APPENDIX A

Table A.1 Laplace transforms of some functions

Function, $\theta(\mathrm{Fo})$	**Transform,** $\theta(s)$
1	$1/s$
Fo	$1/s^2$
$\mathrm{Fo}^{n-1}/(n-1)!$, $(n = 1, 2, 3,\ldots)$	$1/s^n$
$1/\sqrt{(\pi \mathrm{Fo})}$	$1/\sqrt{s}$
$2\sqrt{(F/\pi)}$	$s^{-3/2}$
$\dfrac{2^n \mathrm{Fo}^{(n-1/2)}}{[1.3.5-(2n-1)]\sqrt{\pi}}$ $(n = 1, 2, 3,\ldots)$	$s^{-(n+1/2)}$
Fo^{m-1}, $m > 0$	$\Gamma(m)/s^m$
$e^{\pm aFo}$	$1/(s \mp a)$
$\mathrm{Fo}\, e^{aFo}$	$1/(s-a)^2$
$[1/(n-1)!]\, \mathrm{Fo}^{n-1}\, e^{aFo}$, $(n = 1,2,3,\ldots)$	$1/(s-a)^n$
$\cos a\mathrm{Fo}$	$s/(s^2 + a^2)$
$\sin a\mathrm{Fo}$	$a/(s^2 + a^2)$
$\cosh a\mathrm{Fo}$	$s/(s^2 - a^2)$
$\sinh a\mathrm{Fo}$	$a/(s^2 - a^2)$
$\dfrac{e^{bFo} - e^{aFo}}{2\sqrt{(\pi Fo^3)}}$	$\sqrt{(s-a)} - \sqrt{(s-b)}$
$erfc\left(\dfrac{a}{2\sqrt{\mathrm{Fo}}}\right) - \exp(ba)$	$\dfrac{b\exp(-a\sqrt{s})}{s(b+\sqrt{s})}$
$x\exp(b^2\mathrm{Fo})\,\mathrm{erf}(b\sqrt{\mathrm{Fo}} + a/2\sqrt{\mathrm{Fo}})$	
$\exp(ab)\exp(b^2\mathrm{Fo})\, erfc(b\sqrt{\mathrm{Fo}} + a/2\sqrt{\mathrm{Fo}})$	$\exp(-a\sqrt{s})/[\sqrt{s}(b+\sqrt{s})]$, $b > 0$
$\sqrt{\left(\dfrac{b}{\pi \mathrm{Fo}}\right)}\exp\left(-\dfrac{a^2}{4\mathrm{Fo}}\right) - b\exp\left(a\sqrt{b} + bFo\right)\times$ $erfc\left[\dfrac{a}{2\sqrt{\mathrm{Fo}}} + \sqrt{(b\mathrm{Fo})}\right]$	$\dfrac{1}{1+\sqrt{(s/b)}}\exp\left(-a\sqrt{s}\right)\exp(-a\sqrt{s})$
$\dfrac{2}{b}\left(\dfrac{\mathrm{Fo}}{\pi}\right)^{1/2}\exp(-4a^2\mathrm{Fo}) - \left(\dfrac{1+ab}{b^2}\right)\mathrm{erfc}\dfrac{a}{2\sqrt{\mathrm{Fo}}}$ $+\ \dfrac{1}{b^2}\exp(ab + b^2\mathrm{Fo})\mathrm{erfc}\left(\dfrac{a}{2\sqrt{\mathrm{Fo}}} + b\sqrt{\mathrm{Fo}}\right)$	$\dfrac{1}{s^{3/2}(\sqrt{s}+b)}\exp(-a\sqrt{s})$

