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by

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Vikram Sarabhai Space Center
Trivandrum, India

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With a Ph.D. in Applied Mathematics, received in 1973 from Banaras Hindu University, Varanasi, K. N. Shukla has been working as a senior scientist in the Vikram Sarabhai Space Center, Trivandrum, India. He has over 30 years R&D experience in fluid mechanics and heat transfer, with notable contributions in thermal mathematical modeling, analysis and simulation, microgravity fluid dynamics, computational fluid dynamics, phase-change heat transfer, thermal protection systems, heat pipes, drying of solids, diffusion, and reaction. He is a recipient of an Alexander von Humboldt Fellowship at the University of Stuttgart and at the University of Technology, Munich, Germany. He has guided Ph.D. theses on numerical characterization and computer simulation of heat transfer in solid rocket nozzles, and several master theses at the University of Technology, Munich, and Kerala University on high-temperature thermal insulation. He has authored over 60 publications in archival journals and a monograph on *Diffusion Processes During Drying of Solids* (World Scientific, 1990).

As a professional recognition, biographical lines about Dr. Shukla appear in *World's Who's Who in Science and Engineering, Who's Who in the World, Who's Who in Asia and the Pacific Nations*, and several other reference books.

K. N. Shukla

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PREFACE

This book presents an investigative account of Mathematical Principles of Heat Transfer. It is concerned with three aspects of heat transfer analysis: theoretical development of conservation equations, analytical and numerical techniques of the solution, and the physical processes involved in the three basic modes of heat transfer, namely, conduction, convection, and radiation. A concept of mathematical modeling is developed through the use of differential equations. In doing so, the well-posed boundary value problems are constructed and the solutions are attempted.

The analytical solution techniques, such as separation of variables, Integral transforms, Green's function, and some approximate methods, e.g., the integral and variational methods, are described. The finite difference method for the partial differential equation is derived from the first principle. Convergence of the various difference schemes is established through solved examples. The stability and the compatibility of the difference schemes are discussed. For the sake of generality, one chapter is devoted to the similarity theory and the generalized variables; that enables presentation of the solution in the dimensionless form. The physical processes involved in the basic mode of heat transfer are described. The problems of steady and transient heat conduction are presented as boundary value problems and their solutions are obtained for a variety of geometrical shapes and boundary conditions. Also discussed is the process of heat conduction during melting or freezing. The fundamentals of convection are introduced and the equations for convective heat transfer are derived. With simplifications introduced by boundary layer approximations and considering the effects of turbulence, an attempt is made to model actual flow conditions. The process of free and forced convection is described and the problem of laminar free convection on a vertical surface is cast as a well-posed boundary value problem. The fundamental concept of radiative heat transfer is discussed and the method to find the radiative heat exchange between gray surfaces in an enclosure is outlined.

The text material is organized in such a way as to both give an exposure of practical thermal problems to applied mathematicians and introduce advanced mathematical techniques used in solving complex thermal problems to engineers. While emphasizing the formulation of the thermal problems and their solution procedures, the rigors of mathematical abstraction are avoided. In this way, the book attempts to bridge the gap between mathematicians and engineers. Further, a review of the recent literature on each topic and the references provided therein will trigger the curiosity of the reader and advance his understanding of the subject.

The book, designed for students and the research communities of engineering and applied mathematics, may also attract a wide range of readership from practicing mathematicians and engineers in industry.

It gives me great pleasure to acknowledge the help that I have received from a great

number of specialists in the field. I am indebted to Professor R.N. Pandey for introducing me to heat transfer research. I owe a special debt of gratitude to (late) Professor Thomas Irvine and (late) Professor Edgar Winter for their interest in my book proposal. I thank Professor Manfred Groll, University of Stuttgart, and Dr. Jurgen Blumenberg, University of Technology, Munich, for many fruitful discussions. I thank Dr. A.R. Acharya, Dr. V. Adimurthy, and Dr. R.C. Mehta for perusal of the manuscript, and Dr. V. Jones, K. P. Khosla, R.V. Ramanan, and Dr. S.B. Tiwari for their help in the preparation of the manuscript. I also thank the Alexander von Humboldt Foundation for providing me with a research fellowship at the University of Stuttgart and the University of Technology, Munich, during the preparation of the manuscript. I also thank the Director, VSSC, for permission to publish the book.

