# HYPERSONIC AERODYNAMICS AND HEAT TRANSFER

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**HYPERSONIC AERODYNAMICS AND HEAT TRANSFER**
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# *Preface*

The book presents recent theoretical and experimental results achieved in Russia in the field of aerothermodynamics of hypersonic flows over complex configurations. These results include analysis of gas flows over the space shuttle “Buran”, various aircraft configurations and interplanetary vehicles. The entire problem of hypersonic aerothermodynamics is split into two closely linked parts: gas dynamics itself and physical models.

The gas dynamic models, which are considered in the book, include the Navier-Stokes equations along with their asymptotic simplifications at very high Reynolds numbers such as the Thin Viscous Shock Layer (TVSL) equations and Parabolised Navier-Stokes (PNS) equations. The latter equations are written down in special variables to obtain locally self-similar solutions that have sufficient accuracy with regards to the friction and heat transfer coefficients on windward sides of hypersonic aircrafts. An efficient iterative method is described to solve these reduced Navier-Stokes equations. It requires only a few iterations to reach the stationary solution, if it exists, in the case of flows with a predominant direction.

If non-equilibrium chemical reactions are to be taken into account for multicomponent gas mixtures of partially ionized gases, an exact formulation of heat and mass transfer can be important in the case of different binary diffusion coefficients. In the book it is formulated via the Stefan-Maxwell relations. In contrast to the conventional formulation “fluxes via thermodynamic forces”, there relations consider “forces vs fluxes”. This allows us to avoid double inversion of matrices of order $N*\xi$, where $N$ is the number of component, $\xi$ is the number of terms retained in the Sonin polynomials that are used in the Chapman-Enskog method. The obtained formula provides the exact relation for the thermodynamic forces vs the fluxes. It is a foundation for other very useful results such as simpler transport coefficients. In the case of equilibrium chemical reactions these transport coefficients are written out explicitly. To describe hypersonic high temperature flows, the full set of governing equations is formulated for multicomponent gas mixtures with the account of chemical reactions, multitemperature effects and radiation. This system of equations is to be completed by boundary conditions.

The boundary conditions at the wall take into account the effects of slip and temperature jump for the velocity and temperature, and catalytic activity for the species. A new phenomenological description of heterogenic catalytic processes is given in application to low catalytic surfaces of space vehicles in the Earth and Mars atmospheres. The coefficient of catalytic activity for a thermoprotection surface can be successfully calculated using direct numerical simulation methods from molecular dynamics.

It is well known that the system of Navier-Stokes equations with nonequilibrium chemical reactions is stiff if some reactions are close to equilibrium (the problem of partial equilibrium). To overcome such a problem, a new efficient method is described. In this method, one or two “slow” variables are introduced in such a way that they represent a linear combination of fraction of species as well as their diffusion fluxes. As a result, the stiff diffusion equations can be replaced by the condition of the Debye chemical equilibrium. The provided test cases confirm high accuracy of the model in application to hypersonic flows over space vehicles in the Earth and Mars atmospheres. Another series of test cases is related to the interaction of plasma flow with the thermoprotection surface. It is numerically and experimentally studied in the conditions of a discharge plasmatron.

Various computational experiments carried out for real space vehicle configurations allowed us to identify the regimes at which the viscous-inviscous interaction is important for the prediction of heat fluxes. It is also shown that the heat fluxes on a leeward side of space vehicle and the stagnation point can be comparable due to flow tubulization. Ablation effects are also considered in detail in application to composite silicon-based materials.

Rarefaction regimes are considered on the basis of continuum medium models. It is demonstrated that the TSVL model is capable of providing good enough prediction for the heat transfer and friction coefficients while the Navier-Stokes equations fail. This counterintuitive result is verified numerically and analytically at low Reynolds numbers and explained.

The final chapter of the book is devoted to the description of a novel domain decomposition approach for near-wall turbulence modelling. The main idea is that the boundary conditions are transferred from the wall to an interface boundary. Then, the problem needs to be solved in the outer domain. The solution in the inner domain can be obtained afterwards.

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# Introduction

The intense conquering of space by economically developed countries (Russia, United States, France, Germany, China, and Japan) faces the problem of developing new cost-efficient transportation systems for delivering crews and cargo to the near-Earth orbit and their returning to the Earth, as well as reusable planetary probes and expendable space probes designed for various purposes, including investigations of various planets, the Moon, and the solar system in general. An extremely important task both in Russia and in the United States is future landing of astronauts on the Moon and Mars and their return to the Earth, as well as industrial conquering of the Moon. In 2004, the US president put forward the challenge of space research: to walk on the surface of the Moon and Mars.

Since the 1980s, the leading spacefaring nations initiated intense development of principally new hypersonic flying vehicles: transatmospheric, interorbital, highly maneuverable vehicles performing aerodynamic maneuvers and/or hopping at hypersonic velocities in the upper layers of the Earth atmosphere when passing from higher to lower orbits without using engines, but using aerobraking, as well as small-size cost-efficient aerospace planes.

It should be noted that the first experience of using aerobraking in the Earth's atmosphere for reducing the hypersonic velocity of the flying vehicle approaching the planet to the orbital velocity was acquired in the USSR back in 1968 when the Zond vehicles returned to the Earth after flying around the Moon. These vehicles dipped by 50 km into the atmosphere two or three times and then popped out owing to tuning of the lift-to-drag ratio to altitudes greater than 100 km. In-flight experiments demonstrated that the thermal protection system of these vehicles was significantly overestimated because of the lack (at that time) of knowledge with regard to some aerodynamic problems, in particular, ablation of thermal protection coatings.

The above-mentioned projects revealed a number of aerothermodynamic problems that were poorly studied or not studied at all in both experimental and theoretical investigations performed in the 1960s and 1970s, which were mainly aimed at designing and operating conservative (from the current viewpoint) configurations, such as the Space Shuttle, Jupiter probe called Galileo, vehicles with ballistic reentry, and others. The Soyuz and Space Shuttle manned systems currently used to reach the Earth orbit and return back to the Earth will be replaced in the near future by other vehicles both in Russia and in the United States (for instance, the Space Shuttle system was taken out of service in 2012, though there was a probability of its further operation). These new vehicles are expected to combine the reliability of the Soyuz spacecraft, sufficiently low g-loads on the crew during reentry, comfort, reusability typical for the Space Shuttle and its domestic analog Energia–Buran, with smaller operational and temporal expenses of fabricating these vehicles and their post-flight recovery. Most of these requirements are satisfied by the Clipper spacecraft being designed in Russia, which would be able to perform gliding descent in the atmosphere and aircraft-type landing.

Current activities on design and testing of hypersonic demonstrators, which are prototypes of reusable vehicles, are performed within the framework of various national programs, such as the Clipper in Russia, X-38 (transport vehicle for crew rescue in the United States, Pre–X in France, Hyper–X and Hope–X (Hope H-II orbiting plane) in Japan, Hopper in Germany, USV in Italy, and others.

After the development and successful flights of the Space Shuttle (US) and Energia–Buran (Russia) transport systems, all industrially developed countries initiated the development of their own vehicles in the late 1980s: HOTOL (horizontal takeoff and landing) in Great Britain, ZAENGER and HYTEX (Hypersonic Technology Experimental Vehicle—a flying laboratory based on the ZAENGER technology) in Germany, HERMES (minishuttle) in France, and HOPE and HIMES (Highly Manoeuvrable Experimental Space Vehicle) in Japan.

