LASER DIAGNOSTICS
IN FLUID MECHANICS
LASER DIAGNOSTICS IN FLUID MECHANICS

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Abstract

To the bright memory of my parents

Modern the flow parameters measuring techniques are under discussion.

The physical nature of the laser methods as well as the optical systems design are discussed.

Data important for prediction, development, and design of laser devices are presented. The laser methods of fluid, gas and two-phase flows investigating in velocity range from $10^{-8}$ to $10^4$ m/s are described.

The monograph is intended for engineers and scientists concerned with laser-aided investigations as well as students and post-graduate students.
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Preface to Russian Edition

Laser metrology as a link between fundamental science and modern production ranks high among numerous applications of lasers.

Wide use of lasers in different measurements owes to their high monochromaticity, coherence, and narrow radiation beams, high power and possibility of continuous or pulsed generation. These features have promoted quick development of laser metrology, the potential of which is far from being exhausted. Naturally, a transition to a new laser standard of a meter will stimulate improvement of the old and design of new effective measuring methods and systems.

In metrology, laser diagnostics of liquid and gas flows holds a particular place. In this fields, most effectively used are the basic properties of laser radiation which have given rise to such new measurements techniques as holography, Doppler anemometry, speckle-interferometry, etc.

The goal of the present monograph is to help researchers in mastering flow laser diagnostics methods which essentially differ from those applied earlier and are even advantageous with respect to many parameters. The author is among the pioneers in this field of laser application, has a great deal of experience in development and use of laser techniques, in particular, of the Doppler local flow velocities measuring.

The present book is the generalization of the author's multiyear experience and the results produced by the scientific group which he headed. Also, it covers the most important results of research studies conducted in our country and abroad. A particular attention is paid to the analysis of optical aspects of flow laser diagnostics. Alongside the widely used Doppler velocity measurements technique
and the other methods essentially complementing it are under discussion.

In flow diagnostics, laser Doppler anemometers manufactured by many firms are most widely employed. The flow laser diagnostics is recognized as the main experimental technique in some fields of science and technology. Laser Doppler anemometers have become the necessary tool in aerohydrodynamic laboratories. Introduction of the Doppler measuring systems into commercial practice has been restricted by the complexity of their service. At present the situation changes for the better. Use of semiconductor lasers, fiber optics and computing facilities gives a hope that the nearest years will see a new generation of laser Doppler anemometers both reliable in operation and simple in use.

Computing facilities occupy a particular place in development of flow laser diagnostics. Rational combination of optical techniques with high potentialities of computer and, particularly, personal computer are a vivid example of a new generation highly intellectual laser measuring system development.

The book will be useful for scientists developing and using the laser flow diagnostics techniques in research practice of their laboratories. It may be recommended to all those who tend to master and bring into practice the new advanced measurement technique. The monograph will be of help as well to students and post-graduate students of the relevant specialties.

Professor V. A. Fabrikant
At present, the laser diagnostics of fluid and gas flows has evolved into a self-sustained area of laser metrology. Methodologically, the laser diagnostics is based on probing of a fluid flow by a laser beam and measuring of transmitted or scattered laser light characteristics. Next, by solving an appropriate inverse problem, one obtains the desired fluid flow parameters. Scientific intuition supported by a profound theoretical knowledge, experimental skill, and a clear understanding of the potentialities of adopted techniques constitute the basis of any physical research. All this holds in full measure for such a scientific area as laser diagnostics.

The purpose of this book is to describe the methods of laser diagnostics of fluid and gas flows that are currently finding wide application in aero- and hydrodynamic studies in various fields of science and technology - power engineering, space and aircraft industry, chemical engineering, metallurgy, biology, environmental science – to name but a few.

In the first part of the book (Chapters 1, 2, 3), the general aspects of physical optics, laser techniques and coherent radiation scattering theory are considered that are essential for understanding of anemometry, interferometry, refractometry, ellipsometry, particle image velocimetry, high-speed filming and other methods used in laser flow diagnostics.

The second part of the book (Chapters 4, 5, 6) is mainly concerned with the Doppler technique of fluid flows local velocities measuring and its use in the analysis of various flow characteristics. Described here are the optical layouts and signal shaping techniques for laser Doppler anemometers and their applica-
tion in fluid flows diagnostics are illustrated by numerous examples.

