

**COLLISIONS IN PARTICLE-LADEN  
GAS FLOWS**

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A.Yu. Varaksin, **Collisions in Particle-Laden Gas Flows**, Moscow, FIZMATLIT, 2008

The book is devoted to the problems of modeling turbulent gas flows carrying a dispersed impurity in the form of solid particles. Particular attention is directed to consideration of various collisional processes (particle–particle, particle–wall, particle–body) occurring in heterogeneous flows.

With the use of a large file of experimental and numerical data, dimensionless criteria responsible for the presence and intensity of the indicated interactions have been suggested and verified. The characteristics of turbulent heterogeneous flows in channels (pipes), as well as near streamlined bodies and in boundary layers are considered in detail. The results of physical and mathematical modeling of particle-laden gas flows obtained in the last few years by both Russian and foreign research workers are described and analyzed.

The book is intended for research workers dealing with the study of the gas dynamics and heat and mass transfer of multiphase flows, as well as for teachers, post-graduates and students of universities.

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## **Preface**

In the last 20 to 30 years, a steady interest of numerous groups of researchers in the study of multiphase (heterogeneous) turbulent flows can be observed. Among the most widespread forms of heterogeneous flows there are gas flows which carry solid particles. "Gas–solid particles" heterogeneous flows take place in such natural phenomena as sandstorms, forest fire, volcanic eruptions, etc. Such flows also appear due to the human activity (formation of dust, exhaust of harmful substances during motion of transport facilities, atmospheric pollution by effluents of industrial enterprises).

Examples of technical facilities which use such flows are: facilities of sand- and shot-blasting treatment of various surfaces; pneumatic conveyers of loose materials; particle-size classifiers of polydisperse materials; different types of dust collectors; apparatuses of thermal preparation of coal in the schemes of power-technological use of a fuel; combustion chambers of heat engines; facilities for thermal treatment of loose materials; heat exchangers with two-phase working media, and so forth.

Hence, the study of gas flows carrying solid particles and the construction of the mechanics of heterogeneous media become very topical. By now, quite a voluminous theoretical and experimental material has been gathered devoted to most different aspects of the hydrodynamics and thermophysics of "gas–solid particles" heterogeneous flows (Abramovich et al., 1984; Babukha and Rabinovich, 1968; Babukha and Shraiber, 1972; Boothroyd, 1971; Clift et al., 1978; Crowe et al., 1998; Deich and Filippov, 1981; Dyunin et al., 1965; Friedlander, 1977; Fuks, 1955, 1962; Gorbis and Kalenderiayn, 1975; Gorbis and Spokoinyi, 1995; Gorbis, 1964, 1970; Grishin and Fomin, 1984; Hetsroni, 1982; Mednikov, 1981; Mikhatulin et al., 2007; Nigmatulin, 1978, 1987; Saltanov, 1972, 1979; Shraiber et al., 1980, 1987; Soo, 1967, 1989; Sternin and Shraiber, 1994; Sternin et al., 1980; Sternin, 1974; Sukomel et al., 1977; Volkov et al., 1994; Voloshchuk and Sedunov, 1975; Voloshchuk, 1971; Wallis, 1969; Yanenko et al., 1980). Despite this fact, it can be said with confi-

dence that the currently available theory of multiphase turbulent flows is far from completion.

Addition, into a turbulent flow, of a dispersed impurity in the form of particles makes the flow pattern more complicated. This is associated with the great diversity of properties (first of all, inertia ones) and concentrations of the particles introduced which leads to implementation of numerous regimes (classes) of gas suspension flows. Mathematical simulation of heterogeneous flows is complicated only because of the fact that the introduction of a dispersed impurity sharply raises the question as to the permissibility of describing the gas phase motion within the framework of the mechanics of continua. Physical modeling of heterogeneous flows is also difficult because of the inability of using contact methods of measurements (e.g., hot-wire anemometry), whereas the application of optical methods (laser anemometry, digital tracer visualization) is considerably limited at moderate and high dispersed phase concentrations. Thus, the methods of experimental and computational-theoretical investigations that have shown a good performance over the course of tens of years in studying single-phase flows are frequently inapplicable in principle for studying heterogeneous flows. The aforesaid hinders the development of the physics of multiphase flows. Despite this fact, the demands of the practice and the logic of science development call for the constant improvement of the theory of heterogeneous flows.