Though the implementation of many of these projects was suspended for a number of reasons (mainly financial ones), investigations on these topics are continued at university laboratories and national research institutes. West European countries, which started their own research and engineering studies of hypersonic vehicles in the 1990s, considered the possibility of creating multinational research companies to compete with the United States in this field. An example is the common research of 17 European countries within the framework of the European Space Agency (ESA) with the center at the European Space Research and Technology Center (ESTEC) located in Noordwijk (The Netherlands). ESA organizes regular conferences and workshops on aerothermodynamics of space vehicles and also symposia on aerothermochemistry of space vehicles and corresponding hypersonic flows under the umbrella of International Union of Theoretical and Applied Mechanics (IUTAM). The main goal of these forums, which was formulated by Prof. R. Brun, the chairman of one of such symposia, is to elucidate the relationship between mechanical, physical, and chemical phenomena in high-enthalpy hypersonic flows.

Chinese activities in the field of manned space missions became known worldwide after the successful launch of the Shenzhou unmanned vehicle (November 21, 1999), the subsequent launch of a real space vehicle, and finally, a manned space vehicle. The first space walk of a Chinese astronaut took place in 2008.

Japanese specialists also performed investigations aimed at creating a two-stage space transportation system; later on, the design of a single-stage space vehicle is planned. Financing of research on hypersonic aerothermodynamics, heat and mass transfer, has permanently increased in Japan.

In Russia, there is also a national governmental program aimed at developing a new generation of manned transportation systems. The S. P. Korolev Rocket and Space Corporation “Energia” was asked to develop a new manned space vehicle capable of Moon missions. The design of this vehicle was started. A new prospective piloted transport system (PPTS) will replace the Soyuz vehicle. It is planned that the PPTS structure will be able to bring a crew of six people to the Earth’s orbit and a team of four astronauts to the Moon (the crew of the Soyuz spacecraft being in operation for more than 40 years consists of three people). This will be a capsule-type vehicle, in analogy with the new space vehicle being developed now in the United States. A larger size of the space vehicle was required because of the plan to increase the permanent crew of the International Space Station to six astronauts and to prepare Moon missions.

The maiden launch of the manned vehicle is planned for 2018. In addition to the capsule-type vehicle having a low lift-to-drag ratio, lifting-body configurations and transformers changing their shape during the flight are also investigated. There are also numerous activities in the field of hypersonic propulsion, including supersonic combustion and scramjet technology.

A new generation of transatmospheric vehicles, i.e., the National Aero-Space Plane (NASP), is being developed in the United States. These projects are motivated by the desire to create a single-stage system capable of launching the spacecraft to the Earth orbit with the use of a hydrogen-driven air-breathing engine, i.e., with the use of oxygen contained in air as an oxidizer, in contrast to liquid-propellant engines, which use the oxidizer stored onboard. Moreover, a new aeroassisted orbital transfer vehicle (AOTV) is being developed in the United States for the last 20 years. It is an orbital transport vehicle that employs the aerodynamic lift force when maneuvering in the upper layers of the atmosphere. It is expected to move from the geocentrical orbit to the high or low orbits of Earth satellites with velocities of 7–11 km/s and perform aerobraking in the atmosphere at altitudes of 70–100 km. The use of aerodynamic forces in this maneuvering instead of propulsion was recognized to be more cost-efficient.

It is also planned to use the AOTV for returning probes from the Moon and Mars. Thus, automatic or manned vehicles returning from the Martian orbit to the Earth will enter Earth's atmosphere with velocities up to 16 km/s. Probes designed for studying planets and small bodies in the solar system will also move with superorbital velocities in planetary atmospheres, for instance, the HUYGENS project designed for atmospheric measurements and subsequent landing on Titanium surfaces, the ROSETTA project aimed at spacecraft landing on a comet and subsequent reentry with comet nucleus material samples, and the MARSNET and MSR (Mars sample return) Martian probes designed to collect Martian soil samples and bring them to Earth. The Deep Impact project (USA) involves an impact of a special probe on the Tempel-I comet, and one of the ESA projects includes the Don Quixote experiment with impact on an asteroid.

The frontal shield of planetary probes returning to the Earth is protected by an ablative coating made of phenolic impregnated carbon ablator (PICA), which is a matrix of carbon fibers bound (impregnated) by a phenol-formaldehyde (bakelite) resin. This material was used in 2006 for thermal protection of the Stardust planetary probe, which collected dust samples near the Wild-2 comet nucleus and returned to the Earth with a reentry velocity of 13 km/s (this was the fastest controlled descent).

Thus, the aerothermodynamic problems to be solved by the worldwide scientific community are versatile and important. The state of the art in these fields is described in this book.

As there were no required computational resources and effective numerical methods for solving the Euler, Navier–Stokes, and Reynolds equations during the early stage of the design and development of space transportation systems in the 1960s and 1970s, approximate engineering analytical methods based on the Prandtl scheme (inviscid external flow plus boundary layer) were developed. In this methodology, which experienced the maximum development in the United States and Russia during the design of the Space Shuttle and Energia–Buran vehicles, respectively, the real three-dimensional spacecraft structure was divided into a number of sections, which could be approximated by simple geometric shapes.

The boundary conditions on the boundary layer (BL) edge were determined from approximate analytical solutions (e.g., tangential wedges or cones) or numerical solutions of the Euler equations. If it was necessary (in situations with high Mach numbers and low or moderate Reynolds numbers, i.e., in regimes of high-altitude hypersonic aerodynamics), these solutions were refined with allowance for the second-order effects of the boundary layer theory, in particular, the effect of inviscid-viscous interaction. Correspondingly, the heat flux, temperature distribution over the spacecraft surface, and ablation rate were determined either from the numerical solution of the BL equations or from their approximate solutions: self-similar solutions, methods of the effective length, axisymmetric analogy, mean-mass quantities, consecutive approximations, etc.

In contrast to the above-mentioned early stage of development of hypersonic aerodynamics and heat- and mass-transfer research, computational fluid dynamics (CFD) now plays a key role in the development of codes used to design modern space vehicles with calculations of aerodynamic and thermal characteristics of real spacecraft configurations with allowance for all basic physical and chemical processes in the shock layer and on the vehicle surface. This is explained by impressive and continued increase in computer power (a 350-fold increase in supercomputer performance from 1 teraflops in 1996 to 350 teraflops in 2006, with simultaneous 60-fold reduction of the computation cost) and the hardware and software progress achieved in the last decades. In 2011, the Lomonosov supercomputer based at the Moscow State University was expected to reach a peak performance of 1 petaflops. Such a computer is available nowhere else, neither in Japan, Canada, or France.

The key to success (by 80–90%) is the development of codes for solving rationally posed problems, parallel algorithms, and mathematical apparatus adopted to petaflops-class computers with a parallel architecture. For today, this is the most difficult aspect of the problem and the most important one. Without exaggeration, we can say that computational fluid dynamics (numerical simulation) turned out to be a key technology, which offers unlimited possibilities for effective design and further development of space vehicles, in particular, for modeling the flow field around the entire vehicle structure. This became possible owing to the recent progress in geometry modeling, generation of surface and volume difference grids, and development of effective numerical methods.

The hypersonic flow around space vehicles includes a large set of atomic and molecular high-temperature physical phenomena, such as rarefaction, relaxation of internal degrees of freedom, multispecies diffusion, dissociation and recombination both in the flow and on the wall, ionization, radiation, and thermal, chemical, and thermodynamic nonequilibrium. In the macroscopic form, these phenomena are manifested as wide-range variations of the governing similarity criteria: Mach number M, Reynolds number Re, Knudsen number Kn, Damköler number Dam, Schmidt number Sc, Lewis number Le (sometimes called the Lewis–Semenov number), etc. It follows from here that effective experimental modeling of hypersonic high-altitude flows in ground-based facilities has considerable constraints. On the other hand, reasonable and reliable extrapolation of numerical results to in-flight conditions requires in-flight experiments to be performed.