Table A.1 Continued

Function, $\theta(\mathrm{Fo})$	**Transform,** $\theta(s)$
$\frac{1}{(-b)^n}\exp(ab+b^2\mathrm{Fo})\operatorname{erfc}\left(\frac{a}{2\sqrt{\mathrm{Fo}}}+b\sqrt{\mathrm{Fo}}\right)$ $-\frac{1}{(-b)^n}\sum_{m=0}^{n-1}\left(-2b\sqrt{\mathrm{Fo}}\right)^m i^m\operatorname{erfc}\left(\frac{a}{2\sqrt{\mathrm{Fo}}}\right)$	$\frac{1}{s^{(n+1)/2}\sqrt{(s+b)}}\exp\left(-a\sqrt{s}\right)$
$\frac{1}{2}\exp(b\mathrm{Fo})\left[\exp\left(-\frac{a}{\sqrt{b}}\right)\operatorname{erfc}\left(\frac{a}{2\sqrt{\mathrm{Fo}}}-\sqrt{b\mathrm{Fo}}\right)\right]$ $+\exp\left(\frac{a}{\sqrt{b}}\right)\operatorname{erfc}\left[\frac{a}{2\sqrt{\mathrm{Fo}}}+\sqrt{b\mathrm{Fo}}\right]$	$\frac{1}{s-b}\exp\left(-a\sqrt{s}\right)$
$\frac{2\mathrm{Fo}^{(n+2)/2}}{2n+1}\Pi\left(1+\frac{n}{2}\right)(2i)^{(n+2)}\,erfc\left(\frac{a}{2\sqrt{\mathrm{Fo}}}\right)$	$\frac{\Pi\left(\frac{1}{2}n\right)}{\sqrt{s}^{(4n+1)/2n}}\exp\left(-a\sqrt{s}\right)$
$\frac{1}{2}\left[\exp\left(-a\sqrt{b}\right)\operatorname{erfc}\left(\frac{a}{2\sqrt{\mathrm{Fo}}}-\sqrt{2b\mathrm{Fo}}\right)\right.$ $\left.+\exp\left(a\sqrt{b}\right)\operatorname{erfc}\left(\frac{a}{2\sqrt{\mathrm{Fo}}}+\sqrt{b\mathrm{Fo}}\right)\right]$	$\frac{1}{s}\exp\left(-a\sqrt{s}+b\right)$
$\frac{1}{\sqrt{\pi\mathrm{Fo}}}\exp\left(\frac{a}{2\sqrt{\mathrm{Fo}}}+2b\mathrm{Fo}\right)+\frac{\sqrt{2b}}{2}[\exp\left(-a\sqrt{2}b\right)$ $\times\operatorname{erfc}\left(\frac{a}{2\sqrt{\mathrm{Fo}}}-\sqrt{2b\mathrm{Fo}}\right)-\exp\left(a\sqrt{2}b\right)$ $\times\operatorname{erfc}\left[\frac{a}{\sqrt{2\mathrm{Fo}}}+\sqrt{2b\mathrm{Fo}}\right]$	$\frac{1}{s\sqrt{(s+2b)}}\exp\left[-a(s+2b)^{1/2}\right]$
$\left[\frac{1}{\sqrt{\mathrm{Fo}}}\right]-ae^{a^2\mathrm{Fo}}\operatorname{erfc}a\sqrt{\mathrm{Fo}}$	$1/\left(a+\sqrt{s}\right)$
$\frac{1}{\sqrt{(\pi\mathrm{Fo})}}+ae^{a^2\mathrm{Fo}}erfa\sqrt{\mathrm{Fo}}$	$\sqrt{s}\,/(s-a^2)$
$\frac{1}{\sqrt{\pi\mathrm{Fo}}}-\frac{2a}{\sqrt{\pi}}\exp(-a^2\mathrm{Fo})x\int_0^{a\sqrt{\mathrm{Fo}}}\exp(x^2)dx$	$\sqrt{s}\,/(s+a^2)$
$\frac{1}{a}\exp(-a^2\mathrm{Fo})erf\ a\sqrt{\mathrm{Fo}}$	$1/\sqrt{s}(s-a^2)$
$\frac{2}{a\sqrt{\pi}}\exp(-a^2\mathrm{Fo})\int_0^{a\sqrt{\mathrm{Fo}}}\exp(x^2)dx$	$1/\sqrt{s}(s^2+a^2)$