Investigations of heterogeneous flows are aimed at the solution of two basic classes of problems. The first (or direct) problem consists in studying the behavior of disperse particles suspended in a gas flow. The solution of this problem presupposes determination of the dispersed phase characteristics, namely, of the sizes of particles (in the case of a polydisperse flow), the fields of their velocities, temperatures, and concentrations. The second (or inverse) problem consists in studying the influence of particles on the characteristics of the gas flow carrying them. The solution of this problem presupposes determination of gas characteristics in the presence of particles: velocity and temperature fields, coefficients of friction and of heat transfer, and so on.

Various collisional processes can take place in heterogeneous flows. Relating to them are: collisions of particles among themselves (“particle–particle” interaction), collisions of particles with a body immersed in a heterogeneous flow (“particle–body” interaction), as well as collisions of particles with the walls that bound a heterogeneous flow (“particle–wall” interaction). All the above-mentioned collisional processes can play a substantial part in the formation of the statistical characteristics of the motion of particles and, consequent-

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ly, exert their influence on the characteristics of the gas flow carrying them. As a result, investigation of the role of contact interactions in the context of solving the two basic problems of studying heterogeneous flows seems to be extremely urgent.

The present monograph considers the problems associated with the role of collisional processes in particle-laden gas flows. The closest to the content of this book is the monograph (Varaksin, 2007). The problems considered there have gained further development in the present book.

The first, introductory, chapter contains brief information on single-phase and heterogeneous turbulent flows prerequisite for understanding the problems considered in what follows. The equations which describe the processes of transfer in single-phase incompressible flows are presented. The principal characteristics of single-phase flows and distributions of them over the section of channels are described. This information was adopted from the well-known literature sources and in no way claim originality. Also presented in the first chapter are the principal characteristics of gas flows which carry solid particles. The special features of the mathematical and physical modeling of heterogeneous flows responsible for the difficulties in studying them are considered briefly. Two basic classes of problems dealing with the investigations of particle-laden gas flows are described, and the current interest in studying various collisional processes in heterogeneous flows is shown. The quantities that characterize the processes of interparticle collisions and collisions of particles with channel walls are presented. Five dimensionless numbers are described: the Stokes numbers which characterize the dispersed phase inertia in the processes of relaxation of particle and gas velocities during averaged motion, large-scale fluctuation motion, fine-scale fluctuation motion, as well as the relaxation of the velocities of particles after their collisions amongst themselves and with the channel wall. In the final part of the chapter, the turbulent particle-laden gas flows are classified in terms of the volume concentration and inertia of particles; this classification is of great methodological value.

Chapter 2 gives consideration to the approaches and methods of mathematical and physical modeling of particle-laden gas flows. The entire history of the development of natural sciences confirms the reciprocal significance of theoretical and experimental methods of investigation. In constructing the theory of any physical phenomenon (however complex or simple it might seem to be at first glance), one must not underestimate the importance of particular methods of investigation. The process of constructing the theory of multiphase flows will be more efficient if the means of mathematical and physical model-

ing could complement each other. The possible ways of such “cooperation” are also evident. Thus, the results of measurements are widely used to verify the mathematical models being developed. In turn, the results of computations are intended to minimize the needed amount of experimental information, as well as to assist in deeper interpretation of the data obtainable in the course of measurements.

The second chapter contains the description of the principles of mathematical simulation of particle-laden gas flows. The characteristic features of simulation of different classes of heterogeneous flows are considered. The possibilities of calculating the motion of particles on the basis of the Lagrangian trajectory and Eulerian continuum approaches are described. The problems of accounting for turbulent gas pulsations and collisions of particles among themselves when calculating the motion of the latter are considered within the framework of the stochastic Lagrangian approach. Equations of gas motion with account for the inverse effect of particles on the carrier medium characteristics are presented. The presence of numerous regimes of gas suspension flow, the classification of which is attempted in Chapter 1, has led to the development of a large number of mathematical models of two-phase flows.