All this methodology (analytical methods, numerical methods, on-ground experiments, and in-flight experiments) is described in this book.

Implementation of such large-scale space programs requires a profound theoretical analysis of aerothermochemical problems in wide ranges of the Mach (1–30) and Reynolds ($\mathrm{Re} \geq 1$) numbers, which includes investigations in the continuum, transitional, and free-molecular flow regimes around the space vehicle with allowance for various physical and chemical processes, because these processes exert an appreciable (sometimes determining) effect on the gas parameters in the shock layer, aerodynamic characteristics, heat and mass transfer, plasma formation, and radiation spectrum. It is also important to take these processes into account in estimating the effect of the flow on control surfaces (elevons, flaps, thruster jets, etc.). At high altitudes (60–100 km), these processes proceed in thermochemical nonequilibrium, sometimes with large deviations from the local thermochemically equilibrium state, and it is not always possible to model these processes in available on-ground facilities.

In this book, the leading Russian specialists in hypersonic flows and heat and mass transfer describe the theoretical basis of real nonequilibrium physical and chemical processes in the shock layer on the space vehicle in the continuum and transitional flow regimes and a large number of codes for numerical modeling of the flow around real hypersonic vehicles in wide ranges of the flight Mach numbers M (1–30) and Reynolds numbers ($\mathrm{Re} \geq 1$).

This book deals with the fundamental research direction (aerothermodynamics plus aerothermochemistry) associated with studying hypersonic flows of thermochemical equilibrium and nonequilibrium multispecies mixtures of gases and plasma with different diffusion properties of the species in the vicinity of reusable thermal protection coatings possessing finite catalicity and also in the vicinity of ablative thermal protection coatings in wide ranges of altitudes and flight velocities, including the transitional flow regime. In particular, a new simple and precise form of transport relations (constitutive relations) was obtained for the mass and heat of the species in the form of “thermodynamic forces via fluxes.” In calculations performed by this scheme, the computation time is proportional to the number of species rather than to the squared number of species or more, as is the case if the complete classical form of the transport relations (fluxes via thermodynamic forces) with multispecies coefficients of diffusion and thermodiffusion is used.

Original effective computational technologies were developed which allow one to determine the aerodynamic parameters of reentry vehicles by solving Navier–Stokes (NS) equations, Reynolds equations, and their asymptotically simplified mathematical models of aerothermodynamics—BL equations, viscous shock layer (VSL) equations, and parabolized NS equations—to perform calculations for experimental investigations in aerodynamic and high-enthalpy facilities of the Central Research Institute of Machine Building (TsNIIMASh), Central Aerohydrodynamic Institute (TsAGI), Institute for Problems in

Mechanics of the Russian Academy of Sciences (IPMech), and other research organizations. The goal of these investigations is to determine the catalytic properties of thermal protection materials of reusable vehicles and effective enthalpies of ablation of various thermal protection coatings for various reentry trajectories in the Earth and Martian atmospheres, and finally, to design, develop, and optimize the shapes of new vehicles having realistic geometric configurations with the minimum number of expensive experiments, which are often difficult to perform in practice.

The developed computational technologies allow the computation performance to be appreciably increased, while the duration and cost of design activities in the aerospace industry have been substantially reduced. These technologies have already been used to perform detailed analysis of heat and mass transfer for existing and promising aerospace vehicles, including those developed at the Moscow Institute of Thermal Technology, Military-Industrial Corporation “NPO Mashinostroeniya,” Progress State Research and Production Space Center (TsSKB-Progress), Production Corporation “Polyot,” the S. P. Korolev Rocket Space Corporation “Energia,” and numerous academic and departmental institutes. The developed calculation techniques are widely used for planning complicated experiments, processing results of these experiments, and extrapolation of data to in-flight conditions (IPG-4 induction plasmatron based at IPMech and TsNIIMASh).

In particular, a concept was developed to choose the aerodynamic configuration of a shuttle, tentatively called Clipper, designed to return the crew from the Earth’s orbit to the Earth surface.

Investigations performed at the S. P. Korolev Rocket and Space Corporation “Energia” and TsNIIMASh with the use of the codes for numerical simulation, which are described in this book, made it possible to calculate the reentry trajectory of the Clipper vehicle from Earth’s orbit with a necessary lift-to-drag ratio to ensure a given side range, acceptable g-loads, and aircraft-type landing. The aerodynamic characteristics and heat transfer for missile-type vehicles were calculated. The results were compared with experimental data.

Original physicomathematical models were developed which take into account the details of the basic thermally nonequilibrium homogeneous and heterogeneous catalytic processes. The effect of these parameters on aerodynamic forces, moments, and heat fluxes was found by modeling space vehicles descending from the Earth’s orbit along ballistic, gliding, and aerobraking trajectories, as well as space probes returning to near-Earth space after flying around various planets of the solar system.

The use of these methods and codes, which are at the top level of aerospace science and engineering, made it possible to significantly improve the quality of design calculations in the field of aerodynamics and heat transfer and the performance of engineers involved in the calculations, to ensure better design characteristics of vehicles, and to reduce the costs associated with the development of these vehicles in many aeronautic and astronautic research centers and design bureaus of Russia.

Let us briefly describe the basic results of research of the authors of this book.

**1. Exact relations and transport coefficients in the form of “thermodynamic forces via fluxes.”** A new exact and simple form of closing relations for heat and mass transfer in multispecies multitemperature mixtures of gases and plasma with different coefficients of binary diffusion in the presence or absence of external electromagnetic fields was obtained by methods of the rigorous kinetic theory of gases and thermodynamics of irreversible processes in the 1980–1990s. The old "classical" form of the transport relations ("fluxes via thermodynamic forces") is rather complicated (double inversion has to be applied to matrices of the order of $N\xi$, where $N$ is the number of the species and $\xi$ is the number of terms retained in the expansion of the disturbed distribution function in the Sonin polynomials); moreover, the matrix elements in the second inversion are determinants of the order of $N\xi$. For this reason, the "classical" form of the transport relations in the full formulation is not used in solving particular problems. Similar transport relations were obtained with allowance for excitation of internal degrees of freedom of particles and for moderately dense mixtures of gases.

Thus, the equations of motion of a multispecies and multitemperature gas combined with the original form of the transport relations, "thermodynamic forces via fluxes," with exact and simpler (as compared with the "classical") transport coefficients can be considered at the moment as a reliable scientific basis for solving gas-dynamic problems associated with studying thermochemically nonequilibrium flows of multispecies, multitemperature viscous heat-conducting ideal and weakly nonideal mixtures of gases and plasma, with and without external electromagnetic fields. These equations allow one to calculate flows in wide ranges of pressures and temperatures (up to hundreds of thousands of degrees). They form the basis for solving a large number of problems of physical and chemical hydrodynamics.

**2. Equations for thermochemically nonequilibrium flows of mixtures of gases and plasma.** Based on the new form of the transport relations, equations were derived for thermochemically equilibrium flows: the translational temperature is equal to the temperatures of all internal degrees of freedom (thermal equilibrium); moreover, fast chemical and ionization reactions occur in the flow region considered, i.e., there is also local chemical equilibrium. Such a flow regime is observed during spacecraft motion at altitudes below 55 km above the Earth's surface.

A complete set of all effective transport coefficients was determined (rather than only one effective thermal conductivity, as it was previously done).