At present, a vast variety of measuring devices for studying of fluid flows are commercially available, and to make their practical use most effective, one needs a solid professional background to assess the strong and weak points of a particular technique chosen for a particular experimental research. I hope that this book will be suited both for a designer of novel advanced techniques and a user of commercially available instrumentation.

Undoubtedly, this aim has been achieved only in part. Out of the vast wealth of available materials, preference has been given to those topics which, in the author's opinion, may be of particular importance for the reader. At need for more detailed information an interested reader may refer to cited sources.

The English edition of this book differs in many respects from the former Russian version. Actually, all chapters have been revised. Chapter 1 has been supplemented with Section 1.8 in which the principle of CCD camera technique is outlined and its use for optical radiation recording is discussed.

In Chapter 2 major changes have been made in Section 2.2, where the description of laser effect is given in a more concise form. This was done in view of the vast literature that is currently concerned with this topic.

Chapter 3 was supplemented with Section 3.5 where optical methods of particle sizing are described.

Chapter 4 has been substantially extended by adding of three new sections: "Flow Visualisation Using Laser Sheet", "Doppler Global Velocimetry", and "Particle Image Velocimetry".

Also, three sections have been added to Chapter 5. Section 5.7 is concerned with the integral Doppler spectrum for laminar, turbulent and two-phase flows. Section 5.8 is dedicated to an analysis of the Doppler signal frequency measuring conventional techniques over a wide range from a few Hz to a few GHz units. Section 5.9 is focused on computer-assisted methods of Doppler signal processing.

Chapter 6 has been augmented by Section 6.6 where the laser hydrophone principle is described. Bibliography of the English edition has been extended by adding the recently reported books and journal articles.

Before submitting this book to the readers approval, the author deems it his duty to mention the name of V. A. Fabrikant, the author's teacher, co-author, and the editor of the Russian edition of this book. Regrettably, V. A. Fabrikant has not lived to the appearance of the English edition of this monograph.

V. A. Fabrikant was a continuer in the field of research of the famous Russian physicists S. I. Vavilov, G. S. Landsberg and L. I. Mandel'shtam. His studies into the gas-discharge optics and quantum electronics and his educational practice obtained recognition both in this country and abroad. The contribution of V. A. Fabrikant's works to the emergence of quantum electronics has been vividly demonstrated in the book by M. Bertolotti "Masers and Lasers. A Historical
Approach". Intellectual by birth, he shared the fate of working intelligentsia in the tragically years of the Soviet Russia. Academician A. D. Sakharov, the world-famous scientist and humanist, who maintained friendly with V. A. Fabrikant and who was Fabrikant's coworker over a certain period of time, wrote: "His fate in science was full of dramatism... He proposed the principle of laser and maser effects, ... but the delight of implementing those remarkable ideas -- and the fame of it -- have become possessions of other people".

In 1951 V. Fabrikant, together with M. Budynskii and F. Butaeva, filed an application for a patent on a new electromagnetic radiation amplification method. It was shown that a medium with population inversion causes an exponential growth in passing radiation intensity. From this application, first a certificate of authorship (1959) and then a discovery diploma (1964) came.

Thus, V. Fabrikant not only suggested the inverse population idea, but he was also able to propose a number of experimental methods for achieving inversion, practically implemented today. Furthermore, he discovered electromagnetic radiation amplification in media with population inversion -- a concept which has enabled quantum devices, most notably the laser, to be designed.

Following the advent of the laser in 1960, most of V. Fabrikant's activity focused on gas discharge optics and on developing the physical principles which now underlie the use of lasers in fluid flow diagnostics -- an entirely novel and promising application area for the so-called laser Doppler effect. Under his guidance a number of dissertations were written on the subject of flow diagnostics, in which some problems of laser Doppler anemometry have been resolved and which enabled experimental facilities for carrying out research in a wide range of flow velocities (from superflow ones in expanding solids to hypersonic ones in industrial wind tunnels) and flow regimes (laminar, turbulent, and self-glowing) to be developed.

I wish to express my sincere thanks to my colleagues and post-graduate students who have given permission to have some of their results included in the book. Furthermore, this book would have never had the chance to appear without the kind attitude of many co-workers of the V. A. Fabrikant Department of Physics at the Moscow Power Engineering Institute (Technical University). Also, I would like to thank Dr. P. de Groot from the DSM Research (Netherlands) for fruitful and stimulating discussions that contributed much to a higher quality of the book. I highly appreciate the enthusiasm and benevolence of the Publisher, Mr. W. Begel, whose patience and never-failing kindness have made it possible for the book to see the light.

