Appendix C

Some Form-Factor Formulas

C.1 Point Form Factors

In the following formulas for $F_{\mathrm{d}1-2}$, elemental area dA_1 is located at the origin facing up, so that its inward normal is $-\widehat{\mathbf{k}}$. The titles of the sections below relate to the nature of surface 2. Note that the entire surface 2 must be above the xy plane.

Rectangle in a Plane Parallel to the xy Plane

Let A_2 be a horizontal $a \times b$ rectangle on a plane c units up the z-axis from the xy plane. First consider the case where the rectangle has one vertex on the z-axis. The vertices of the rectangle are $(0,0,c)$, $(a,0,c)$, $(0,b,c)$, and (a,b,c), respectively. Then

$$F_{\mathrm{d}1-2} = \frac{1}{2\pi}\left[\frac{X}{X_1}\tan^{-1}\left(\frac{Y}{X_1}\right) + \frac{Y}{Y_1}\tan^{-1}\left(\frac{X}{Y_1}\right)\right] \tag{C.1}$$

where $X = a/c$, $Y = b/c$, $X_1 = \sqrt{1+X^2}$, $Y_1 = \sqrt{1+Y^2}$. By adding, subtracting, or multiplying values given by Eq. (C.1), one can obtain $F_{\mathrm{d}1-2}$ for cases where a vertex is not on the z-axis. For example, if the vertices are (a,b,c), $(-a,b,c)$, $(-a,b,c)$ and $(-a,-b,c)$, then $F_{\mathrm{d}1-2}$ will equal four times the value given by Eq. (C.1).

Rectangle in a Plane Perpendicular to the xy Plane

First consider the case where the bottom side of the rectangle is on the xy plane. Let the vertices be $\mathrm{P}_1 = (0,c,0)$, $\mathrm{P}_2 = (b,c,0)$, $\mathrm{P}_3 = (0,c,a)$, and $\mathrm{P}_4 = (b,c,a)$.Then

$$F_{\mathrm{d}1-2} = \frac{1}{2\pi}\left[\tan^{-1}\left(\frac{1}{Y}\right) - \frac{Y}{\sqrt{Y^2+X^2}}\tan^{-1}\left(\frac{1}{\sqrt{Y^2+X^2}}\right)\right] \tag{C.2}$$

where $X = a/b$ and $Y = c/b$. By adding, subtracting, or multiplying values given by the above formula, one can obtain $F_{\mathrm{d}1-2}$ for other cases. For example, if the vertices are $\mathrm{P}_1 = (-b, c, 0)$, $\mathrm{P}_2 = (b, c, 0)$, $\mathrm{P}_3 = (b, c, a)$, and $\mathrm{P}_4 = (-b, c, a)$ then $F_{\mathrm{d}1-2}$ will equal twice the value given by Eq. (C.1).

Circle in a Plane Parallel to the xy Plane

Let the radius of the circle be r and its center be at $(a, 0, h)$, making its normal $\widehat{\mathbf{k}}$. Then

$$F_{\mathrm{d}1-2} = \frac{1}{2}\left[1 - \frac{1 + Y^2 - X^2}{\sqrt{(1 + Y^2 + X^2)^2 - 4X^2}}\right] \tag{C.3}$$

where $Y = h/a$ and $X = r/a$. By a suitable choice for the direction of the x-axis, this formula can be made to apply for any circle in a plane parallel to the xy plane.

Circle in a Plane Perpendicular to the xy Plane

Let the radius of the circle be r and its center be at $(a, 0, h)$, (with $a \leq r$), making its normal $\widehat{\mathbf{j}}$. Then

$$F_{\mathrm{d}1-2} = \frac{X}{2}\left[\frac{1 + X^2 + Y^2}{\sqrt{(1 + X^2 + Y^2)^2 - 4Y^2}} - 1\right] \tag{C.4}$$

where $X = h/a$ and $Y = r/a$. By a suitable choice for the orientation of the x-axis, this formula can be made to apply for any circle perpendicular to the xy plane, provided $a \geq r$.

Sphere

Let the radius of the sphere be R and let the center of the sphere be H units away from the origin and lie along a line that makes an angle φ with the z-axis. [Alternatively, let the center be at $= (H \sin\varphi, 0, H \cos\varphi)$]. Then

$$F_{\mathrm{d}1-2} = X^2 \cos\varphi \tag{C.5}$$

where $X = R/H$.

C.2 Form Factors

Directly Opposed, Parallel Identical Rectangles

Let A_1 and A_2 both be $a \times b$ rectangles with A_2 lying c units directly above A_1. More precisely, if the vertices of A_1 are at $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, and $(a, b, 0)$, then the vertices of A_2 are at (0,0,c), (a,0,c), $(0, b, c)$, and (a, b, c). We then have

$$F_{1-2} = \frac{2}{\pi XY}\left\{\ln\left[\frac{X_1Y_1}{Z}\right] + XY_1\tan^{-1}\left[\frac{X}{Y_1}\right] + YX_1\tan^{-1}\left[\frac{Y}{X_1}\right]\right\} - \frac{2}{\pi}\left\{\frac{\tan^{-1}X}{Y} + \frac{\tan^{-1}Y}{X}\right\} \tag{C.6}$$

where $X = a/c$, $Y = b/c$, $Z = \sqrt{1 + X^2 + Y^2}$, $X_1 = \sqrt{1 + X^2}$, $Y_1 = \sqrt{1 + Y^2}$.

Perpendicular Rectangles with a Common Side

Let A_1 be an $a \times b$ rectangle, let A_2 be a $c \times b$ rectangle perpendicular to A_1, and let them share a side of length and b. More precisely, if the vertices of $A_1$are $(0, 0, 0)$, $(b, 0, 0)$, $(0, a, 0)$ and $(b, a, 0)$, then the vertices of A_2 are at $(0, 0, 0)$, $(b, 0, 0)$, $(0, 0, c)$, and $(b, 0, c)$. We then have

$$F_{1-2} = \frac{1}{\pi X}\left\{X\tan^{-1}\frac{1}{X} + Y\tan^{-1}\frac{1}{Y} - Z\tan^{-1}\frac{1}{Z}\right\} + \frac{1}{4\pi X}\ln\left(\frac{X_1Y_1}{Z_1}\left[\frac{X^2Z_1}{X_1Z^2}\right]^{X^2}\left[\frac{Y^2Z_1}{Y_1Z^2}\right]^{Y^2}\right) \tag{C.7}$$

where $X = a/b$, $Y = c/b$, $Z = \sqrt{X^2 + Y^2}$, $X_1 = 1 + X^2$, $Y_1 = 1 + Y^2$, and $Z_1 = 1 + Z^2$.

Coaxial Parallel Circles

Let A_1 be a circle of radius a and let A_2 be a circle of radius b, which is coaxial with A_1 and in a plane parallel to that of A_1, h units way. In other words, if $(0, 0, 0)$, $(a, 0, 0)$, and $(0, a, 0)$ are three points on A_1, then $(0, 0, h)$, $(b, 0, h)$, and $(0, b, h)$ are three points on A_2. Then we have

$$F_{1-2} = \frac{1}{2}\left\{1 + X^2 + Y^2 - \sqrt{(1 + X^2 + Y^2)^2 - 4Y^2}\right\} \tag{C.8}$$

where $X = h/a$ and $Y = b/a$.