Chapter 2 contains the description of the algorithm suggested by the present author for a generalized computer model and intended for orientating specialists in selecting specific mathematical models for calculating applied and research problems. Considered in the chapter are also the principles of physical modeling of particle-laden gas flows with the use of the optical methods of diagnostics undergoing intense development. The characteristic features of the experimental investigation of two-phase flows are described. The principles of the laser Doppler anemometry (LDA) method are considered. A diversity of metrological problems of the use of LDA in studying the behavior of solid particles suspended in a gas flow are described. Among these is the optimization of the optoelectronic system for measuring instantaneous velocities of large particles, the development of the procedure of measuring the velocities of polydisperse and bidisperse particles, the development of the techniques of measuring the concentration of particles, and so on. The metrological problems of studying the inverse effect of particles on the carrier gas flow characteristics are described. Among them there is the procedure for estimating the cross disturbance, development of the principles of selecting signals, the procedure of estimating the efficiency of the amplitude selection of signals, and so forth. Considered in one of the sections are the possibilities and limitations of LDA in studying highly dusty two-phase flows in which the motion

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of particles is determined to a large measure by the process of interparticle collisions. Along with the description of the techniques of the LDA diagnostics of heterogeneous flows, much attention is also given to the problems of controlling the accuracy of measurements by computational–theoretical and experimental methods. Considered briefly are the principles of the particle image velocimetry (PIV). The application of the PIV method (the commonly accepted name of the method for determining the velocity of particles from their images) makes it possible to determine outright the instantaneous field of the velocities of particles. In conclusion, examples of experimental facilities for studying particle-laden air flows are given, and the principle of the selection of solid particle characteristics used in investigations of heterogeneous flows as a dispersed phase is described.

Chapter 3 is devoted to consideration of the processes of collisions of particles nearly identical in size (“particle–particle” interaction). The characteristic features of the motion of particles and the intensity of the occurring interphase processes depend largely on the inertia of the dispersed phase and its concentration in a flow. With increase in the concentration of particles, the probability of collisions between them increases; in this case, the physical mechanisms leading to collisions of particles among themselves can vary greatly. The beginning of the chapter is devoted to the theoretical description of the process of collisions of two particles on the basis of the laws of classical mechanics. Relationships for calculating the linear and angular velocities of particles after collision are given in a general form and for different particular cases. Two basic approaches used to analyze collisions of particles, viz., spherical and cylindrical ones are described. The processes of the collisions of particles of different sizes and densities during their gravitational deposition are considered. An analysis of the mechanisms of interparticle interactions during motion of particles in a turbulent flow is carried out. Collisions of a dispersed phase both in homogeneous isotropic turbulence and in the presence of averaged velocity shear and gravity action are considered. At the end of the chapter, the results of experimental and computational–theoretical investigations are presented which demonstrate the influence of interparticle collisions on the characteristics of motion of a heterogeneous flow.

In Chapter 4 a description and analysis of the processes of collisions of particles are given for the case where one of the particles has infinitely large dimensions (the “particle–wall” interaction). Along with the concentration constraint, the motion of particles can also experience geometrical constraint. The collisional interaction of particles with a wall can lead to an appreciable

change in the characteristics of both the dispersed phase and of the carrier gas. In the beginning of the chapter, the physical mechanisms of the deposition of particles on a wall are considered, as well as the influence of gravity, turbulent pulsations of the carrier gas velocity, inhomogeneity of the averaged gas velocity profile on the process of the displacement of particles in the radial direction. The estimation of the contribution of the indicated mechanisms to the deposition of particles at various heterogeneous flows parameters are made. The characteristics of a flow at which the presence of a wall exerts a dominating influence on the motion of particles are found. Relationships for calculating linear and angular velocities of particles after their collision with a wall, as well as relationships for various specific cases are given in a general form. Much attention is given to the description and analysis of available data on the values of the velocity recovery factor of particles and the coefficient of friction. Consideration is given to various models of the account for the wall roughness which is the most important parameter that exerts a considerable influence on the process of interaction of particles with the channel walls. In the final part of the chapter, the results of experimental and computational-theoretical investigations are given that demonstrate the influence of the collisions of particles with a wall, as well as of interparticle collisions on the characteristics of heterogeneous flow motion in horizontal and vertical channels.