Moscow, July 1996 Bronius Rinkevichius
1

Fundamental Laws of Optics

1.1 Electromagnetic Optical Waves

Optical radiation represents propagation of electromagnetic waves with a wavelength of 1 mm to 1 μm which corresponds to a frequency range from $3 \times 10^{17}$ to $3 \times 10^{11}$ Hz. This radiation in a wavelength range between 0.38 and 0.76 μm registered by an unaided man's eye is called visible light or, simply, light. In dependence on perception light may be separated into colors from violet to red. A wavelength, μm, of violet radiation is 0.380 to 0.455, of blue 0.455 to 0.485, blue-green 0.485 to 0.505, green 0.505 to 0.550, yellow-green 0.550 to 0.575, yellow 0.575 to 0.587, orange 0.587 to 0.610, red 0.610 to 0.760. Ultraviolet radiation is the invisible radiation, a spectral region of which lies between visible and X-radiation within the limits of wavelengths from 1 nm to 0.38 μm. Infrared radiation is also invisible and spans a spectrum with wavelengths of 0.78 to 1000 μm.

In laser diagnostics of fluid flows, visible and infrared radiation with a wavelength of 0.4 to 11.0 μm is mainly employed. In this range many media are transparent (radiation with a wavelength shorter than 0.28 μm propagated only in vacuum). Besides, for the given wavelength range high-quantity laser sources and sensitive photodetectors are available.

Optical radiation may be characterized not only by wavelength $\lambda$ and oscill-
Fig. 1.1 The scale of optical radiation parameters. 1 – blackbody radiation spectrum at $T = 6000 \text{ K}$; 2 – eye sensitivity curve; UV – ultraviolet; V – visible; IR – infrared spectrum.

...tion frequency $\nu$ but also such related parameters as quantum energy $h\nu$, J, and a quantity reciprocal to a wavelength, i.e. $\lambda^{-1}$, cm$^{-1}$. The former parameter is mainly adopted in an ultraviolet range, while the latter refers to infrared radiation. A scale of electromagnetic optical waves is shown in Fig. 1.1.

In a homogeneous, isotropic, non-absorbing steady-state medium an electric field vector of an electromagnetic wave satisfied the following wave equation [1]:

$$\nabla^2 E - \frac{1}{\nu^2} \frac{\partial^2 E}{\partial t^2} = 0$$ (1.1)

where $E$ is the electric field intensity, $\nu$ is the wave propagation velocity. Of keen interest are solutions of Equation (1.1) for plane and spherical waves. In the first case, a solution of the wave equation is of the form:

$$E(\mathbf{r}, t) = A \cos(\omega t - \mathbf{k}\cdot\mathbf{r} + \delta)$$ (1.2)

where $A$ is the electric field amplitude, V/m; $\omega$ is the circular frequency, rad/s; $\mathbf{k}$ is the wave vector, $|\mathbf{k}| = 2\pi/\lambda$, rad/m; $\lambda$ is the wavelength; $\delta$ is the initial phase; $\mathbf{r}$ is the radius-vector of a space point.
A surface, for which \( kr = \text{const} \), determines a wave front. In the given case a wave front is a plane which a wave vector \( k \) is perpendicular to.

For a homogeneous spherical wave:

\[
E(r, t) = \frac{B}{r} \cos(\omega t - kr + \delta)
\]

An electric field amplitude of a spherical wave, \( B/r \), decreases with a distance, a wave front is of a spherical form. A phase velocity of propagation of an electromagnetic wave depends on medium properties in the following manner:

\[
\nu = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}
\]

where \( \varepsilon \) is the relative dielectric constant of a medium; \( \varepsilon_0 = 8.85 \cdot 10^{-12} \) F/m is the electric constant; \( \mu \) is the relative magnetic permeability, for an optical range in the majority of cases \( \mu = 1 \); \( \mu_0 = 1.257 \cdot 10^{-6} \) H/m is the magnetic constant. An expression for a velocity may be written in a more familiar form:

\[
\nu = \frac{c}{\sqrt{\varepsilon}} = \frac{c}{n}
\]

where \( c = 1/\sqrt{\varepsilon_0 \mu_0} = 2.9979246 \cdot 10^8 \) m/s is the light velocity in vacuum, \( n \) is the refractive index of a medium.