The proposed new form of the transport relations for both thermochemically nonequilibrium and equilibrium flows allowed the basic equations of aerothermodynamics (BL equations, VSL equations, parabolized NS equations, and full NS equations) to be written in the form resolved with respect to the first derivatives along the normal-to-surface coordinate for all sought quantities: species concentrations (for thermochemically nonequilibrium flows), concentrations of chemical elements (for thermochemically equilibrium flows), temperature, heat flux, and diffusion flux, i.e., in the Cauchy form. This made it possible to develop effective high-order numerical methods (of the fourth order with respect to the crossflow coordinate) for the above-mentioned gas-dynamic models, where the calculation time is proportional to the number of species rather than to the squared number of species, as it happens in the old "classical" relations of heat and mass transfer.

Owing to the exact form of the transport relations, which take into account different diffusion properties of the species in the mixture, the effect of separation of chemical elements in thermochemically equilibrium mixtures of gases and plasma was discovered. The boundaries of domains where it is necessary to take into account the higher approximations (the number of coefficients of the expansion of the disturbed distribution functions into series in the Sonin polynomials for reaching a required accuracy level) for the high-temperature air plasma were obtained.

**3. Model of partial thermochemical equilibrium for solving problems of a hypersonic viscous heat-conducting gas flow around bodies.** The possibility of using the concept of partial chemical equilibrium is analyzed, which is extremely useful for applications. For the class of chemically nonequilibrium models of the gas medium, owing to the use of the differences in the time scales of the chemical processes proceeding in the flow, this concept allows one to simplify the problem and to reduce the stiffness of the system of equations of chemical kinetics. It is demonstrated that introduction of the so-called "slow" variables for mixtures modeling the Earth and Martian atmospheres within the framework of the partial chemical equilibrium model makes it possible to obtain solutions that almost coincide with the solutions of problems in the full diffusion-kinetic formulation for areas with high heat loads over gliding trajectories of vehicle descent in the Earth and Martian atmospheres.

**4. Thermochemically nonequilibrium hypersonic flow regimes.** At high temperatures of the gas behind the bow shock wave (tens of thousands degrees) in the viscous shock layer, the relaxation times of vibrational degrees of freedom of molecules become comparable with the dissociation reaction times. In this case, dissociation occurs on the background of relaxation of vibrational degrees of freedom. The so-called vibrational-dissociation coupling is observed, which increases the temperature in the shock layer (SL) and therefore the heat flux up to 26% and decreases the bow shock-wave stand-off distance up to 20%, as compared with the model of thermally equilibrium constants of chemical reactions. This theory was first developed at the laboratory of physicochemical gas dynamics of the Institute of Mechanics at the Moscow State University by a team headed by G. A. Tirskiy and was implemented for calculating the heat fluxes to the Buran reentry vehicle. At the moment, this theory is commonly used in the worldwide practice of aerodynamics and heat-transfer calculations in the transitional flow regime.

A similar theory was developed for the electron-ionization coupling, where the ionization reactions proceed on the background of relaxation of excited electron degrees of freedom of atoms and molecules. This is a complicated theory, both in terms of its physicochemical formulation and in terms of the actual numerical solution of problems of the hypersonic flow around reentry vehicles entering Earth's atmosphere with superorbital velocities.

**5. Boundary conditions of slipping on a catalytic wall in a multispecies multi-temperature chemically reacting gas flow with excited internal degrees of freedom of particles.** As the gas rarefaction (Knudsen number) increases, the flow near the surface (in the Knudsen layer with the thickness of the order of the mean free path) is no longer correctly described by continuum (gas-dynamic) models. Kinetic equations are solved in this layer on the wall, and effective boundary conditions of slip velocity and temperature and concentration jumps are imposed for solving continuum equations.

In this book, the slip conditions are derived for a surface with an arbitrary catalytic activity with respect to recombination and deionization of a partly dissociated and ionized multispecies multitemperature reacting gas with excited vibrational degrees of freedom of molecules by the Maxwell flux method and by a more precise Loyalka's method. Different degrees of accommodation of vibrational degrees of freedom on the wall are taken into account.

The following general conclusion should be drawn: In the flow regime with the slip condition and in the transitional regime, the aerodynamic and thermal characteristics are more sensitive to the accuracy of setting the transport coefficients than in the regime with the no-slip condition, i.e., in the purely continuum regime.

At altitudes of the transitional flow regime (above 100 km for the bluntness radius of 1 m), the slip boundary conditions appreciably affect the drag force (especially friction drag, which makes a significant contribution to the total drag force at high altitudes) and heat transfer for vehicles performing long-time flights (maneuvering, aerobraking), and they must be taken into account if the problem is solved within the framework of the VSL or NS equations. As was elucidated in publications of the authors of this book, owing to correct allowance for the slip effects, the area of applicability of the VSL model can be substantially extended to altitudes of about 140 km, i.e., the drag and heat-transfer coefficients in the transitional flow regime can be adequately predicted.

**6. Effect of heterogeneous catalytic processes on low-catalytic thermal protection coatings of the space vehicle on its heat flux.** When the space vehicle moves along a gliding reentry trajectory, which has lower heat loads than the ballistic trajectory, the vehicle descent is usually accompanied by significant ablation. In this case, thermal protection with low-catalytic tiles is usually used to retain the shape of the windward part of the vehicle (Buran and Space Shuttle). The heat fluxes toward the surfaces with different catalytic properties with respect to atomic recombination differ severalfold. The calculation of the catalytic properties of the thermal protection system requires fundamental knowledge of physical and chemical processes of interaction of a partly or fully dissociated gas with low-catalytic surfaces. At the moment, there are no direct methods for measuring the coefficients of recombination and accommodation of recombination energy, even for a surface at room temperature. There are no comprehensive theories that would be able to predict the catalytic properties of a particular material *a priori*.

Kinetic models based on the detailed allowance for the mechanisms of physical and chemical adsorption of particles on active centers, and interaction of adsorbed components with each other in the Langmuir–Hinshelwood reactions (associative mechanism of recombination) and Eley–Rideal reactions (shock mechanism of recombination) were developed for the first time in the 1980s at the Institute of Mechanics at the Moscow State University. Formulas for recombination rates on the surface, which take into account both recombination mechanisms, were derived for dissociated air and carbon dioxide. It was demonstrated that catalytic deionization of charged particles appreciably affects the level of ionization on the surface.

A significant effect on heat fluxes to thermal protection coatings of space vehicles is exerted by incomplete accommodation of energy of heterogeneous recombination associated with the formation of particles with excited internal degrees of freedom on the surface. At a high level of ionization, the binary diffusion model can overpredict the heat flux to a noncatalytic wall by 30% and more. For this high-temperature flow regime in the shock layer, a modified model of binary diffusion was proposed, which yields heat fluxes that almost coincide with the exact solution.

Models of heterogeneous catalytic processes for vehicles moving in the Martian atmosphere were developed. It was shown that the influence of heterogeneous catalysis on heat transfer in the rarefied $CO_2$-based Martian atmosphere is even more pronounced than in the case of Earth atmosphere reentry.

The heat fluxes in the case of the Mars Miniprobe descent in the Martian atmosphere were calculated for various coatings. The borosilicate coating was demonstrated to have a low catalytic activity in carbon dioxide, as well as in dissociated air. This coating allows the maximum heat flux to be reduced by a factor of 2.5, as compared with the heat flux to a fully catalytic surface, which corresponds to a decrease in the equilibrium radiative temperature of the wall by approximately 500 K.

The heat fluxes to low-catalytic surfaces were also studied analytically in the case of a chemically nonequilibrium flow in a multispecies boundary layer on the wall with an arbitrary catalytic activity. The analytical solution was used to interpret the experimental data on the probability of recombination on the Space Shuttle thermal protection coating; moreover, the effect of diffusion separation of chemical elements of the mixture owing to different selectivities of the catalytic action of the surface on recombination of oxygen and nitrogen atoms was discovered and explained.

