in a heterogeneous medium	369
22.2.3. A set of devices for creating a micorbubble medium	370
22.2.4. Technical characteristics	371
22.5. Elements of the Theory of Propagation of Sonic Waves in a Heterogeneous Medium	373
22.6. References	375

WORK 1

MEASUREMENT OF FLAME TEMPERATURE BY THE SPECTRAL-LINE REVERSAL METHOD

V. P. Vakatov and A. P. Zuev

1.1. Introduction

The urgent problems of science and technology require measurements of the temperature of emitting bodies and radiative heat fluxes from them. Conventional contact methods are of limited utility because of the simultaneously arising complex problem of convective heat transfer. Optical methods of measurement have been placed in the forefront. The major advantages of these methods are that they do not need direct contact with a measured object and do not distort its parameters.

In the present laboratory work, the temperature of heated bodies is measured by two techniques: (i) a pyrometric one, with the aid of which the temperature of glowing metals is determined (the description of a pyrometer is given in Section 1.6) and (ii) the spectral-line reversal method, with the aid of which the temperature of the flame of a propane burner is determined. Section 1.2 presents the theory of radiation on which the proposed methods of measurements are based. Section 1.3 introduces the readers to the characteristic features of the radiative properties of real media.

The main content of this laboratory work can be extended by studying the processes of heat exchange between the flame of a propane burner and a metal plate inserted into the flame. The reference sources needed for processing experimental data on heat transfer are given in Sec. 1.7.

At the end of Work 1 the literature is cited that may be needed for further study of optical methods and radiation theory. The majority of illustrative experimental results given have been borrowed from Refs. [1–3].

The presentation employs the International System of Units (SI). The sole exception is the use of the following units: $1 \mu\text{m} = 10^{-6} \text{ m}$ to measure the radiation wavelength and $1 \text{ kPa} = 10^3 \text{ Pa}$ to measure pressure.

1.2. Some Information on Radiation

Emission and absorption of thermal radiation. Kirchhoff's law. Blackbody and Planck distribution function. The Stefan–Boltzmann law.

A material medium at any temperature is capable of emitting electromagnetic radiation due to the fluctuation of charges caused by thermal motion. Such a medium is also capable of absorbing outer radiation incident on it, and in this case the energy of electromagnetic waves may again be converted into thermal energy. Electromagnetic waves, being the carriers of thermal energy, differ from waves that correspond to other modes of radiation only by wavelength; it is precisely the wavelength on which the action of radiation depends when the latter is incident on a substance.

The spectrum of emitted radiation embraces a wide range of frequencies; this is attributed to a large number of the degrees of freedom of particles that form such media, to their thermal motion, and to the forces of interaction between them. Consequently, for the theoretical study of thermal radiation, it is necessary to apply statistical methods. The theory of radiation can be presented most sequentially from the standpoints of quantum mechanics or electromagnetic wave theory. However, for many practically important cases, one may adhere to a phenomenological approach, which has been favored by us.

We introduce the concept of *spectral radiation intensity*, $I_\lambda dS dt d\lambda d\Omega$, as the quantity of energy transferred by radiation through an element of unit area, the normal to which coincides with the direction of radiation propagation in unit time, in the unit spectral wavelength interval, and in the unit solid angle. In what follows, the differentials $dS dt d\lambda d\Omega$ are not written but they should be kept in mind. If the normal of the selected area

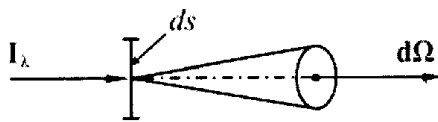


Figure 1.1. Toward the notion of radiation intensity.