A refraction index is one of the main optical characteristics of a medium. It depends on a wavelength of optical radiation and parameters of a medium itself, namely, a temperature, a pressure, a density, a chemical composition. For instance, a refractive index of dry air \( n_0 \) depends on a light wavelength, a pressure and a temperature [2]. At \( p = 1.01 \cdot 10^5 \) Pa (760 mm Hg), \( t = 15^\circ \text{C} \):

\[
n_0 = 1 + 10^{-6} \left[ 64.328 + \frac{29498.1}{146 - 10^6/\lambda_0^2} + \frac{255.40}{41 - 10^6/\lambda_0^2} \right]
\]

where \( \lambda_0 \) is the wavelength of light in vacuum, nm.

In a general case:

\[
n_{tp} - 1 = (n_0 - 1) \frac{p[1 + (1.049 - 0.0157t) \cdot 10^{-6}p]}{720.883(1 + 0.003661t)}
\]

A refractive index of moist air is
### Table 1.1 Refractive indices of some gases, fluids and solids

<table>
<thead>
<tr>
<th>Medium</th>
<th>$\lambda$, $\mu$m</th>
<th>$n$</th>
<th>$dn/dt$, °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.0002724</td>
<td>1.0002793</td>
<td>0.927·10^{-6}</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1.0002531</td>
<td>1.0004197</td>
<td>0.864·10^{-6}</td>
</tr>
<tr>
<td>Oxygen</td>
<td>1.0002354</td>
<td>1.3314</td>
<td>-0.985·10^{-4}</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.6328</td>
<td>1.3253</td>
<td>-4.0·10^{-4}</td>
</tr>
<tr>
<td>Steam</td>
<td></td>
<td>1.4950</td>
<td>-6.40·10^{-4}</td>
</tr>
<tr>
<td>Glass LK1</td>
<td></td>
<td>1.4398</td>
<td></td>
</tr>
<tr>
<td>Glass K8</td>
<td>0.5893</td>
<td>1.5163</td>
<td></td>
</tr>
<tr>
<td>Glass TBF3</td>
<td></td>
<td>1.7557</td>
<td></td>
</tr>
<tr>
<td>Quartz glass</td>
<td>0.6563</td>
<td>1.4566</td>
<td></td>
</tr>
<tr>
<td>Ditto</td>
<td>1.050</td>
<td>1.4500</td>
<td></td>
</tr>
<tr>
<td>Ditto</td>
<td>3.500</td>
<td>1.4062</td>
<td></td>
</tr>
<tr>
<td>Germanium</td>
<td>2.48</td>
<td>4.08</td>
<td></td>
</tr>
<tr>
<td>Ditto</td>
<td>10.00</td>
<td>4.0943</td>
<td></td>
</tr>
<tr>
<td>Silicon</td>
<td>10.00</td>
<td>3.4215</td>
<td></td>
</tr>
<tr>
<td>Gallium arsenide</td>
<td>10.3</td>
<td>3.2727</td>
<td></td>
</tr>
<tr>
<td>Optical ceramics</td>
<td>10.0</td>
<td>2.1985</td>
<td></td>
</tr>
</tbody>
</table>

$$n = n_0 - 4.15 \cdot 10^{-10} m$$

where $m$ is the partial pressure of steam, Pa.

Refractive indices of some media often used in laser diagnostics of flows are listed in Table 1.1 [2–4].

Absorbing media are characterized by a complex refractive index

$$\hat{n} = n + j\chi$$

where $\chi$ is the main absorption index [1].

When an electromagnetic wave propagates in an absorbing medium, its amplitude decreases according to the Bouguer law:

$$A(z) = A_0 \exp \left\{-\frac{\chi \omega z}{c}\right\} = A_0 \exp \{-a'z\}$$
where $\alpha' = \chi \omega / c$ is the natural absorption index, cm$^{-1}$. At $\alpha' = 1$ cm$^{-1}$ a wave amplitude decreases in $e$ times at a 1 cm distance. A natural absorption index depends on medium properties and a radiation wavelength. Figure 1.2 represents a refractive index and a main absorption index versus a wavelength for water. Water is most transparent for green radiation, while for ultraviolet and infrared radiation it is opaque. A refractive index of sea water depends also on salinity and temperature [5].