Effective numerical methods for heat-transfer analysis were developed that take into account real heterogeneous and homogeneous physical and chemical processes for both laminar and turbulent flow regimes in the shock layer on reentry vehicles. Detailed comparisons with in-flight and on-ground experimental data showed that these models of heterogeneous catalytic reactions for the Earth and Martian atmospheres allow a correct interpretation of laboratory experiments and prediction of heat transfer to real low-catalytic nondestroyable thermal protection coatings of space vehicles under real conditions. These models were used to design and calculate the heat fluxes for the Buran vehicle.

**7. Numerical simulation of thermally and chemically nonequilibrium flows and heat transfer in underexpanded jets generated by an induction plasmatron.** In this work, verification of physicochemical models, methods of the numerical solution, and determination of catalytic properties of real low-catalytic coatings were based on experimental data obtained in the IPG-4 induction plasmatron (IPMech), which can generate both subsonic and supersonic high-enthalpy flows of various gases in wide ranges of stagnation temperatures and pressures. Surprisingly, the calculation of flows generated by the plasmatron (discharge chamber, nozzle, and test section with the model in underexpanded chemically nonequilibrium jets) is much more complicated than solving the original problem of heat transfer in hypersonic flows around bodies, for which purpose this plasmatron was used. However, a detailed calculation of plasmatron flows is absolutely necessary for a correct interpretation of experimental data and extrapolation of these data to real hypersonic flight conditions. This book describes the numerical solution of this complex problem based on the technology of numerical integration of Navier–Stokes equations with the use of databases of thermodynamic and transport properties of individual gaseous substances (HIGHTEMP) developed at the Institute of Mechanics at the Moscow State University. Nonequilibrium processes of excitation of vibrational degrees of freedom of molecules of gas mixtures, the difference in temperatures of electrons and heavy particles, chemical reactions, and ionization were taken into account. Comparisons of the calculated heat fluxes with those measured in IPG-4 experiments made it possible to determine the catalytic properties of a number of thermal protection materials for descent trajectories of space vehicles in the Earth and Martian atmospheres.

**8. Numerical study of specific features of heat transfer in a hypersonic flow around a blunted cone lying on a triangular flat plate with blunted edges.** This problem was numerically solved within the framework of the Navier–Stokes equations. The flying vehicle was a blunted triangular plate combined with a blunted circular cone. It was important for applications to reveal experimentally and confirm theoretically some specific features of the flow that lead to a local increase in the heat flux level on the model surface. The calculated distributions of heat fluxes revealed local peaks on the windward and leeward sides of the configuration in certain flow regimes. Despite a rather complicated flow pattern, the heat fluxes to the surface completely coincided with the results of experiments performed at TsAGI and TsNIIMASh. Thus, a comparison of numerical and experimental results can be used to predict and explain such anomalous effects.

**9. Heat transfer in the base region of the Martian reentry vehicle.** This study was performed within the framework of the Russian and European research of vehicles that should descend in the Martian atmosphere. When the vehicle descends in the Martian atmosphere, the heat flux in the base region is so small that it may be principally possible to avoid using the thermal protection coating on the base surface, which would allow bringing a greater payload to the Martian orbit. TsAGI performed a unique experiment aimed at determining the heat fluxes in the base region without the base sting. The configuration developed at TsNIIMASh was used as a prototype of the Martian vehicle.

It was experimentally found that the maximum heat flux in the base region of the examined vehicle at high Mach numbers and low Reynolds numbers typical for the segment with high heat loads of the Martian descent trajectory at a zero angle of attack is 6–7% of the heat flux at the frontal stagnation point. In a turbulent flow, the maximum heat flux in the base region of the examined model is commensurable with the heat flux at the frontal stagnation point. The results of numerical simulations performed for a turbulent flow agree with experimental data.

The experimental technique described in this chapter is used at TsAGI to study heat transfer and other characteristics of the flow for a typical model of the Martian vehicle developed by the European Space Agency.

**10. Numerical simulation of a viscous gas flow and heat transfer at hypersonic velocities.** This book describes the results of numerical simulations of external and internal aerodynamics problems with the use of computational technologies developed by the authors in their organizations. The developed technologies make it possible to solve problems in two-dimensional and three-dimensional formulations within the framework of the full Navier–Stokes equations (V. I. Sakharov, I. V. Egorov, V. I. Vlasov, and A. B. Gorshkov) and the Reynolds equations (I. V. Egorov). Effective numerical methods for solving viscous shock layer equations were also developed (V. G. Gromov, N. E. Afonina, V. L. Kovalev, B. V. Rogov, and G. A. Tirskiy), which were successfully used in important applications. The MPI technology allowed these codes to be adapted to computations on modern multiprocessor systems.

The developed technologies were used to study viscous gas flows with allowance for nonequilibrium physical and chemical processes (chemical reactions and ionization reactions) that play an important role in the motion of aerospace planes on the flight trajectory segments with high heat loads. Good agreement of the number density of electrons in the shock layer with the RAM-C in-flight experiment was achieved. Vibrational relaxation models were verified through comparisons of the radiation intensity predicted by computations and measured in the Bow Shock-2 in-flight experiment.

Systematic computations are used to accompany wind-tunnel experiments at TsAGI. These are computations of the model of the Atlas II launcher, flow around bodies with cavities, and other problems, as well as the flow around various bodies used in experiments in hypersonic wind tunnels based at TsAGI: T-117, UT-12, IT-2, and VAT-104.

Specific features of heat transfer in a hypersonic flow around a blunted cone lying on a triangular flat plate with blunted edges were numerically studied. Comparisons of numerical and experimental results revealed good agreement of the anomalous heat flux distributions on the windward and leeward surfaces of the configuration in certain flow regimes.

The problem of turbulization of near-wall flows is one of the basic problems of aerodynamics. In this work, the direct simulation Monte Carlo (DSMC) method was applied to study the early stage of the laminar-turbulent transition, including excitation and development of unstable disturbances in a hypersonic boundary layer. The evolution of disturbances in a hypersonic flow was found to be qualitatively different from the growth of disturbances in a subsonic flow.

**11. Calculation of a hypersonic chemically nonequilibrium air flow around flying vehicles.** Software systems for modeling high-temperature flows of a multispecies gas and plasma, as well as laminar and turbulent heat transfer and temperature regimes of thermal protection on the basis of solving Navier–Stokes and Reynolds equations were developed within the framework of the multitemperature thermochemically nonequilibrium model with detailed refinement and application of experimental data on rate constants of homogeneous and heterogeneous reactions.

The software system developed for solving NS equations was tested by performing a numerical study of a laminar near wake behind slender cones in a hypersonic perfect gas flow.

Verification of the numerical method for turbulent flow modeling was based on calculating the distributions of parameters in the base region of a blunted body and their comparisons with experimental data obtained in a wind tunnel based at TsNIIMASh. Comparisons of various turbulence models with experimental data revealed reasonable agreement—the difference between the calculated and experimental heat fluxes was smaller than 10%. The operation of the method of calculating complicated spatial flows was checked by studying the flow on the windward side of a blunted delta wing with a sweep angle $\lambda = 75^\circ$. A comparison with experiments revealed good agreement in terms of pressures and heat fluxes on the wing.

For verification of the physicomathematical model of air and codes for calculating flows of nonequilibrium gas mixtures, the flow patterns, and heat transfer for conditions of the OREX and RAM-C in-flight experiments were numerically simulated. The calculated and experimental data were found to agree well at altitudes greater than 92 km. Some differences observed at lower altitudes can be attributed to the finite catalytic activity of the vehicle surface with respect to heterogeneous recombination of oxygen and nitrogen atoms.