Chapter 5 is devoted to consideration of the processes of collisions of particles differing greatly in size (the “particle–body” interaction). In this case, a larger particle can be considered as a body moving in a gas flow laden with a large amount of fine particles. The basic problems of studying heterogeneous flows around bodies involve investigation of the motion of particles near the body surface (determination of their trajectories), as well as determination of the influence of particles on a gas flow. At the beginning of the chapter, one of the most important physical characteristics of particle-laden flow around bodies is considered, namely, the coefficient of deposition of particles onto a surface. The dependence of the deposition coefficient on the inertia of particles (the Stokes numbers in averaging motion) is given and analyzed. The basic factors (the gravity, potentiality, nonisothermicity, and axisymmetry of a flow) that exert their influence on the value of the deposition coefficient are considered. The value of the concentration of particles near a surface can multiply exceed the “initial” value in an undisturbed flow. In one of the sections of Chapter 5, a description and analysis are given of the main mechanisms underlying the increase of the concentration of particles: deceleration of particles on approach to a body, interaction of particles with a wall (the

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appearance of the “phase” of reflected particles), as well as collisions of incident and reflected particles. It should be noted that precisely the value of the concentration of the dispersed phase that determines the presence and intensity of the processes of interparticle collision, as well as collisions with the surface of the body immersed in a flow. Further, the results of the computational-theoretical and experimental investigations of the characteristics of two-phase flows around differently shaped bodies are presented and analyzed. The characteristic features of flow around a sphere, lateral and longitudinal flow past a cylinder and plane wedges, lateral flow past a plate and turbine blades are considered. When particles move along the body surface in a boundary layer where there are appreciable velocity and temperature gradients (in the case of a nonisothermal flow), the distribution of their characteristics is frequently of complex character, with the concentration exceeding its value in the stream incident on the body. At the end of the chapter much attention is given to the description of the characteristic features of a heterogeneous flow in a boundary layer developing along the body surface. Considered and analyzed are the data of experiments on the distributions of the velocities of a “pure” air, air in the presence of particles, and of the particles themselves in all of the regions of the boundary layer developing along the model surface: laminar, transient, and turbulent. It is shown that the presence of particles in a flow can lead to an earlier beginning of the laminar–turbulent transition. Consideration is given to the results of the influence of particles on the intensity of the carrier air turbulence in a turbulent boundary layer.

The author wishes to express deep appreciation to A. I. Leontiev, Member of the Russian Academy of Sciences, V. M. Batenin, Corresponding Member of the Russian Academy of Sciences, Yu. V. Polezhaev, Corresponding Member of the Russian Academy of Sciences, for their longstanding support of this study and attention to it, as well as to A. F. Polyakov and L. I. Zaichik, Doctors of Technical Sciences, T. F. Ivanov and M. V. Protasov, Candidates of Sciences (in Physics and Mathematics), for the assistance in some investigations the results of which are used in the book.

## NOMENCLATURE

### Dimensional quantities

$a$	thermal diffusivity of gas, $\text{m}^2/\text{s}$
$C_p$	heat capacity of gas, $\text{J}/\text{kg}\cdot\text{K}$
$C_{p_p}$	heat capacity of the particle material, $\text{J}/\text{kg}\cdot\text{K}$
$D$	pipe diameter; diameter of the sphere (cylinder) of collisions, $\text{m}$
$d_p$	particle diameter, $\text{m}$
$f_c$	frequency of interparticle collisions, $\text{Hz}$
$f_{cw}$	frequency of collisions of particles with the pipe (channel) walls, $\text{Hz}$
$g$	free fall acceleration, $\text{m}/\text{s}^2$
$k$	turbulent energy of gas, $\text{m}^2/\text{s}^2$
$k_0$	turbulent energy of gas in the absence of particles, $\text{m}^2/\text{s}^2$
$k_p$	energy of particle velocity fluctuations, $\text{m}^2/\text{s}^2$
$L$	characteristic length, $\text{m}$
$l$	Prandtl–Nikuradze mixing length, $\text{m}$
$l_c$	mean free path of particles between successive collisions, $\text{m}$
$l_{cw}$	mean free path of particles between successive collisions with the pipe (channel) walls, $\text{m}$
$m_p$	particle mass, $\text{kg}$
$N$	number density of particles, $\text{m}^{-3}$
$p$	instantaneous pressure, $\text{Pa}$
$P$	averaged pressure, $\text{Pa}$
$p'$	fluctuation pressure, $\text{Pa}$
$R$	pipe radius; radius of the sphere (cylinder) of collisions, $\text{m}$
$r$	distance from the pipe axis, $\text{m}$
$r_p$	particle radius, $\text{m}$
$t$	instantaneous temperature of gas, $\text{K}$
$T$	averaged temperature of gas, $\text{K}$
$t'$	fluctuation temperature of gas, $\text{K}$
$T_f$	characteristic time of gas in averaged motion, $\text{s}$