A velocity of an electromagnetic wave in anisotropic media depends on a direction of its propagation and orientation of crystals. For single-axis crystals a refractive index of a non-ordinary wave depends on a direction of its propagation:

$$n(\theta) = \frac{n_0 n_e}{\sqrt{n_e^2 \cos^2 \theta + n_0^2 \sin^2 \theta}} \quad (1.4)$$

where $\theta$ is the angle between a direction of wave propagation and an optical axis of a crystal, $n_e$ – the main refractive index of a non-ordinary wave, $n_0$ – the refractive index of an ordinary wave.

Values of $n_0$ and $n_e$ for some crystals used in laser devices [6, 7] are given in Table 1.2. A light beam incident on a crystal produces two beams at the outlet because of birefringence (a birefringence phenomenon is not observed if a light beam propagates along an optical axis of a crystal).

In the case of powerful laser beams it is necessary to take into consideration
Table 1.2  Refractive indices of crystals

<table>
<thead>
<tr>
<th>Crystal</th>
<th>$\lambda, \mu m$</th>
<th>$n_0$</th>
<th>$n_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>0.1855</td>
<td>1.6758</td>
<td>1.6899</td>
</tr>
<tr>
<td>Ditto</td>
<td>0.6278</td>
<td>1.5428</td>
<td>1.5519</td>
</tr>
<tr>
<td>Ditto</td>
<td>3.0000</td>
<td>1.4996</td>
<td>1.5070</td>
</tr>
<tr>
<td>Iceland spar</td>
<td>0.4861</td>
<td>1.6678</td>
<td>1.4907</td>
</tr>
<tr>
<td>Calcite</td>
<td>0.4861</td>
<td>1.6678</td>
<td>1.4898</td>
</tr>
<tr>
<td>Ditto</td>
<td>0.6563</td>
<td>1.6544</td>
<td>1.4846</td>
</tr>
<tr>
<td>Tourmaline</td>
<td>0.5893</td>
<td>1.669</td>
<td>1.638</td>
</tr>
<tr>
<td>Saphire</td>
<td>0.5893</td>
<td>1.769</td>
<td>1.760</td>
</tr>
<tr>
<td>Ditto</td>
<td>0.6328</td>
<td>3.019</td>
<td>2.739</td>
</tr>
<tr>
<td>Ditto</td>
<td>1.014</td>
<td>2.8264</td>
<td>2.5901</td>
</tr>
<tr>
<td>Ammonium dihydrophosphate (ADP)</td>
<td>0.5461</td>
<td>1.5266</td>
<td>1.4808</td>
</tr>
<tr>
<td>Potassium dihydrophosphate (KDP)</td>
<td>0.6328</td>
<td>1.5217</td>
<td>1.4768</td>
</tr>
<tr>
<td>Lithium niobate (LiNbO$_3$)</td>
<td>0.6000</td>
<td>2.3002</td>
<td>2.2083</td>
</tr>
</tbody>
</table>

the non-linear character of interaction between radiation and a medium since the latter is characterized in this case by a refractive index dependent on a power density of incident radiation.

Now consider a vector character of an electromagnetic wave. If a plane wave propagates along a z-axis, then projections of an electric field vector are determined by the following relations:

\[
E_x(z, t) = A_x \cos(\omega t - kz + \delta_x)
\]

\[
E_y(z, t) = A_y \cos(\omega t - kz + \delta_y)
\]  

(1.5)

where $A_x, A_y$ are the amplitude components of an electric field vector for $x$- and $y$-axis; $\delta_x, \delta_y$ are the initial phases of the components of an electric field vector. The component $E_z$ equals to zero since electromagnetic waves are transverse. At $A_y = 0$ a wave is linearly polarized along the $x$-axis, while at $A_x = 0$, along the $y$-axis.