The original physicomathematical model was used to perform a detailed study of the electron density distribution in the shock layer on a 9° blunted cone with a bluntness radius of 0.1524 m and a length of 1.295 m entering the atmosphere with a velocity of 7.65 km/s (RAM-C experiments).

The developed computational technique was used in unique parametric computations of distributions of heat fluxes and gas-dynamic parameters of the flow for various geometric configurations of the reusable reentry vehicle called Clipper, which is developed at the S. P. Korolev Rocket and Space Corporation “Energia,” in wide ranges of the governing parameters, Mach numbers M, and Reynolds numbers Re. After these computations, the geometry of the Clipper vehicle was optimized to ensure an admissible level of heat loads. The levels of laminar and turbulent heat transfer at a flight altitude of 50 km were compared, where the laminar-turbulent transition on the windward surface of the Clipper vehicle is expected to occur. When passing to the turbulent flow regime, the equilibrium-radiative temperature of the surface on areas with high heat loads increases by approximately 200–400 K. Based on a numerical analysis of this complicated spatial problem, changes in the geometry of the reentry vehicle Clipper were proposed to reduce heating at the vehicle surface areas with the highest heat loads.

**12. Theoretical basis of calculations of heat and mass transfer during aerothermochemical destruction of thermal protection materials.** A new effective methodology of solving a complex problem of the flow of a multispecies, partly ionized, selectively emitting and absorbing gas in the shock layer above the surface of a destroying thermal protection material was developed. This unique methodology ensures significant reduction of computer time needed for calculations, and thus, it is possible to perform a greater volume of high-accuracy computations necessary for research and design activities.

A new research direction was developed, which is based on using large software systems at all stages of design of modern aerospace vehicles. These software systems were developed for parallel adjoint computations of problems of motion dynamics, aerodynamics, gas dynamics, thermal processes, and thermal strength of the structure of flying vehicles. An effective technology of constructing such systems was developed. Based on this technology, several large software systems were developed for various purposes. The use of this approach made it possible

- to reduce the labor effort of design computations and the design time;
- to increase the quality of design computations;
- to reduce the costs associated with numerical and theoretical research at all stages of the design process.

In some cases, for instance, by using a large specialized software system developed for computing the dynamics of motion of blunted bodies of revolution in the Earth's atmosphere at a zero angle of attack, their aerodynamic and mass-inertial characteristics, convective heat transfer, heating, and ablation of thermal protection materials, one can obtain a rigorous adjoint numerical solution of the entire set of problems at all stages of the design process, including, in particular, the influence of ablation of the thermal protection material on aerodynamic, thermal, and dynamic characteristics of the vehicle.

**13. New transportation systems for space conquering.** One of the key issues of creating a small-size reusable winged vehicle is to ensure admissible heating of surfaces experiencing the highest heat loads (nose bluntness and windward edges of the wings) and simultaneous provision of a high lift-to-drag ratio. As the characteristic size of this vehicle is planned to be much smaller than that of the Buran aerospace vehicle, the equilibrium-radiative temperatures are expected to be higher by more than 300 degrees.

Based on investigations performed at the S. P. Korolev Rocket and Space Corporation "Energia," a principle of constructing the nose and windward parts of the fuselage was developed that decreases heat-transfer intensification on the wing edges. This principle is based on additional deflection of the free stream in the lateral direction owing to appropriate contouring of the nose and windward parts of the vehicle.

Investigations performed at the S. P. Korolev Rocket and Space Corporation "Energia" and TsNIIMASh with the use of software systems made it possible to choose and optimize the shape of the aircraft-type vehicle Clipper. The computations showed that the chosen vehicle shape ensures acceptable aerodynamic characteristics in all flight regimes and admissible surface temperature levels that can be endured by modern thermal protection materials.

**14. Numerical modeling of hypersonic heat transfer on the windward side of the Buran vehicle.** Numerical simulations were performed to model the flow around the Buran aerospace vehicle descending from the orbit along a gliding reentry trajectory with a large angle of attack ($\alpha$ = 35–40°) at altitudes of 80–65 km, where the entire windward surface of the vehicle experiences the maximum heat fluxes, which are insufficient for initiating phase transitions of the thermal protection coating, resulting in ablation. The unchanged shape of the vehicle is ensured by using low-catalytic coatings (tiles with a silicon carbide coating), which appreciably decelerate heterogeneous exothermic reactions on the surface, thus reducing the heat flux toward this surface. The method of the numerical solution of the spatial problem is based on using a system of viscous shock layer equations with allowance for real properties of high-temperature air and allows the solution to be obtained with significantly reduced computational requirements. The earlier activities of the authors of these studies were the basic theoretical investigations used to develop the Buran vehicle in the Soviet Union.

**15. Modeling of turbulent compressible near-wall flows.** Semiempirical turbulence models used for numerical simulation of near-wall flows of a compressible gas are reviewed. Algebraic and differential classes of turbulence models, the basic hypotheses of turbulence, and systems of differential equations supplemented with boundary conditions are considered. Examples of various variants of taking into account the processes of interaction of laminar and turbulent flows near the solid surface of the body, which are used in turbulence models both for an incompressible fluid and under conditions of a compressible gas and heat transfer, are given. Various issues of modification of both algebraic and differential models initially developed for low-velocity flows for studying supersonic and hypersonic near-wall flows under conditions of intense heat exchange with the body surface, the limits of applicability of these models determined by the free-stream parameters, and the temperature factor of the surface are studied. The models considered allow closing the systems of averaged boundary-layer and Reynolds equations for their subsequent solution by numerical methods.

**16. Iterative-marching method of solving viscous shock layer equations in a wide range of Reynolds numbers.** A new iterative-marching method of the numerical solution of VSL equations was developed and mathematically justified. This algorithm is based on an original method of splitting the marching pressure gradient responsible for upstream transportation of disturbances into hyperbolic and elliptical components. The elliptical component is calculated by an original formula that minimizes the part of the streamwise pressure gradient along which the global iterations are performed. The use of this splitting allows the number of iterations to be reduced to one or two for obtaining the basic characteristics (drag and heat transfer) with acceptable accuracy for applications. A similar algorithm was developed for solving the direct problem of the Laval nozzle.

**17. Analytical method of determination of heat transfer, friction, and pressure on blunted bodies in a hypersonic rarefied gas flow in the transitional flow regime.** The existence of a unique continuum model [i.e., the model of a thin viscous shock layer (TVSL)], which yields a correct solution for the drag and heat-transfer coefficients in the transitional flow regime with a limiting transition to the free-molecular regime as Re $\to$ 0 is justified. An analytical method of solving the TVSL equations at low Reynolds numbers Re was developed; this method involves the integral method of consecutive approximations and asymptotic expansion into series. Similarity parameters of a hypersonic rarefied gas flow around bodies were identified. Simple analytical solutions were obtained for heat-transfer, friction, and pressure coefficients as functions of the free-stream parameters and the body geometry for axisymmetric and plane flows, as well as for three-dimensional flows in the vicinity of the stagnation point and in the vicinity of the plane of symmetry of the body at low Re values. It was demonstrated that these solutions are sufficiently accurate in the major part of the transitional flow regime and approach the corresponding free-molecular values as Re tends to zero. The analytical dependences derived in this study can be useful in obtaining numerous estimates of heat transfer and drag with variations of flow parameters, which are necessary for optimizing the shape of new designed vehicles.

An important property of TVSL equations (predicting a correct limiting transition to free-molecular values for the drag and heat-transfer coefficients as Re $\rightarrow 0$) offers a possibility of a unified continuum approach to solving problems of hypersonic flows around blunted bodies in the entire range of Re, with VSL or NS equations being solved at high and moderate values of Re and TVSL equations being solved at low values of Re. Such an approach is an alternative to continuum-kinetic methods, which require significantly greater computational resources.