## NOMENCLATURE

$T_L$	characteristic time of gas in large-scale fluctuation motion, s
$t_p$	instantaneous temperature of particle, K
$T_p$	averaged temperature of particle, K
$t'_p$	fluctuation temperature of particle, K
$u_*$	dynamic velocity, m/s
$u_i$	projections of instantaneous velocity of gas, m/s
$U_i$	projections of averaged velocity of gas, m/s
$u'_i$	projections of fluctuating velocity of gas, m/s
$v_i$	projections of instantaneous velocity of particle, m/s
$V_i$	projections of averaged velocity of particle, m/s
$v'_i$	projections of fluctuation velocity of particle, m/s
$w_i$	projections of instantaneous velocity of particles relative to gas and of colliding particles relative one another, m/s
$W_i$	projections of averaged velocity of particles relative to gas and of colliding particles relative one another, m/s
$w'_i$	projections of fluctuation velocity of particles relative to gas and of colliding particles relative to one another, m/s
$x, r, \varphi$	axial, radial, and azimuthal cylindrical coordinates, m, deg
$x, y, z$	axial, radial, and azimuthal Cartesian coordinates, m

### Greek symbols

$\beta$	core of collisions, m <sup>3</sup> /s
$\varepsilon$	rate of dissipation of turbulent energy, m <sup>2</sup> /s <sup>3</sup>
$\eta$	Kolmogorov scale, m
$\lambda$	thermal conductivity of gas, W/m·K
$\lambda_p$	thermal conductivity of the particle material, W/m·K
$\mu$	coefficient of dynamic viscosity, N·s/m <sup>2</sup>
$\nu$	coefficient of kinematic viscosity, m <sup>2</sup> /s
$\rho$	density of gas, kg/m <sup>3</sup>
$\rho_p$	density of the particle material, kg/m <sup>3</sup>
$\tau$	time, s
$\tau_c$	time between consecutive collisions of particles, s
$\tau_{cw}$	time between collisions of particles with the pipe (channel) walls, s
$\tau_K$	characteristic time of gas in small-scale fluctuation motion (Kolmogorov time scale of turbulence), s
$\tau_p$	time of dynamic relaxation of particle, s
$\tau_{p0}$	time of dynamic relaxation of Stokesian particle, s

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$\tau_t$	time of thermal relaxation of particle, s
$\tau_{t0}$	time of thermal relaxation of Stokesian particle, s
$\tau_w$	shear stress on the wall, Pa

### Dimensionless quantities

$C_D$	coefficient of aerodynamic resistance of particle
$C_{x0}$	body resistance coefficient
$C_{xp}$	coefficient of body resistance due to the effect of particles
$c_f$	coefficient of friction
$m$	instantaneous mass concentration of particles
$M$	averaged mass concentration of particles
$m'$	fluctuation mass concentration of particles
$Re_D$	Reynolds number
$Re_p$	averaged value of the Reynolds number of particle
$\tilde{Re}_p$	instantaneous value of the Reynolds number of particle
$Re'_p$	fluctuation value of the Reynolds number of particle
$Re_x$	local value of the Reynolds number based on longitudinal coordinate
$Stk_c$	Stokes number for the process of interparticle collisions
$Stk_{cw}$	Stokes number for the process of collisions of particles with the pipe (channel) wall
$Stk_f$	Stokes number in averaged motion
$Stk_K$	Stokes number in small-scale fluctuation motion
$Stk_L$	Stokes number in large-scale fluctuation motion
$\gamma$	Kármán constant
$\eta$	coefficient of particle deposition
$\varphi$	instantaneous volume concentration of particles
$\Phi$	averaged volume concentration of particles
$\varphi'$	fluctuation volume concentration of particles