In a general case the end of the vector $E$ circumscribes a complex curve. A projection of the curve into a plane perpendicular to the direction of propagation has an elliptical form. In particular cases when a phase difference $\delta = \delta_y - \delta_x$ is
Table 1.3 Kinds of polarization of the electromagnetic waves

<table>
<thead>
<tr>
<th>$\delta = 0$</th>
<th>$0 &lt; \delta &lt; \pi/2$</th>
<th>$\delta = \pi/2$</th>
<th>$\pi/2 &lt; \delta &lt; \pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = \pi$</td>
<td>$\pi &lt; \delta &lt; 3\pi/2$</td>
<td>$\delta = 3\pi/2$</td>
<td>$3\pi/2 &lt; \delta &lt; 2\pi$</td>
</tr>
</tbody>
</table>

equal to $m\pi$ ($m = 0, \pm 1, \pm 2, \ldots$) an ellipse transforms into a straight line (linear polarization); when $\delta = m\pi/2$, ($m = \pm 1, \pm 3, \pm 5, \ldots$) and $A_x = A_y$, an ellipse becomes a circle (circular polarization).

Thus, an electromagnetic wave may be characterized by linear, circular and elliptic polarization in dependence on a phase difference $\delta$, and a relationship between amplitude components (Table 1.3).

The presented description of polarization of electromagnetic waves is complete and unambiguous. However, the other ways of description are also known [8].

Consider the most general kind of polarization, i.e., the elliptic polarization characterized by three parameters, namely, amplitude components $A_x, A_y$ and a phase difference $\delta$. In practice, such parameters as azimuthal angle $\theta$, ellipticity
Fig. 1.3 Parameters of an elliptically polarized wave.

\( e \), direction of rotation, amplitudes \( a \) and \( b \), absolute phase \( \delta \) are often used (Fig. 1.3).

**Azimuthal angle** \( \theta \) is the angle between the \( x \)-axis and the major axis of an ellipse which determine orientation of an ellipse and takes values \(-\pi/2 \leq \theta \leq \pi/2\).

**Ellipticity** \( e \) is the length ratio of minor \( a \) and major \( b \) semiaxes of an ellipse; \( e = a/b \); it is obvious that \( 0 \leq e \leq 1 \).

**Direction of rotation:** it is the right polarization if vector \( \mathbf{E} \) clockwise and is the left polarization if it rotates counterclockwise. Usually the definition of direction of vector \( \mathbf{E} \) rotation is combined with that of ellipticity \( e \): at right polarization \(-0 < e \leq 1\), at left polarization \(-1 \leq e < 0\), for the sake of convenience let us introduce an angle of ellipticity, \( \gamma \). Here \( e = \tan \gamma \), with \(-\pi/4 \leq \gamma \leq \pi/4\).

**An amplitude** of an elliptically polarized wave is determined from the relation:

\[
A = \sqrt{a^2 + b^2}
\]

**An absolute phase** \( \delta \) is the angle between the initial position of a vector \( \mathbf{E} \) and the major axis of an ellipse at the moment \( t = 0 \) which varies within the limits \(-\pi \leq \delta \leq \pi \). Parameters of an elliptically polarized wave are related [1] as follows:

\[
a^2 + b^2 = A_x^2 + A_y^2
\]

\[
tg 2\theta = tg 2\alpha \cos \delta
\]

\[
\sin 2\gamma = \sin 2\alpha \sin \delta
\]
\[ \tan \alpha = \frac{A_y}{A_x} \]

Optical radiation, as compared to radio-frequency radiation, is characterized by high frequency and, therefore, a small period of oscillations. Thus, for red light with \( \lambda = 0.6 \, \mu m \) an oscillation frequency is \( \nu = 5 \cdot 10^{14} \, Hz \) with a period \( T = 2 \cdot 10^{-15} \, s \). At present no receiver are available which could respond to such rapid changes in an intensity of electric or magnetic fields. Therefore all receivers are sensitive not to a field intensity but to a mean square value of its amplitude, i.e. to intensity which is an energetic characteristic of optical radiation.

Energy transfer by optical radiation is described by the Pointing vector determined by the relation: \( S = [EH] \), where \( H \) is the magnetic vector of optical radiation, \( A/m \).

Thus, the Pointing vector is the vector magnitude, a direction of which coincides with that of propagation of radiation energy, while the absolute magnitude is equal to a ratio of radiation power passing through a surface perpendicular to the vector direction to an area of this surface. \( F = |\langle S \rangle| \) will be henceforth called a power density of optical radiation, \( W/m^2 \). A power density at \( T_0 \gg T \) is related with light intensity \( I \) as follows:

\[
F = QI = Q \frac{1}{T_0} \int_0^{T_0} E^2(t) dt
\]

(1.6)

where

\[
Q = \sqrt{\varepsilon \varepsilon_0 / \mu_0}
\]

for vacuum \( Q = 2.65 \cdot 10^{-3} \, A/V \). A power density and intensity of monochromatic wave are:

\[
F = \frac{QA^2}{2}
\]

(1.7)

\[
I = A^2 / 2
\]

where \( A \) is the wave amplitude.

