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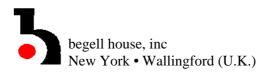
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by

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To the memory of Professor Yuri A. Buyevich, brilliant scientist and originator of numerous scientific ideas, who always kept the love for life and science dear.

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Introduction

A vast number of specialized investigations concerned with different aspects of transport processes in heterogeneous and, in particular, granular media has been published. This stems from the exceptionally great variety of such media and processes and also from their extremely extensive prevalence in the most varied fields of energy generation, metallurgy, chemical technology, mining thermophysics and other practical activities. A large number of monographs and other surveys were also published for the purpose of systematizing in a relatively complete form the most important advances and results attained during the past decades in disciplines of quite portentous import

For this reason the first question that arises immediately, upon an even casual perusal of the boundless ocean of journal articles and the numerous monographs on the subject, is that of the need of still another such monograph. It is our opinion that the motivation for this will become evident upon recognizing the wide methodological chasm between the many engineering-type studies and attempts of general-physical analysis of transport processes on the basis of rigorous methods of theoretical physics and applied mathematics. Studies of the first type contain an enormously large body of empirical material, pertaining to different aspects of heat and mass transfer in engineering practice; these are absolutely necessary for efficient design and optimization of most of industrial devices and equipment. However, they do not always or insufficiently explain the underlying physical factors and mechanism of transport and the very abundance of these studies and their excessive detailing frequently only complicate analytic correlation. On the other hand, the second type of studies suffers from excessive

model simplifications, which usually are very far from the actual situation. As a result they are too complicated both for direct use and even for gaining insight into the processes, which makes them poorly accessible and not too interesting to the practicing engineer.

The existence of this chasm reduces to a large extent the value both of applied and theoretical studies and overcoming it requires, first of all, developing a consistent system of physical concepts on the principal features of transfer of heat, mass and electrical discharge in heterogeneous and multiphase media of different structure and under different conditions. This applies, in the first place, to gaining insight and incorporation of only principal factors that affect these processes, while neglecting secondary and unimportant details and, on the other hand, developing sufficiently simple and understandable general principles of modeling them, which would have a sufficiently wide applicability and would not involve excessively refined analytical methods.

The attempt to formulate certain unified approaches to describing different aspects of the different transport processes in dispersed media is, in fact, the main purpose of this book. This goal did not consist so much in providing a general description of these approaches, based on the technique of averaging over an ensemble and illustrating their workability with respect to elementary applied problems, but in broadening the range of methodologies which could then be used in analyzing more complex and constantly arising problems.

This goal left its imprint on the level and style of presentation. Firstly, given the extensive bibliography and surveys of empirical correlations on heat and mass transfer in various specific media, the authors have completely forgone systematizing these data in a more or less complete form. Publications were used primarily only to the extent to which they appeared to be useful for better mastering the idea and methods of their implementation and for this reason reference to these are somewhat arbitrary or random. Secondly, since our objective consisted in presenting these ideas and methods in a sufficiently simple form, which could be actually used by engineering personnel in solving the problems that they face, ensuring better understanding required at times sacrificing elements of rigor and preexistent validity of the assumed representations and approaches, leaving the matter to the intuition of the reader. This feature of individual chapters of the book may apparently cause some lack of internal satisfaction not only to the adherents of rigor, but also to fully sensible theoreticians. However, in our opinion, this is compensated for by the obvious practical applicability of the methods developed here for practical solution of new problems of the theory of heat and mass transfer, including those coupled with phase and chemical transitions.

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I

Heat Transfer Equations

In engineering practice one usually deals with situations in which the characteristic minimal linear scale L of a significant variation of some of the quantities that represent the average properties of the heterogeneous medium or the course of heat transfer in it, exceeds highly the specified linear scale l of the internal structure of this medium. In the case of heat transfer in a macroscopically homogeneous granular layer, L apparently represents the characteristic distance at which a change occurs in the mean temperature of significance to this process, whereas the mean dimension of the particles making up the layer can be used as the structural microscale. In dilute dispersions (suspensions, emulsions, composite materials) this microscale is determined by the mean distance between the centers of adjoining discrete inclusions.