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Also, please note that the utmost care has been taken in checking the lengthy derivations, but it is quite possible that some of them might have gone unnoticed. I express my gratitude to the readers in advance for all suggestions for further improvement of the book.

K. N. Shukla
July 22, 2004

CHAPTER 1

Basic Concepts of Heat Transfer

This book presents a descriptive analysis of the mathematical development of heat transfer. In any discussion on heat transfer, it is appropriate to recall the definition of heat. Up to the beginning of the nineteenth century, heat was considered an invisible elastic fluid, known as caloric, which could neither be created nor destroyed. Caloric was the first really useful tool for describing heat transfer processes. Later, Count Rumford (1753–1814) concluded from his famous cannon-boring experiments that the source of heat generated by friction appeared evidently to be inexhaustible and cannot be a material substance. It was extremely difficult, if not impossible, to form any distinct idea of anything capable of being excited and communicated in those experiments, except that it was motion. While the caloric description of heat was widely accepted, James Clark Maxwell (1831–1879) provided a precise description of the mechanism of heat propagation in gases. By predicting how energy was passed from molecule to molecule during collisions, he showed that heat was really a mode of motion. Thus, the movement of something from a hot body to a cold body can be called heat. An appropriate definition is: heat is that which is transferred between a system and its surroundings as a result of temperature difference only.

1.1 Basic Modes of Heat Transfer

Heat is transferred by three basic modes: conduction, convection, and radiation. In two of these modes, a medium is required to transfer heat from one point to the other, whereas in the third mode it is transmitted through empty space as well as through certain materials transparent to thermal radiation. In reality, temperature distribution in a medium is controlled by the combined effects of these three basic modes of heat transfer, and it is difficult to entirely isolate one mode of heat transfer from interactions with the other two modes. However, one mode may be dominant with respect to the other modes and, for simplicity, one can isolate this mode for analysis while neglecting the influence of the other two modes. Before giving a brief qualitative description of these three basic modes, we will introduce the concepts of temperature and its gradient.

1.1.1 Temperature Field

Temperature is a measurement of the energy level within a molecule. A point in a body at a higher temperature with respect to another point in that body means that the former is at a higher energy level in comparison to the latter. Thus, the physical phenomenon associated with the energy transfer is described by a change of its physical properties with respect to space and time. The temperature at a point in a body is thus described as

$$T = T\,(x,y,z,t) \tag{1.1}$$

Equation (1.1) is the mathematical description of the temperature field of a body that defines the temperature at a point (x,y,z) at any instant of time t.

The scale of measurement of temperature is based on the triple point of water (at which solid, liquid, and vapor can coexist in equilibrium under its own vapor pressure), which is used to establish the size of a degree on an absolute temperature scale. This temperature is defined to be 273.16 on the Kelvin temperature scale. The more familiar Celsius (previously known as centigrade) and Fahrenheit temperature scales were defined in terms of the ice point of water, the temperature at which melting or fusion takes place under a total pressure of one atmosphere. This temperature is 0°C or 32°F and is known to be 0.01 K below the triple point.

1.1.2 Temperature Gradient

If we join all of the points of a body with the same temperature, we obtain a surface of equal temperature. This surface is known as an isothermal surface. Thus, an isothermal surface on a body represents the locus of the points having the same temperature field. Figure 1.1 shows the isothermal surfaces for the temperature fields $T - \Delta T$, T, and $T + \Delta T$, respectively. These isothermal surfaces cannot intersect each other because no one point on the body will be at two different temperatures at a time. Intersection of isothermal surfaces by a plane gives a family of isotherms on this plane. The family of isotherms possesses the following properties: The isothermal surfaces do not intersect each other, they are continuous, and they originate or end at the surfaces or within the body.