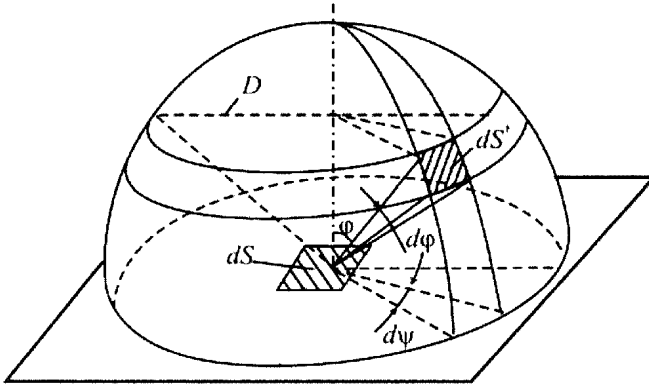


Figure 1.2. Determination of the solid angle from the elemental area dS' .

dS makes an angle φ with the direction of radiation and $d\Omega$, the energy transmitted through this inclined element of area in time dt will be

$$dU = I_\lambda \cos(\varphi) d\lambda dS d\Omega dt. \quad (1.1)$$

If we consider that the selected area dS is located on the surface of an emitting medium (the temperature of which is T), the intensity of emission from this surface is called *the emissive capacity of the body surface* or *the spectral surface density of radiation in the direction of the normal*, and it is designated by $E_\lambda(T)$.

In the case of isotropic or, as it is said, diffuse radiation from the surface dS , the energy emitted in the time dt inside the solid angle $d\Omega$ within the interval of wavelengths $d\lambda$ is equal to

$$dU = E_\lambda(T) \cos(\varphi) d\lambda dS d\Omega dt, \quad (1.2)$$

which is Lambert's law.

In order to calculate the total amount of energy emitted by the element of diffuse surface dS in the wavelength interval $d\lambda$, it is necessary to integrate expression (1.2) over the solid angle $d\Omega$ over the hemisphere. If in integration we make use of spherical coordinates, as shown in Fig. 1.2, we obtain

$$d\Omega = \frac{dS}{r^2} = \sin(\varphi) d\varphi d\psi, \quad (1.3)$$

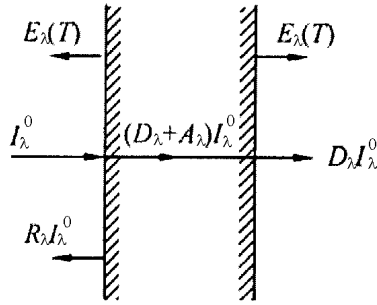


Figure 1.3. Various processes connected with radiation.

$$\frac{U}{dt} = K_{\lambda}(T) dS d\lambda = E_{\lambda}(T) dS d\lambda$$

$$\times \int_0^{2\pi} d\psi \int_0^{\pi/2} \sin(\varphi) \cos(\varphi) d\varphi = \pi E_{\lambda}(T) dS d\lambda. \quad (1.4)$$

The quantity $K_{\lambda}(T)$ is sometimes called *the hemispherical spectral radiant flux surface density*. Relation (1.4) is very important in measurements of radiation; it shows that the total energy emitted by the surface element dS into a semispace is π times greater than the energy emitted by it in the direction of the normal to the surface within the limits of a unit solid angle.

When some radiation I_{λ}^0 is incident on the body, one can consider several processes shown in Fig. 1.3. Part of the incident radiation I_{λ}^0 may penetrate to the interior of the body and part of it, I_{λ}^R , be reflected from it.

Moreover, part of the radiation that penetrates inside a body can be absorbed, I_{λ}^A (and converted into a new form of energy, e.g., thermal), and part of it will pass through the body intact, I_{λ}^D . From the energy conservation law it is clear that

$$I_{\lambda}^0 = I_{\lambda}^R + I_{\lambda}^A + I_{\lambda}^D. \quad (1.5)$$

We introduce the following notions:

- $R_{\lambda} = I_{\lambda}^R / I_{\lambda}^0$ — spectral *reflectivity*,
- $A_{\lambda} = I_{\lambda}^A / I_{\lambda}^0$ — spectral *absorptivity*,
- $D_{\lambda} = I_{\lambda}^D / I_{\lambda}^0$ — spectral *transmissivity*.