From the table of contents of this book and numerous references, the reader can obtain an idea about the current state in the field of hypersonic aerothermodynamics and heat and mass transfer of modern and promising (currently being developed) expendable and reusable space vehicles and planetary probes, both in Russia and abroad.

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*G. A. Tirskiy*

# CHAPTER 1

# *Asymptotically Simplified Gas-Dynamic Models of Supersonic and Hypersonic Aerodynamics and Heat Transfer*

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## 1.1. INTRODUCTION

During reentry of space vehicles from the Earth's orbit, during the return of space probes to the near-Earth space after flying around various planets in the solar system and the Moon or after landing on these space objects, and during flying of meteoroids through the atmosphere of the Earth and other planets, all these bodies consecutively experience various regimes of hypersonic (high-velocity) flow from low to high flight Reynolds numbers (from high to low Knudsen numbers, respectively), as follows:

1. Free-molecule regime
2. Regime of the first and second collisions of particles (molecules, atoms, ions, and electrons)
3. Transitional regime [with Knudsen numbers $\mathrm{Kn} = \mathrm{O}(1)$]
4. Regime where the bow shock wave (SW) starts to form
5. Regime with a smeared bow SW
6. Regime of a viscous shock layer with effects of viscosity, thermal conductivity, and diffusion in the entire shock layer, including the region directly behind the SW replaced by generalized Rankine-Hugoniot conditions
7. Regime of weak and strong vortex interaction of an inviscid gas with the boundary layer (BL)
8. Finally, at high Reynolds numbers, regime of the boundary layer with an external inviscid flow

In each regime, the gas flow is traditionally described by an adequate mathematical model, though all these regimes can be rigorously described by one model based on solving the Boltzmann kinetic equation for a one-particle distribution function, which is a nonlinear integrodifferential equation, in the general case, of six independent variables of the phase space [three coordinates and three components of particle velocity (momentum)] and time with a quintuple collision integral (for a monatomic gas or a polyatomic gas without allowance for excitation of internal degrees of freedom). The multiplicity of the collision integral increases if excitation of internal degrees of freedom of particles (molecules, atoms, or ions) is taken into account. Because of the high dimension of the phase space of independent variables of the Boltzmann equation, the solution of informative problems of hypersonic aerodynamics and heat transfer with the use of this equation is a difficult computational task [1–3].

An aspect important for practice is construction of simpler model kinetic equations. The first model was constructed by Krook [Bhatnagar-Gross-Krook (BGK) model] [4, 5] for the Boltzmann equation with a rather simple relaxation term approximately replacing the Boltzmann collision integral. The Krook equation retains all features of the Boltzmann equation associated with free motion of particles and approximately (mean-statistically) describes their collisions. Quantitative results obtained by solving the Krook equation, except for few rare cases, differ from the corresponding results obtained by solving the Boltzmann equation. In particular, in passing to a continuous medium (as Kn $\rightarrow 0$), the Krook equation yields the Prandtl number equal to unity, whereas its exact value for a monatomic gas is 2/3 [5]. Molecular transport equations obtained from the Krook equation do not contain terms with thermodiffusion. The solution of this model equation, however, yields a physically correct flow pattern and a fairly exact solution for the drag force, which allows researchers to use this equation for obtaining qualitative and approximate solutions.

Since the 1960s, more detailed model Boltzmann equations have been constructed. The model equation of the incomplete third approximation, which was called the S-model, is widely used now [6, 7]. Though this model kinetic equation is simpler than the exact Boltzmann equation, it is also a fairly complicated integrodifferential equation with a high dimension. A typical problem of the numerical solution of both the model and exact Boltzmann kinetic equations at high values of Kn is the necessity to take into account the discontinuities of the distribution function in the flow, which appreciably complicates the numerical algorithm and its implementation [8]. On the other hand, the numerical solution of kinetic equations at small values of Kn (approaching the continuum flow regime) requires construction of completely conservative methods with a high order of approximation (at least, the second order). Both the Boltzmann equation and its model kinetic analogs are mainly used for solving problems of hypersonic aerodynamics and heat transfer at Kn = O(1).

An alternative for kinetic equations in solving the above-mentioned problems at Kn $=$ O$(1)$, which is particularly popular now, is based on approximate direct simulation Monte Carlo (DSMC) methods [9–14]. These methods have become the main mathematical tool for studying complicated two-dimensional and three-dimensional hypersonic flows for the following basic reasons: comparative simplicity of the transition from one-dimensional to two-dimensional and three-dimensional problems, possibility of using various models of particle interaction with excitation of internal degrees of freedom, possibility of taking into account chemical reactions without significant complication of the computational algorithm, and, finally, possibility of effective implementation of the method on modern computers with parallel and vector architecture.

Despite widespread applications of the DSMC methods, they have some drawbacks. In modeling near-continuum flows, additional computations on a fine grid with a large number of model particles become problematic and a question of the accuracy of results arises: How close are the results to the solution of the Boltzmann equation? An analysis of the solution accuracy is complicated by the presence of statistical errors induced by discretization in space and time and errors caused by limited capabilities of definition of a large number of model particles. The relation between the DSMC method and the solution of the Boltzmann equation has not been completely clarified yet.

The presence of the Knudsen number at the convective term (at the total derivative of the distribution function in the phase space of coordinates and velocities) in the dimensionless Boltzmann equation allows one to construct asymptotic solutions of this equation, which can effectively solve problems of hypersonic aerodynamics and heat transfer at fairly high and low Knudsen numbers, thus, eliminating mathematical difficulties inherent in solving the Boltzmann equation at $\mathrm{Kn} = \mathrm{O}(1)$. In the first (free-molecular) flow regime ($\mathrm{Kn} \to \infty$), the Boltzmann equation admits an exact solution in the form of an equilibrium (in terms of velocities and internal degrees of freedom of particles) Maxwell-Boltzmann distribution function determined from the free-stream temperature and density. In this flow regime, the main difficulty of solving boundary value problems of aerodynamic and heat transfer is correct quantification of the boundary conditions for the distribution function [15, 16].

Obtaining these conditions reduces to a quantum mechanics problem of calculating collective interaction of particles incident onto the body surface with a prescribed crystal lattice of the body; solving this problem finally leads (at the level of the distribution function) to determination of the boundary transform, i.e., the probability density of particle reflection from the surface, which is further used to find the distribution of velocities of these particles and, thus, the momentum and energy imparted by the particles to the body.

This is largely a quantum mechanics problem, and it is solved for simple model presentations of the crystal lattice of the body surface. Aerodynamic calculations usually involve the use of a rough (but applicable in practice) phenomenological scheme of specular-diffuse reflection, where the boundary transform is expressed via only two macroscopic quantities: diffusion or accommodation coefficient (separately for the tangential and normal momentum of incident particles) and energy accommodation coefficient, which are obtained from experiments.

In problems of aerodynamics and heat transfer, the more rarefied the gas, the more pronounced the influence of the law of interaction of particles (molecules, atoms, ions, and electrons) with each other and with body surfaces. In the continuum flow regime, i.e., at sufficiently high Reynolds numbers (sufficiently low Knudsen numbers at finite

Mach numbers), these difficulties with the boundary conditions do not arise because the molecule located near the surface collides with the latter many times and finally almost completely loses its tangential momentum (the no-slip condition is satisfied) and transfers its energy (the energy accommodation coefficient is equal to unity). When the distribution function satisfying imposed boundary conditions is found, determination of aerodynamic forces and heat fluxes toward the wall reduces in rarefied gas aerodynamics to quadratures, which can be calculated explicitly for simple shapes (plate, cylinder, sphere, wedge, cone, etc.) and numerically for complicated surfaces.