In the above situations it is natural to analyze heat transfer in the continuum approximation as occurring simultaneously in several fictitious homogeneous continua with their specific effective properties — specific heat and thermal conductivity. Each such continuum simulates a separate phase or component of the original heterogeneous medium; the possibility of heat transfer between the continua is allowed for. For steady-state processes, when the phase or component temperatures equalize, one may introduce a single continuum, which corresponds to a single-temperature model of the heterogeneous medium. In the case of a granular layer or of a moving disperse system it is natural to introduce two fictitious continua that simulate respectively the dispersed phase formed by the particles and the continuous medium between them. The latter may be a gas or a dropwise liquid, as well as a solid substance, which occurs in com-

posite materials with discrete inclusions. These inclusions may be in any state. If the composite material, granular layer or dispersed system contain particles or inclusions with highly differing thermophysical and other properties, it may turn out advisable to introduce a large number of homogeneous continua — one of the continuous medium between the particles and several for the different kinds of particle systems.

The continuum method of describing heat transfer in heterogeneous media involves two fundamental interrelated problems. The first consists in formulating a set of equations for the mean temperatures of these continua on the basis of the law of conservation of energy. These equations always contain effective heat fluxes in the individual continua and also terms describing the heat transfer between these continua. All these quantities must be expressed as functions or functionals of unknown variables of the equations, i.e., of the mean temperatures of the continua and of their derivatives with respect to independent variables, and also of parameters and physical characteristics that describe the structure of the medium and the properties of its phases. This requires attaining closure of this set of equations, which is absolutely necessary for their applicability of the study of applied problems. The finding of such functions and functionals comprises the second fundamental problem.

An entirely similar problem arises also in analyzing transfer of some physical quantity (mass, electric charge, momentum), rather than of heat, in heterogeneous systems. Given the identical physical meaning and mathematical formulation of problem of diffusion mass transfer, all the results obtained for heat transfer also apply to transfer of mass. In the latter case the mean heat flux densities are replaced by impurity mass fluxes, whereas the effective thermal conductivities are replaced by effective diffusion coefficients, etc. In the case of momentum transfer, equations for the mean temperatures of continua are replaced by equations for their average velocities or displacements and the obtaining of closure equations for effective forces and stresses in continua is the principal concern of rheology within the framework of which one must find effective viscosities and elastic moduli, and also all the coefficients in equations for the forces of interaction between the continua. The present monograph is not concerned with rheology problems and it focuses primarily on the transport of a scalar physical quantity — such as heat or the mass of a diffusing admixture.

This chapter is concerned with the fundamental solution of the first of the above problems — rigorous derivation of effective equations for the mean temperatures of the dispersed phases (or of the continua modeling them) on the basis of the general Gibbs representations over the ensemble of possible versions of the states of this medium. However, before describing the properties of the configuration ensemble of a system of particles of a moving dispersion or of a stationary granular layer and the mathematical tools of the averaging procedure needed for this purpose, we shall present a brief description of the traditional methods of analysis of heat transfer in heterogeneous and, in particular, in granular systems.

I.1 Equations of the Two-Temperature Model

The equations of heat transfer are usually postulated within the framework of the phenomenological or semi-empirical method or are obtained as a result of averaging over

representative spatial objects (volume or elements of differently oriented surfaces). In studies of the former kind, which apparently were started several decades ago for the cases of analysis of temperature distributions in rocks, oil-bearing strata and also in dense granular beds of different types [1–3], it is necessary to postulate both the very form of the equations and the functional representations of the quantities making them up. Exhaustive surveys and examples of practical application of different versions of the phenomenological approach are contained in a number of monographs [4–9] and in a large number of journal articles. Lately these methods were further developed by Antonishin and his coworkers [10, 11], who paid particular attention to different aspects of their application in describing heat transfer in granular beds of different types (quiescent, rotating, ordinary- and vibro-fluidized).

The technique of spatial averaging has attained a high level of perfection as a result of work by a large number of investigators. A survey of usually employed representations and methods and also of the principal results can be found in [12–18]. The effective heat transfer equations obtained from spatial averaging of ordinary Fourier or convective heat transfer equations, which are valid in materials of the individual phases, have the same basic form as some of those postulated from empirical considerations. This, obviously, can be regarded as important evidence of the goodness of fit of the phenomenological theory. However, approaches based on spatial averaging do not yield anything virtually new with respect to derivation of an "equation of state" for effective heat fluxes and rate of interphase heat transfer, i.e., they do not contribute to the solution of the second of the above fundamental problems.