Table A.1 Continued

Function, θ(Fo)	**Transform,** θ(*s*)
$\exp(a^2\mathrm{Fo})\operatorname{erfc} a\sqrt{\mathrm{Fo}}$	$1/\sqrt{s}\left(\sqrt{s}+a\right)$
$\frac{1}{\sqrt{(b-a)}}\exp(-a\mathrm{Fo})\operatorname{erf}[(b-a)\mathrm{Fo}]^{1/2}$	$1/(s+a)\sqrt{s+b}$
$\frac{a}{2\sqrt{(\pi\mathrm{Fo}^3)}}\exp(-a^2/4\mathrm{Fo}), a>0$	$\exp(-a\sqrt{\ }s)$
$1-\operatorname{erf}\left(a/2\sqrt{\mathrm{Fo}}\right)=\operatorname{erfc}\left(a/2\sqrt{\mathrm{Fo}}\right)$ $=\int_{a/2\sqrt{\mathrm{Fo}}}^{\infty} e^{-x2}dx, a>0$	$\frac{1}{\sqrt{s}}\exp\left(-a\sqrt{s}\right)$
$\frac{1}{\sqrt{(\pi\mathrm{Fo})}}\exp\left(-\frac{a^2}{4\mathrm{Fo}}\right) a\geq 0 \quad a\geq 0$	$\frac{1}{\sqrt{\ }s}\exp(-a\sqrt{\ }s)$
$\frac{1}{(n-1)!}\int_0^{\mathrm{Fo}}\frac{1}{\sqrt{(\pi x)}}(\mathrm{Fo}-x)^{n-1}\exp(-a/x)dx, a\geq 0$	$\frac{1}{s^{(n+1/2)}}\exp\left(-2\sqrt{(as)}\right)$
$J_0(a\mathrm{Fo})$	$\frac{1}{\sqrt{(s^2+a^2)}}$
$I_0(a\mathrm{Fo})$	$\frac{1}{\sqrt{(s^2-a^2)}}$
$\exp[-\frac{1}{2}(a+b)\mathrm{Fo}]Io\left[\frac{1}{2}(a-b)\mathrm{Fo}\right]$	$\frac{1}{\sqrt{[(s+a)(s+b)]}}$
$J_0\left(2\sqrt{a\mathrm{Fo}}\right)$	$\frac{1}{s}e^{-a/s}$
$\frac{1}{\sqrt{(\pi\mathrm{Fo})}}\cos 2\sqrt{a\mathrm{Fo}}$	$\frac{1}{\sqrt{s}}e^{-a/s}$
$\frac{1}{\sqrt{(\pi\mathrm{Fo})}}\sin 2\sqrt{a\mathrm{Fo}}$	$\frac{1}{s^{3/2}}e^{-a/s}$
$\frac{1}{a\sqrt{\pi}}\exp(-\mathrm{Fo}^2/4a),\ a>0$	$\exp(a^2s^2)\operatorname{erfc} as$
$\operatorname{erf}\frac{\mathrm{Fo}}{2a},\ a>0$	$\frac{1}{s}\exp(a^2s^2)\operatorname{erfc} as$
$\sqrt{a}/\pi\sqrt{\mathrm{Fo}}(\mathrm{Fo}+a),\ a>0$	$e^{as}erf\sqrt{as}$
$\frac{1}{2Fo}\exp\left(-\frac{a^2}{4\mathrm{Fo}}\right)$	$Ko(a\sqrt{s})$

* The error function *erfx* and the integration of *erfx* are defined as

$$\operatorname{erf} x=(2/\sqrt{\pi})\int_0^x\exp(-x^2)dx \qquad \operatorname{erfc} x=(2/\sqrt{\pi})\int_x^{\infty}\exp(-x^2)dx$$

$$i\operatorname{erfc} x=\int_x^{\infty}\operatorname{erfc}\varepsilon d\varepsilon=\frac{1}{\sqrt{\pi}}\exp(-x^2)-x\operatorname{erfc} x \quad i^n\operatorname{erfc} x=\frac{1}{2n}[i^{n-2}erfcx-2xi^{n-1}erfcx]$$