The free-molecular flow regime is one of the few examples in gas mechanics where the drag force and heat transfer can be obtained from the exact solution for the distribution function by using quadratures. The final results are presented with accuracy to the coefficients of accommodation of the normal and tangential momenta and the energy accommodation coefficient. Available experimental and theoretical data give the boundary of the free-molecular flow regime approximately at $\mathrm{Kn}_\infty = l_\infty/L \geq 10$, where the subscript $\infty$ denotes free-stream conditions.

In the other extreme regime, i.e., continuum flow, where the Knudsen number is sufficiently low, there is another singular-asymptotic solution of the Boltzmann equation (the small parameter is at the higher derivatives): expansion of the distribution function in the neighborhood of local thermodynamic equilibrium (in the neighborhood of the equilibrium Maxwell function in terms of velocities) in integer powers of Kn (Enskog method) and subsequent finding the normal solution, which yields the Euler equations in the zeroth approximation and the Navier-Stokes (NS) equations in the first approximation. However, it is now impossible to describe processes proceeding during times of the order of the mean time between the particle collisions ($\sim 10^{-9}$ s under standard conditions) and in domains of the order of the mean free path, for instance, in Knudsen layers, in SW structures, and near sharp leading edges and tips.

In contrast to hydrodynamic equations, the original Boltzmann equation describes processes proceeding at spatial and temporal scales of the Knudsen layers but does not describe particle collision processes in time and space. The collision occurs instantaneously and at a point. Comparing the Boltzmann equation to the Euler and NS equations, we should also note that the latter admit analytical solutions of boundary value problems in some cases, whereas solving initial boundary value problems within the framework of the Boltzmann equation requires application of labor-consuming numerical methods.

In the second approximation of the Enskog method, we arrive at the Burnett equations and then at the super-Burnett equations [17]. There is increasing recent interest in taking into account the highest approximations of the Enskog method, in particular, the Burnett equations, aimed at extending the area of applicability of continuum models in hypersonic flow problems toward higher Knudsen numbers (or lower Reynolds numbers). This interest is primarily caused by successful application of these equations in the problem of ultrasound and satisfactory solution of the problem of the SW structure [17] (at high Mach numbers, the NS equations predict a much smaller SW thickness than that observed in experiments).

Nevertheless, attempts to use the Burnett equations for solving initial boundary value problems face principal difficulties. First, the presence of the second derivatives of temperature with respect to the spatial coordinates in the expression for the stress tensor and the second derivatives with respect to the velocity vector in the heat flux term finally leads to the third derivatives (those of pressure and temperature in the momentum equation and of velocity in the energy equation), which results in the necessity of imposing additional boundary conditions (as compared to the number of boundary conditions in the NS equations), which do not follow from the mechanical formulation of the problem.

Moreover, because of the emergence of SW instability in the Burnett equations [18, 19], it is necessary to develop additional measures for stabilizing the numerical solution in the case of obtaining this solution on a fine grid [20]. Thus, for suppressing instability in solving stationary problems by the relaxation method, "expanded" ("corrected") Burnett equations are used, which include some deliberately chosen out-of-order terms [21].

At the same time, additional studies of differential approximations of the Burnett equations in the flow regime with slipping show that these approximations can violate the second law of thermodynamics [21]. Moreover, the numerical solution [22] of the problem of the flow past a flat plate with a sharp leading edge shows that the Burnett equations yield a less exact description of the flow field than the NS equations do. It was shown [9, 23, 24] that the allowance for the Burnett terms leads to worse agreement of theoretical and experimental results at Knudsen numbers approaching the boundary of applicability of the NS equations. This boundary in hypersonic flow problems depends on the governing parameters of the problem, the flow domain near the body, and the method of the solution; as was found in numerical calculations, this boundary lies at $\mathrm{Kn}_\infty = 0.1$–0.8. The Burnett equations can improve the accuracy of the solution only in those areas where the NS equations ensure acceptable accuracy (i.e., where Kn is sufficiently small), but the Burnett equations are also inapplicable in those areas where the NS equations are inapplicable [24]. Thus, the hope for improving the solution of supersonic and hypersonic flow problems in the range of low Reynolds numbers by using the Burnett equations over the results obtained by solving the NS equations was not justified. The same refers to the super-Burnett equations. The modern status of the Burnett equations was reviewed in detail in Ref. 17.

In the very beginning (1940–1950s) of the scientific investigations of problems of external high-velocity flows of low-density gases, which were described in detail in 1946 in the famous paper of Tsien [25], the interest was in the free-molecular hypersonic flow regime. Later studies were performed to clarify the possibility of extension of the continuum approach to solving problems of hypersonic flow at moderate and low Reynolds numbers (Re) [26–28].

Numerical studies [26] showed that the BL equations solved in the entire shock layer on the body with the classical Rankine-Hugoniot equations on the bow SW yield better results on heat transfer at low Reynolds numbers than the full NS equations. Subsequent numerical solutions of such problems [29] showed that the NS equations and their asymptotically simplified variants, i.e., parabolized NS equations and viscous shock layer (VSL) equations (these models were discussed in Refs. 30 and 31), in situations with Re tending to zero yield unlimitedly increasing values for the heat transfer and friction coefficients, which exceed the free-molecular limit (i.e., these methods yield physically incorrect results beginning from certain sufficiently low values of Re). Allowance for the slip velocity and temperature jump on the body surface and on the SW (generalized Rankine-Hugoniot conditions [32–34]) somewhat reduce these coefficients, thus, expanding the area of applicability of these continuum models to lower values of Re, but do not eliminate the tendency to an unlimited increase in these coefficients with a further decrease in Re. Contraction of the area of applicability of continuum models with decreasing Re (increasing Kn) also agrees with the asymptotic derivation of the NS equations from the Boltzmann equation at low Knudsen numbers with allowance for terms of the order of O(1) and O(Kn) in the Enskog method. Moreover, the parabolized NS equations and the VSL equations were asymptotically derived from the full NS equations at $\text{Re} \gg 1$; naturally, we cannot expect these simplified continuum models to give satisfactory and physically correct results at low Reynolds numbers.

## 1.2. THEORY OF THE BOUNDARY LAYER IN THE SECOND APPROXIMATION

The development of the theory of the boundary layer in the second approximation was started in late 1950s and early 1960s [the first approximation refers to the classical Prandtl scheme: external inviscid flow and the BL flow, which is valid as $\text{Re} \to \infty$, and retaining of terms O(1) in the BL equations]. The essence of the BL theory in the second approximation is as follows. As compared to the classical BL theory, effects of the second order, i.e., terms of the order of $\text{O}(\text{Re}^{-1/2})$, are taken into account in the NS equations as $\text{Re} \to \infty$. These effects include the following factors:

1. Curvature of the body surface
    a. Streamwise curvature
    b. Transverse curvature
2. BL interaction with the external (inviscid) flow
    a. Effect of the BL displacement thickness on the inviscid flow
    b. Gradient of entropy in the external (with respect to the BL) flow
    c. Gradient of stagnation enthalpy in the external (with respect to the BL) flow
3. Non-continuum (kinetic) effects near the surface
    a. “Slip velocity” on the surface
    b. “Temperature jump” on the surface
    c. “Concentration jump” on the surface

approaches based on performing iterations over the pressure gradient component in the chosen flow direction (see Chapter 16).

There are numerous publications on the numerical solution of the full NS equations as applied to problems of the hypersonic flow around real configurations of space vehicles [106]. This monograph describes three numerical methods of solving the NS equations as applied to solving three-dimensional problems of the flow around real space vehicles and probes (Sakharov, Borovoy, Egorov, Skuratov, Vlasov and Gorshkov, Kovalev, and Lunev).

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