A beam power \( P \) depends on a power density \( F \) and a beam cross-section \( S \), i.e.
\[ P = \int_S FdS \]  \hspace{1cm} (1.8)

In optics instead of actual values the complex values are often used which essentially simplifies calculations. A plane homogeneous monochromatic wave may be described by the equation:

\[ E(z, t) = A \exp\{ -j(\omega t - kz + \delta) \} \]

At linear operations with such complex signals it is sufficient to take a real part of a complex signal at the outlet in order to obtain actual values. On multiplying two signals, it is necessary to manipulate with actual values in a general case. However, often of interest are only time-averaging products of signals and averaging time is appreciably larger than a period of oscillations. Then in order to obtain actual output signals, it is sufficient to take a real part of a product of one signal by a complex-conjugate function of the second signal:

\[ \langle \text{Re}E_1 \text{Re}E_2 \rangle = \frac{\text{Re}\langle E_1E_2^* \rangle}{2} \]

An electromagnetic wave depends on a spatial and temporal coordinates. In some cases it may be presented as a product of temporal \( E(t) \) and spatial \( E(r) \) functions, i.e., \( E(r, t) = E(t)E(r) \).

Consider a temporal part of the function \( E(r, t) \) in more detail. Representation of optical radiation as a monochromatic wave without the beginning and the end is the mathematical simplification. A more adequate mathematical model of optical radiation is the time-limited harmonic function:

\[
E(t) = \begin{cases} 
A \exp\{-j\omega_0 t\} & \text{at } -T_1/2 \leq t < T_1/2 \\
0 & \text{at } -T_1/2 > t > T_1/2 
\end{cases}
\]

whence it follows that a wave amplitude is constant in a time interval from \(-T_1/2\) to \(+T_1/2\). Such wave is not monochromatic, it is characterized by a definite spectral width, i.e., the time-limited function is obtained by summation of the infinite number of monochromatic waves:
\[ E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(\omega) \exp\{-j\omega t\} d\omega \]

where a spectral density of amplitude \( E(\omega) \) is expressed in terms of \( E(t) \) as follows:

\[ E(\omega) = \int_{-\infty}^{\infty} E(t) \exp\{j\omega t\} dt \]

For a time-limited process a spectral density of an amplitude is:

\[ E(\omega) = AT_1 \text{sinc}\left[\frac{(\omega - \omega_0)T_1}{2}\right] \]

A spectrum width of frequencies \( \Delta \omega \) between the points where \( E(\omega) \) turns into zero is equal to \( 4\pi/T_1 \), where \( T_1 \) is duration of process.

For a wave, an amplitude of which depends on time according to the Gauss law

\[ E(t) = \exp\left\{-\frac{t^2}{T^2}\right\} \exp\{-j\omega_0 t\} \]

a spectral density of an amplitude is

\[ E(\omega) = T\sqrt{\pi} \exp\left\{-\frac{(\omega - \omega_0)^2 T^2}{4}\right\} \]

where \( T \) is the time for which the amplitude decreases \( e \) times. Plots of the functions \( \text{Re} E(t) \) and \( E(\omega) \) are shown in Figure 1.4. A spectrum width \( \Delta \omega \) where an amplitude \( E(\omega) \) decrease \( e \) times is equal to \( 4/T \).

As it follows from the cases under consideration, a spectrum of an amplitude is in neighborhood of a certain frequency and with a decreasing the process duration a frequency band increases. Effective duration of a process, \( T_p \), and a spectrum width \( \Delta \nu = \Delta \omega/(2\pi) \) are related as \( \Delta \nu T_p = a \), where \( a \) is the coefficient dependent on the method used to determine the process duration and a spectrum width approximately equal to unity.
In a general case an amplitude, a frequency, a phase, polarization are the random functions of time and coordinates [9, 10]. In a laser diagnostics of flows the quasi-monochromatic radiation is employed, a spectrum width of which is considerably smaller than a medium frequency, i.e., $\Delta \omega / \omega_0 \ll 1$.