### Indices

$\langle \dots \rangle$	averaging over the cross-sectional area of the pipe (channel)
$\overline{(\dots)}$	time averaging, relative value
$(\dots)'$	fluctuation value

### Subscripts

0	value in external flow, value in the absence of particles
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## NOMENCLATURE

$c$	value on the pipe (channel) axis
$i, j$	characteristic of the species of particle
$m$	modified value
$w$	value on the pipe (channel) wall

## **CHAPTER 1**

### **CONCISE INFORMATION ON SINGLE-PHASE AND HETEROGENEOUS TURBULENT FLOWS**

#### **1.1. Preliminary Remarks**

In subsequent chapters, in describing the characteristics of the motion of particles in heterogeneous flows, as well as the characteristics of a gas in the presence of particles, we will refer to the data on the corresponding characteristics of single-phase (in the absence of particles) flows considered in the present chapter. The information given in Sections 1.2 and 1.3 is borrowed from (Bradshaw et al., 1980; Bradshaw, 1971; Comte-Bellot, 1968; Dyban and Epik, 1985; Frost and Moulden, 1977; Hinze, 1959; Kuznetsov and Sabelnikov, 1986; Monin and Yaglom, 1965; Petukhov and Polyakov, 1986; Schlichting, 1969) where more detailed information on the problems treated is given.

Section 1.4 is devoted to a brief description of the specific features of the mathematical and physical modeling of heterogeneous flows responsible for the difficulty of studying this type of flows. Also considered in this section are two basic classes of problems related to investigations of particle-laden flows and the current interest in the study of various collisional processes in heterogeneous flows is shown.

Section 1.5 contains the characteristics of heterogeneous flows. The basic quantities that characterize the processes of interparticle collisions and collisions of particles with the channel walls are described. Dimensionless criteria — the Stokes numbers — that characterize the dispersed phase inertia relative to some physical processes including collisional interactions are given.

Section 1.6 presents the description of the suggested classification of “gas–solid particles” turbulent flows which is of great methodological impor-

tance due to the above-noted distinctive features of the study of heterogeneous flows.

## 1.2. Equations of Single-Phase Turbulent Flows

Given in this section are basic equations which describe turbulent single-phase flows.

**1.2.1. Actual equations.** The continuity, motion, and energy equations of an incompressible gas in actual variables in the absence of external mass forces have the form

$$\sum_j \frac{\partial u_j}{\partial x_j} = 0, \quad (1.2.1)$$

$$\frac{\partial u_i}{\partial \tau} + \sum_j u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \sum_j \frac{\partial^2 u_i}{\partial x_j \partial x_j}, \quad (1.2.2)$$

$$\frac{\partial t}{\partial \tau} + \sum_j u_j \frac{\partial t}{\partial x_j} = a \sum_j \frac{\partial^2 t}{\partial x_j \partial x_j}, \quad (1.2.3)$$

where  $i, j = 1, 2, 3$ .

**1.2.2. Averaged equations.** In conformity with O. Reynolds suggestion, the actual values of the turbulent flow parameters are represented as the sum of two components, namely,

$$\theta_i(\tau) = \Theta_i + \theta'_i(\tau), \quad (1.2.4)$$

where  $\Theta_i$  is the time-averaged local value of this quantity and  $\theta'_i$  is its fluctuation value (the deviation of the instantaneous value from the averaged one).

Time averaging is performed as

$$\Theta_i = \frac{1}{T} \int_0^T \theta_i(\tau) d\tau. \quad (1.2.5)$$

It should be noted that the averaging period  $T$ , on the one hand, must significantly exceed the characteristic time scale of turbulent fluctuations and, on the other, must be much less than the characteristic time of variation of the macroscopic parameters of turbulent flow.

After time averaging of Eqs. (1.2.1)–(1.2.3) according to O. Reynolds, we obtain averaged equations of continuity, motion, and energy in the following form:

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