We shall restrict ourselves, for the purposes of illustration, to analysis of heat transfer in a macroscopically homogeneous infiltrated porous body or granular bed. When using the phenomenological model, the mean temperatures τ_0 and τ_1 of the "continuous" (fluid) and "dispersed" (solid) medium are usually obtained from expressions such as

$$c_{e0}\left(\frac{\partial \tau_0}{\partial t} + \nabla\left(\mathbf{u}\tau_0\right)\right) = \lambda_{e0}\Delta\tau_0 + h_e + j_{e0}\,, \tag{I.1}$$

$$c_{e1}\frac{\partial \tau_1}{\partial t} = \lambda_{e1} - h_e + j_{e1}\,,$$

the general form of which is the same as that of equations obtained by spatial averaging. If it is assumed that the temperatures of the solid and liquid phases equalize virtually instantaneously (which is possible in principle when rate h_e of interphase heat transfer is sufficiently high), then Eqs. (I.1) can be replaced by the expression

$$c_e\frac{\partial \tau}{\partial t} + c_{e0}\nabla\left(\mathbf{u}\tau\right) = \lambda_e\Delta\tau + j_e\,, \quad \tau_0 = \tau_1 = \tau\,, \tag{I.2}$$

$$c_e = c_{e0} + c_{e1}\,, \quad \lambda_e = \lambda_{e0} + \lambda_{e1}\,, \quad j_e = j_{e0} + j_{e1}\,,$$

that is obtained by summing Eqs. (I.1) at $\tau_0 = \tau_1$.

Actually, Eqs. (I.1) and (I.2) implicitly utilize an unproven hypothesis to the effect that the effective mean heat flux densities in both phases satisfy the Fourier law, i.e., that $\mathbf{q}_{e0} = -\lambda_{e0}\nabla\tau_0$, $\mathbf{q}_{e1} = -\lambda_{e1}\nabla\tau_1$. Similarly, they include a convective heat flux stemming from the flow of the fluid phase, which is assumed to be equal to $c_{e0}\mathbf{u}\tau_0$, where $\mathbf{u}$ has the meaning of the mean velocity of the fluid in the pore space (but not to the mean filtration velocity $\mathbf{u}_f = \varepsilon\mathbf{u}$, where ε is the porosity).

Accepting both the above hypotheses, we shall express the effective strengths of the mean heat sources in the liquid and solid phases per unit volume of heterogeneous medium by similar quantities, assumed to be known, for materials of both phases as $j_{e0} = \varepsilon j_0$ and $j_{e1} = \varphi j_1$, where $\varphi = 1 - \varepsilon$ is the mean volumetric concentration of the solid phase. In addition, we shall write the effective specific heats in Eqs. (I.1) as

$$c_{e0} = \varepsilon\rho_0 c_0 \,, \quad c_{e1} = \varphi\rho_1 c_1 \tag{I.3}$$

(ρ_0 and ρ_1 are the densities and c_0 and c_1 are the specific heats of the liquid- and solid-phase materials), the physical meaning of which is obvious. However, even in this case closure is far from being attainable.

First, the meaning of the effective heat transfer coefficients λ_{e0} and λ_{e1} and also their functional dependence on the thermal conductivities λ_0 and λ_1 of the materials and on the mean phase concentrations ε and φ are also highly unclear. The obtaining of this relationship is difficult also because firstly, the effective coefficients depend to a larger extent not only on the mean concentrations, but also on the very nature and specifics of the microstructure of the heterogeneous medium. Secondly, they turn out to be far from identical for heat transfer with different time scales. We shall illustrate the above using quite simple examples.

Let the topological properties of the domains occupied by the solid and liquid phases be approximately the same. This is approximately valid for many fluid-filled capillary-porous media. If heat transfer occurs at close to steady-state conditions (relatively large time scale), then the effective thermal conductivity λ_e of the heterogeneous medium in Eq. (I.2) should be properly expressed by the equation

$$\lambda_e = \varepsilon\lambda_0 + \varphi\lambda_1 \,, \tag{I.4}$$

that follows directly from the theory of mixtures. However, it is virtually impossible to make allowance, in this quasi-steady approximation, for the difference between the mean phase temperatures within the framework of the two-temperature model given by Eq. (I.1) if for no other reason then because it remains highly unclear how to separate λ_e into its two components λ_{e0} and λ_{e1} that are contained in the different equations (I.1). It is obvious that this specification of thermal conductivities for the individual phases is very high from being unique.