From these definitions and from equality (1.5), it follows that

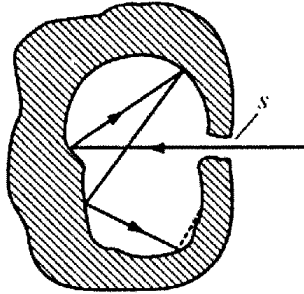


Figure 1.4. Toward the concept of blackbody.

$$R_\lambda + A_\lambda + D_\lambda = 1. \quad (1.6)$$

The properties of media can be different, and two of these quantities separately or both quantities simultaneously can be equal to zero. We will consider in more detail the case $A_\lambda = 1$. The body that at any temperature completely absorbs the radiation of arbitrary frequency incident on it is called the blackbody. There are no such bodies in nature; soot and platinum black seem to be black only in the visible region of the spectrum. The properties of the blackbody are best reproduced by a small hole in a closed cavity whose walls are made of an absorbing material. The radiation that penetrates through the hole S into the cavity will be absorbed almost entirely there (see Fig. 1.4). Consequently, the cross-sectional area of the hole is the absorbing surface of the blackbody. If the hole is taken to be rather small, it practically will not influence the radiation inside the cavity. Moreover, if the cavity walls are made of a homogeneous substance and are kept at temperature T , then in the course of time an equilibrium must set in between the radiation emitted and absorbed by the walls

$$E_\lambda(T) + R_\lambda J_\lambda(T) = I_\lambda(T). \quad (1.7)$$

The equilibrium radiation set inside the cavity is homogeneous and isotropic and may be characterized by a single quantity — the spectral blackbody radiation intensity $B_\lambda(T)$. Thus, it follows from the previous equality that

$$E_\lambda(T) = (1 - R_\lambda) B_\lambda(T) = A_\lambda B_\lambda(T), \quad (1.8)$$

whence

$$\frac{E_\lambda(T)}{A_\lambda} = B_\lambda(T), \quad (1.9)$$

which is Kirchhoff's law. This law shows that the ratio between the emissivity and absorptivity is the same for all substances and is equal to the spectral blackbody intensity; a body that strongly absorbs any radiation emits the same radiation strongly in the case of thermal radiation.

Since in the case depicted in Fig. 1.3 $A_\lambda = 1$, then from Kirchhoff's law we obtain

$$E_\lambda(T) = B_\lambda(T); \quad (1.10)$$

that is, the blackbody radiation intensity (the radiation that will be emitted from the cavity through the hole S) is equal to the intensity of thermal equilibrium radiation at the same temperature.

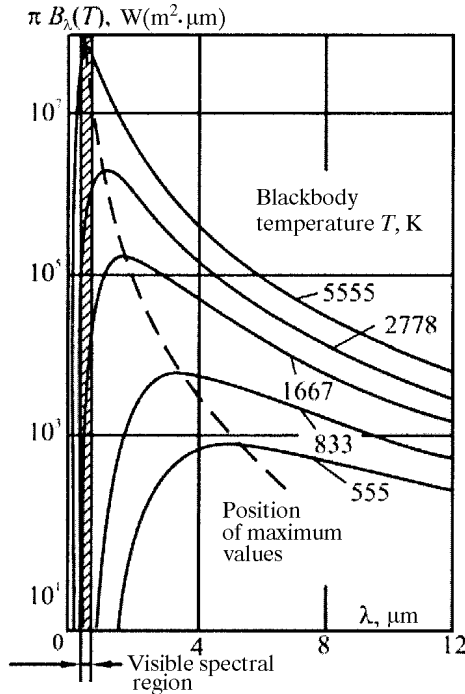


Figure 1.5. Spectral hemispheric blackbody radiation density $\pi B_\lambda(T, \lambda)$ versus the wavelength for several values of temperatures.

