INTRODUCTION

In learning thermal radiation, we need to master a few differential equations expressing the appropriate conservation laws together with the appropriate boundary conditions. In this way, learning thermal radiation is similar to learning heat conduction and convection. But there are several important variations that make radiation decidedly different. Understanding the general nature of these differences should help in the total learning process, and so we will highlight them in what follows.

The fundamental differential equation describing the conservation of the radiant energy in a specified direction, say $\hat{\mathbf{d}}$, is the Radiative Transfer Equation, which for a gray nonscattering medium can be written as

$$\vec{\nabla} \cdot \mathbf{i} = -a\left(i - \sigma T^4/\pi\right) \tag{1}$$

where $\mathbf{i}$ is the intensity vector for radiation in direction $\hat{\mathbf{d}}$, which is a measure of the strength of that radiation, i is the magnitude of $\mathbf{i}$, T is the local temperature, and a is a property for the medium and σ is a constant[2]. If we were to compare this with another conservation equation, say that for the conservation of heat in a stationary medium at steady-state

$$\vec{\nabla} \cdot \mathbf{q} = q''' \tag{2}$$

or the conservation of mass for a flowing fluid:

$$\vec{\nabla} \cdot (\rho \mathbf{V}) = -\frac{\partial \rho}{\partial t} \tag{3}$$

we immediately see some similarities, particularly in the structure of the left-hand side. The important difference is not so much the details of the equations, but the fact that Eq. (1) must be written an infinite number of times, once for each possible direction $\hat{\mathbf{d}}$ in space, whereas Eqs. (3) and (2) need be written only once.[3] Moreover, because very few media are gray, in practice, an equation

[2] We use a slightly different nomenclature for intensity in this Introduction from that used in the main body.

[3] Why do not conduction and convection also require conservation equations in every direction? After all, the medium for Eqs. 3 and 2 may be a gas, and gas molecules travel in all directions in space. The answer is that because a gaseous molecule travels very short paths before colliding with a neighbor, the overall effect of many, many molecules can be captured by a single vector equation like $\mathbf{q} = -k\vec{\nabla}T$.

like Eq. (1) will have to be written an infinite number of times, once for each wavelength, there being a different intensity and value of a at every wavelength. So in a sense there is a double infinity of governing equations, and this is the main thing that makes radiation different. The saving feature of radiation, at least for a nonscattering medium, is that for any one combination of wavelength and direction, the differential equation is relatively easy to solve: it readily transforms into a first-order, linear, ordinary differential equation, the general solution to which can be written down at once.

Another difference is in the boundary condition for Eq. (1), which, as in conduction, is prescribed at an interface between adjacent media. The boundary condition for the intensity in one direction $\hat{\mathbf{d}}$ depends on the intensity in all the other (incident) directions. This means that it also depends on the intensity at all other points on the interface, whereas that for Eqs. (3) or (2) (say zero velocity or a specified heat flux) is independent of what is going on elsewhere on the interface. The net result is that the complete statement of the radiative boundary conditions, written over the entire interface, reduces to an integral equation. Solving this integral equation is one of the more daunting tasks in radiant analysis. (One approximate scheme for solving these integral equations leads to the well-known "form factors.")

Once the differential equations of a conduction or convection problem have been solved, the entire solution is more or less complete—although sometimes one wants to integrate the solution over a bounding area to get the heat flow over a surface. But once a *radiation* problem has been solved for the intensity field, there is always the need for further integration because the intensity itself is not normally of engineering interest. Normally one wants to know the heat flow over a surface, and this requires integration over directions and wavelength, as well as over area. For this reason, multivariable integration plays a major role in radiant analysis.

All of the above assumes that the temperature field is given [notice the appearance of temperature in Eq. (1)], but often one does not know the temperature field beforehand. Rather radiation itself plays an important role in fixing the temperature in the medium. Clearly one needs an additional equation, and this is the familiar Energy Equation for the medium, which must now be generalized to include a new term, one representing the local rate, per unit volume, at which energy is taken out of the radiant field, a quantity that can be positive or negative. The Energy Equation and the Radiative Transfer Equation must be solved simultaneously to establish the temperature field and the intensity field.