Further, if the time scale of the process is sufficiently small (as this happens, among others, at the initial stage of heating an infiltrated porous body by a solid wall), the heat initially propagates only through the phase with the higher thermal conductivity, whereas the second phase virtually either does not succeed in significantly affecting this process or to change its temperature to a significant extent. Under these conditions

Eq. (I.4) loses its meaning and the effective thermal conductivity as a whole at sufficiently small times consists of only one of the terms in the right-hand side of Eq. (I.4), namely the one corresponding to the phase at the higher temperature.

Let us now examine slightly unsteady heat transfer in a granular bed with radically different topological properties. In fact, in this case the continuous phase then exhibits the connectivity property and permits unhindered transfer of heat only through itself. The dispersed phase, conversely, consists of discrete non-connected elements, separated by interstices of the continuous phase. If we neglect the conceivable transfer of heat by direct physical contact between the particles, and also through the aforementioned interstices, then this continuous transport of heat turns out to be impossible in principle. It follows from this that in the last case

$$\lambda_e = \lambda_{e0} = \lambda_0 f(\varphi, \kappa), \quad \lambda_{e1} = 0, \quad \kappa = \frac{\lambda_1}{\lambda_0}, \tag{I.5}$$

where $f(\varphi, \kappa)$ is some function of the mean concentration in the bed and of the ratio of the thermal conductivities of the phase that may, additionally, depend on the bed's structure (type of particle packing, their size distribution and shape). If the particle shape is significantly anisotropic and exhibits an orderly orientation, then in the general case, the single scalar coefficient (I.5) should be replaced by the tensor of effective thermal conductivities.

The existence of even a weak contact conductivity naturally gives rise to heat transfer over the dispersed phase and non-conformance to Eqs. (I.5), which is the more perceptible, the higher the thermal conductivity of the material of this phase as compared with that for the continuous phase in the intervals between the particles.

If we now consider heat transfer in a granular bed with a small time scale, then only the continuous phase participates in this process, whereas there is insufficient time for the discrete particles to react to the rapidly changing temperature situation. In this case the effective thermal conductivity turns out to be much smaller than that formally computed from Eq. (I.5). Such a situation occurs, among others, at the initial stage of heating of a granular bed by a solid wall [10, 11, 19].

We wish to note that Eqs. (I.4) and (1.5) completely ignore the transport component induced by the so-called convective dispersion of heat. This phenomenon is brought about by transfer of heat resulting from mixing of individual jets of liquids flowing over the bed particles or in the course of mixing of liquid flowing in the sectioned porous space of the body. For this reason the above equations are valid approximately only when the thermal Peclet number, based on the linear structural microscale of the heterogeneous medium, is small.

The question of identification of the specific rate h_e of interphase heat transfer in Eqs. (I.1) of the two-temperature model, which was analyzed particularly thoroughly for granular systems by Antonishin with his coworkers [10, 11], is even more complicated. Without going into details, we wish to point out that virtually all the suggestions on defining this quantity reduce to the expression

$$h_e = \alpha_e \rho_1 c_1 S (\tau_1 - \tau_0), \tag{I.6}$$

where S is the specific area of the phase interface, whereas α_e is some effective coefficient per unit area of this surface, and differ only by the manner of definition of α_e.

Upon neglecting the conductive manner of transfer as compared with the convective for the continuous phase and the contact conductivity for the dispersed phase, Eqs. (I.1), with closure provided by Eq. (I.6), were applied to specific problems of heating or cooling granular and porous strata by percolating flows of gas or dropwise liquid as far back as the 20ties [20, 21]. Exhaustive solutions of such problems were suggested in the book [22], which also presents tables of a number of functions that arise in the course of these solutions.

The use of Eqs. (I.3) and (I.6) in the second of equations (I.1), coupled with neglecting the contact conductivity and extraneous heat sources, yields a simple relaxation equation for the mean temperature of the dispersed (solid) phase

$$\frac{\partial \tau_1}{\partial t} = \frac{\alpha_e S}{\varphi} \left(\tau_1 - \tau_0 \right),\tag{I.7}$$

where $\varphi / \alpha_e S$ has the meaning of a characteristic relaxation time. In general, however, the relationship between τ_1 and τ_0 differs perceptibly from that in Eq. (I.7). Many attempts were made to allow for this difference within the assumptions of the model used in [20, 21], i.e., by neglecting the molecular heat transfer by introducing a time-dependent heat transfer coefficient α_e (see, among others, the book by Arens, et al. [9]). A large volume of computational data pertaining to different aspects of mining thermophysics, based on the first of equations (I.1) of the two-temperature model and its simplified versions with closure provided by expressions such as Eq. (I.7) are given in the books [23, 24].