Temperature in a body varies only in directions crossing isothermal surfaces. The temperature difference per unit length is a maximum in the direction normal to the isothermal surfaces. An increase in the temperature difference in this direction is characterized by the temperature gradient. The temperature gradient is a vector normal to the

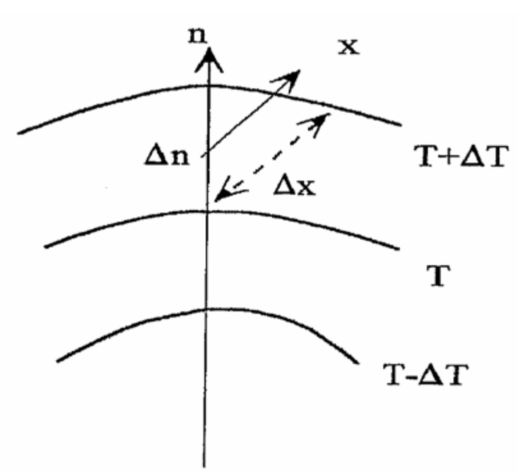

Figure 1.1 Isotherms representation

isothermal surface and is positive in the direction of increasing temperature. It is mathematically described as

$$\text{grad } T = \mathbf{n}\partial T/\partial n \tag{1.2}$$

where **n** is the unit vector normal to the isothermal surface and positive in the direction of increasing temperature and $\partial T/\partial n$ is the temperature derivative along the normal n. The magnitude of $\partial T/\partial n$ is negative in the direction of decreasing temperature.

1.2 Conduction

Conduction is the mode of heat transfer in which thermal energy is transmitted by direct molecular communication, without appreciable displacement of the molecules. A common example of heat conduction is when one touches one end of an iron rod that is heated at the other end. Conduction occurs through solids, liquids, and gases, and from one body to another when they are in physical contact with each other. In the conduction mode of heat transfer, the flow of heat takes place from the region of high temperature to the region of low temperature. The law of heat conduction originates from the experimental observation of Joseph Biot (1774–1862), but it was French mathematician, Joseph Fourier (1768–1830), who formulated the laws governing the flow of heat in solids and it is attributed to him as Fourier's law of heat conduction. As per Fourier, the rate of heat conduction through a solid material is proportional to the temperature gradient across the material and to the area perpendicular to the heat flow. The rate of heat transfer through a unit area of an isothermal surface is determined by the relation

$$\mathbf{q} = -\mathbf{n}\lambda\frac{\partial T}{\partial n}, \ (\text{W/m}^2) \tag{1.3}$$

where a vector quantity **q** represents the rate of heat flow or heat flux and acts in the direction normal to the isothermal surface. The proportionality constant λ is called the thermal conductivity of the material and is always positive. The minus sign in the right-hand side of Eq. (1.3) ensures that **q** is positive because heat always flows from the higher temperature level to a lower temperature level.

The proportionality law between the flux (heat) and the force (temperature gradient) has analogues in electrical conduction, mass diffusion, and fluid flow. For example, Ohm's law of electrical conduction states that the electric current is directly proportional to the potential difference, Fick's law of diffusion in a binary system states that the mass flux of either component is directly proportional to the concentration gradient and, similarly, Newton's law for fluid motion states that stress is directly proportional to the velocity gradient.

1.3 Thermal Conductivity

Thermal conductivity λ, which is analogous to electrical conductivity, is a property of the material. It is defined as an equivalent to the rate of heat transfer between opposite faces of a unit cube of the material that are maintained at temperatures differing by one degree. In the international system of units (SI), which is also referred to as the MKSA system, λ is expressed as (W/m K). It is determined from the relation

$$\lambda = \frac{|\mathbf{q}|}{|\nabla T|} \tag{1.4}$$

These basic units measure quantities that could vary considerably in magnitude. To avoid awkwardly large or small figures, common prefixes representing multiples of 10 are given in Table 1.7 for SI units.

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