Perhaps some comparisons with steady-state conduction will make the above clear. Finding the intensity corresponding to a given temperature distribution is like using Fourier's law: $\mathbf{q} = -k\vec{\nabla}T$ to find the conductive heat flux corresponding to a given a temperature distribution. We note, however, that in conduction, it is the conduction process itself that will fix the temperature distribution, so Fourier's Law is of little use unless it is combined with the Energy Equation, namely Eq. (2). These two equations must be solved simultaneously. It happens, however, that for pure conduction, one equation can be substituted directly into the other to reduce the pair to a single equation: thus $\mathbf{q} = -k\vec{\nabla}T$ substituted into $\vec{\nabla}\cdot\mathbf{q} = q'''$ gives $\nabla^2 T = -q'''/k$, the familiar partial differential equation that is solved routinely in conduction analysis. Such simple substitution and reduction are not normally possible for radiation; nevertheless, the underlying structure is the same.

Because of the complexity of the general case, it is in fact very rare to solve a radiant problem exactly, and there is a heavy reliance on models that simplify the problem and admit tractable solutions. One such model—one with which the student should be familiar, as it is treated in undergraduate texts—is the enclosure containing a transparent medium bounded by a diffuse opaque surface. In fact, there are three models at work here: the "diffuse surface," for which incident radiation is reflected uniformly in all directions; the "transparent medium," for which $a = 0$ in Eq. (1); and finally the "opaque surface," for which the radiant term in the Energy Equation is zero except in a very small volume so close to the surface that for all practical purposes, the radiant exchange happens at the surface itself.

This text may breaks into three principal parts. The first, Chapter 1 to 8, is about the Radiative Transfer Equation: its derivation, some solutions for presumed internal and boundary conditions, and some examples of integrating these solutions over direction and wavelength. This part also includes chapters characterizing scattering and radiation's role in the Energy Equation for the medium.

The second part, Chapter 9 to 14, is mainly about how we determine the boundary conditions to the Radiative Transfer Equation, so it deals with the radiant properties of interfaces and surfaces, and with surface models. When an interface is perfectly smooth and the material on each side is homogeneous, the properties of surfaces can be treated exactly by electromagnetic wave theory, and the second part presents a precis of that theory.

The third part, the remainder of the book, is about solving the Radiative Transfer Equation for a prescribed set of boundary conditions. The first chapters of this part treat transparent enclosures, a situation that leads to an integral equation, which is solved to various levels of approximation. Next the assumption that the gas is transparent (but not the assumption that the gas is nonscattering) is relaxed, and the associated chapters give a model for the radiant properties of the gas, as well as an exposition of the solution methodology for isothermal gases. The final chapter treats some simply defined situations where the Radiative Transfer Equation may be solved in concert with the Energy Equation for a stationary medium. Some of these solutions allow for a scattering medium.

As has been mentioned in the Preface, many problems are solved using the software package Mathcad, and where that is done, a hardcopy of the Mathcad worksheet is included as a figure. Also, the CD in the back of the book includes an Evaluation Version of Mathcad® 11, Single User Edition, which is reproduced with permission[4]. This software is a fully functional trial of Math-

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cad which will expire 120 days from installation. For technical support, more information about purchasing Mathcad, or upgrading from previous editions, see http://www.mathcad.com. The CD also contains the computer files of the Mathcad worksheets that were included in hardcopy in the figures.

The book uses a slightly unorthodox referencing system that needs to be explained. The section "References, Bibliography, and Further Reading" at the back of the book lists many important literature sources. Some of these (but not all) are cited in the main section of the book, with the author, title, and year being mentioned at the point of citation if the citation is to a book and the author, journal, and year if it is to a paper. A more complete reference for these brief citations can be quickly found by searching through the References, Bibliography, and Further Reading Section.

Students are encouraged to review the radiation chapters in their undergraduate heat transfer textbook before starting on this graduate text. While the present text does not require any previous exposure to thermal radiation, the different perspective of the undergraduate text will help to set the framework for the present treatment, which takes a more fundamental approach. It is hoped that it will answer any fundamental questions encountered on reading the undergraduate text, although it will probably raise a few more, as there is much to learn about radiation.