The function $B_\lambda(T)$ is the same for all the substances and it can be found theoretically. This was done by Planck in 1905 as follows:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}, \quad (1.11)$$

where c is the speed of light in vacuum, $c = 3.0 \cdot 10^8$ m/s, h is the Planck constant, $h = 6.63 \cdot 10^{-34}$ J·s, and k is the Boltzmann constant, $k = 1.38 \cdot 10^{-23}$ J/K.

In the literature, use is more often made of the hemispherical spectral surface blackbody radiation density, which, according to Eq. (1.4), is equal to $\pi \cdot B_\lambda(T)$. Figure 1.5 depicts the graphs of the function $\pi \cdot B_\lambda(T)$ for several values of temperatures. For the convenience of computations we give the expression for the hemispherical spectral surface blackbody radiation density in terms of the constants C_1 and C_2 as follows:

$$K_\lambda^p(T) = \pi B_\lambda(T) = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)}, \quad (1.12)$$

where $C_1 = 2\pi hc^2 = 3.74 \cdot 10^{-16}$ W·m², $C_2 = hc/R = 1.44 \cdot 10^{-2}$ K.

Knowing the Planck distribution law, it is not difficult to determine the total energy emitted by one side of the unit surface into the hemisphere in the entire spectral interval (from 0 to ∞),

$$N_S(T) = \int_0^\infty K_\lambda^p(T) d\lambda = \int_0^\infty \pi B_\lambda(T) d\lambda = \sigma_S T^4, \quad (1.13)$$

which is the Stefan–Boltzmann law, where σ_S is the Stefan–Boltzmann constant.

$$\sigma_S = \frac{12\pi^5 k^4}{15c^2 h^3} = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4.$$

Expression (1.13) for $N_S(T)$, in contrast to (1.12), can be called the hemispherical integral blackbody radiation intensity. (In what follows, the term integral will mean that radiation corresponds to all wavelengths.)

Table 1.2. Specific resistance and thermal conductivity of some metals at $T = 273$ K

	$\rho \cdot 10^8 \Omega \cdot \text{m}$	$\lambda, \text{W/m} \cdot \text{K}$
copper	1.6	380
aluminum	2.5	210
tungsten	5.1	170
nickel	7.6	83
steel	10.0	50

$$\lambda_{\max} T = 2.66 \times 10^{-3} \text{ m K} . \quad (1.41)$$

From this equation it follows that at radiator temperatures not exceeding 700 K, the maximum of the energy distribution curve is at a wavelength longer than 4 μm . And, consequently, for temperatures above 700 K the influence of the electromagnetic theory should be considered with caution. A comparison between experimental and calculated values of the integral emissivity shows satisfactory agreement. In some cases, for example, for platinum, relation (1.39) is unexpectedly satisfied at very high temperatures when a considerable portion of radiation is in the region of short wavelengths. For a more accurate determination of a radiant flux from heated metals, one should use experimental values of emissivity instead of relation (1.39).

1.9. References

1. M. A. Mikheev, *Fundamentals of Heat Conduction* [in Russian], Gos. Energ. Izd. Press, Moscow, 1956.
2. G. Gröber, S. Erk, and U. Grigull, *Grundgesetze der Wärmeübertragung* [Russian translation], IL Press, Moscow, 1958.
3. R. Siegel and J. R. Howell, *Thermal Radiation Heat Transfer* [Russian translation], Mir Press, Moscow, 1975.
4. M. Born and E. Wolf, *Principles of Optics* [Russian translation], Nauka Press, Moscow, 1973.
5. S. S. Penner, *Quantitative Molecular Spectroscopy and the Emissivity of Gases* [Russian translation], IL Press, Moscow, 1963.
6. Ya. B. Zeldovich and Yu. P. Raizer, *The Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena* [in Russian], Nauka Press, Moscow, 1966.