Then a quasi-monochromatic wave may be written as a product of two functions:

$$E(t) = A(t) \cos[\omega_0 t - \varphi(t)]$$

where $A(t)$ and $\varphi(t)$ are the functions slowly varying as compared to the function $\cos \omega_0 t$. A statistical relationship of oscillations at the moments of time $t$ and $t + \tau$ is characterized by a degree of time coherence $\gamma(\tau)$, while a statistical relationship of oscillations for two points of space $\mathbf{r}$ and $\mathbf{r} + \mathbf{R}$, by a degree of spatial coherence $\mu(\mathbf{R})$. These functions of coherence allow calculation of time $\tau_c$ and radius coherence $R_c$ [9, 10].

Coherence time and a spectrum width of optical radiation are related as:

$$\tau_c \Delta v = \eta \quad (1.9)$$

where $\eta$ is the constant coefficient approximately equal to unity, dependent on a form of an energy spectrum and a level of counting off $\tau_c$ and $\Delta v$. For approximate estimates we may assume $\eta = 1$, then $\tau_c = \Delta v^{-1}$.

In the course of coherence time a wave propagates for a distance $l_c = c \tau_0$ called a length of coherence of optical radiation.
1.2 Reflection and Refraction

Laser devices used for diagnostics of fluid and gas flows contain many optical elements which transform optical radiation according to the certain laws. Before discussing the operation of these elements, it is necessary to establish what changes an electromagnetic wave undergoes when it passes through an interface of two media with different refractive indices, for instance, a laser beam at first propagates in air, then it impinges on a glass plate or enters into a water layer.

A laser beam at the interface of two media is reflected and refracted. Consider characteristics of the refracted and reflected beams, namely, directions of propagation, an amplitude and a state of polarization.

The directions of propagation of the reflected and refracted beams are determined from the refraction and reflection laws: an angle of reflection is determined by an angle of incidence \( \theta_r = \pi - \theta_i \), while a refraction angle is related with an angle of incidence by the Snellius law as:

\[
 n_1 \sin \theta_i = n_2 \sin \theta_r \tag{1.10}
\]

where \( n_1 \) and \( n_2 \) are the refractive indices of the first and the second media, \( \theta_i \) is the angle of incidence, \( \theta_r \) is the refraction angle. All three beams are in the plane of incidence which is defined as the plane containing an incident beam and a normal to the interface of two media (Figure 1.5).

If a refractive index of the second medium is larger than that of the first medium then, as follows from the Snellius law, a refracted beam is deflected to a normal since \( \sin \theta_r < \sin \theta_i \) (Fig. 1.5a). And, vice-versa, if the second medium is less dense, e.g. a laser beam comes out of glass to air, then a refraction angle is

![Fig. 1.5 Illustrating refraction and reflection of laser beam at an interface of two media: a - reflection from a denser medium boundary; b - reflection from a lower-density medium boundary; c - total internal reflection.](image-url)
fer function ($MTF$):

$$MTF = \left| \frac{\sin[(\pi/2)(f_m/f_N)(w/d)]}{(\pi/2)(f_m/f_N)(w/d)} \right|$$

Here $f_m$ is the modulation frequency of the sinusoidal test pattern, $f_N = 1/2 d$ is the Nyquist frequency. If the elements are arranged right up to each other ($w = d$), then $MTF = 0$ for $f = 2f_N$. A point to be noted is that the modulation depth is different in the vertical and the horizontal direction.

Commercial photoresponsive integrated linear-array silicon systems commonly contain 1024 or 2048 CCD unit cell per chip (of length to 30 mm). Depending on the semiconductor material or dopant used, the spectral range of CCD responsivity is 50–14000 nm. The line resolution of laboratory single-line sensors reaches 20000 pixels. Commercial CCD cameras feature $576 \times 512$ pixels, and laboratory CCD cameras, as high as $2000 \times 2000$ pixels, that is, the degree of integration is in excess of 1000000 active elements per chip. Recently, results have been reported for $4096 \times 4096$ pixels per chip [20]. Methodological topics on the application of linear-array CCD sensors have been dealt with in [21]; as has been shown, the unlike photoresponsivity and nonlinearity of sensor elements can be compensated for by proper software design.

References

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