I.2 Heat Transfer in Systems with Perceptible Thermal Inertia of the Solid Phase

The difficulties in obtaining closure of the equations of the two-temperature model and, in particular, in determining the magnitude of interphase heat transfer in a more rigorous form than given by quasi-steady equation (I.6) have resulted in partially forgoing the continuum method of describing heat transfer in porous and granular media. Under the new approach, transport in the continuous (fluid) phase is, as previously, described as for a continuum, but transfer with elements of the dispersed (solid) phase is analyzed on the basis of classical approaches to the heating and cooling of each such element. This statement of the problem appears to be justified, in the first place, in situation when there is no transport over the dispersed phase, which is frequently the case in granular beds. However, such a situation may occur also in the case of porous materials being infiltrated by fluid. Moreover, it is suitable in familiar simplified models of porous bodies consisting of intermittent plane-parallel layers of a solid substance, separated by similar free cracks that contain only a fluid. A survey of solved problems of this type is given by Kitayev [8].

The applications of this method to problems of heating of granular beds consisting of identical, regularly shaped particles are of greater interest. Apparently, Ivantsov and Lyubov [25] were the first to formulate and solve this problem for a bed of solid

ertia. In these cases it is advisable to introduce several different continua, that model particles of different types and, consequently to use a multi-temperature model of the starting medium. Without going into details, the form of the equations of this model is

$$\rho_0 c_0 \varepsilon \left(\frac{\partial \tau_0}{\partial t} + \mathbf{v}_0 \nabla \tau_0 \right) = - \nabla (\mathbf{q} + \mathbf{q}_{f_0}) + h + \sum_{k=1}^{K} h_k + \sum_{k=1}^{K} \sigma_k + \varepsilon j_0 , \qquad (I.57)$$

$$\rho_k c_k \varphi_k \left(\frac{\partial \tau_k}{\partial t} + \mathbf{v}_k \nabla \tau_k \right) = - \nabla \mathbf{q}_{f_k} - h_k + \varphi_k j_k , \quad k = 1, ..., K ,$$

where

$$h_k = \int\limits_{|\mathbf{r}-\mathbf{r}'| \le a_k} n_k (t, \mathbf{r}') \nabla_{\mathbf{r}} \mathbf{q}^{(k)} (t, \mathbf{r}|\mathbf{r}') \, d\mathbf{r}' , \qquad (I.58)$$

$$\sigma_k = \int\limits_{|\mathbf{r}-\mathbf{r}'| = a_k} \mathbf{n} (\mathbf{Q}_0 - \mathbf{Q}_1) n_k (t, \mathbf{r}') \, d\mathbf{r}' ,$$

$$\mathbf{q} = - \lambda_0 \nabla \tau - \sum_{k=1}^{K} (\lambda_k - \lambda_0) \int\limits_{|\mathbf{r}-\mathbf{r}'| \le a_k} n_k (t, \mathbf{r}') \nabla_{\mathbf{r}} \tau^{(k)} (t, \mathbf{r}|\mathbf{r}') \, d\mathbf{r}' +$$

$$+ \lambda_0 \sum_{k=1}^{K} \int\limits_{|\mathbf{r}-\mathbf{r}'| = a_k} (T_0^{(k)} - T_1^{(k)}) n_k (t, \mathbf{r}') \, d\mathbf{r}' , \quad \varepsilon = 1 - \sum_{k=1}^{K} \varphi_k ,$$

where integration is carried out over the locations of center $\mathbf{r}'$ of a particle of the kth kind such that point $\mathbf{r}$ lies within this particle, $n_k(t, \mathbf{r}')$ represents the conditional denumerable concentration of particles of the kth kind in the vicinity of a fixed particle of the same kind, whereas the superscripts (k) in the integrands simply designate the type of particle within which the conditional distributions of temperature $\tau^{(k)}$ and heat flux $\mathbf{q}^{(k)}$ are calculated.

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