CHAPTER 4

The Distinctive Features of Sound Diffraction by Some Complex Bodies

“You know my methods. Apply them!”
Arthur Conan Doyle, “The Hound of the Baskervilles”

4.1 SOUND WAVES DIFFRACTION BY PARABOLIC REFLECTOR

The parabolic reflector has unique wave properties: reflector can transform a plane incident wave into a spherical wave, which concentrates in its focus. And vice versa—it transforms a spherical wave (if there is a source of spherical wave in its focus) into a plane wave. The parabolic reflector is widely used in the modern physics and engineering due to these properties. For example, the parabolic reflector is used in the receiving antennas of satellite television in radio engineering. There are a lot of location stations of different application in the military and civil spheres. The receiving antennas with parabolic reflectors are used in such stations. The parabolic reflectors are also used in medical acoustics. They are used, in particular, in the powerful sound radiators, which are applied to break the stones by noninvasive method (so called lithotripsy method). In hydro-acoustics the parabolic reflectors are often used as a part of the antennas of hydro-acoustic stations, used for navigation and submarines detection.

We are concerned with the reflectors used in acoustics. There are a lot of quantitative data in literature (see, for example, [1–6]) for the cases, when the reflector material is ideal (absolutely non-transparent). Reflector diameter $D$ exceeds by far the wave length $\lambda$. There is not enough data in literature about the reflectors made of real (partially transparent) materials (see, for example, work [7]). This is particularly so with the reflectors, the dimensions of which are comparable with the wave length.

The purpose of this paragraph is to find the quantitative relations between the acoustical reflector parameters in the form of paraboloid of rotation and the degree of its material transparency. The range of the wave dimensions of reflector is rather wide.

Figure 4.1 shows reflector cross section with focus in the point $O$ and all the geometrical parameters are indicated. Plane wave $\Phi_0$ is propagated along the axis $z$. We assume that the reflector thickness $h$ is rather small (the conditions $h \ll D$ and $h \ll \lambda$ are fulfilled). The reflector surface is characterized by local impedance

$$Z = -i\omega M f(\theta),$$

(4.1)
where \( M = \rho_1 h \)—area density, \( \rho_1 \)—the density of reflector material, \( f(\theta) \)—the function of value \( M \) distribution on the reflector surface. Condition (4.1) characterizes the properties of a real reflector of small wave thickness, when the wave impedance of its material far exceeds the wave impedance of surrounding medium [8–10]. We confine ourselves to the consideration of so called shallow\(^*\) reflector, when \( \alpha < \pi/2 \). The approximation of parabolic reflector by spherical reflector is possible then. And it is possible to use the spherical coordinate system with point \( O' \) as center (Fig. 4.1).

We will solve the problem by partial domains method. We partition the entire field existence domain into two domains. Domain I \( (0 \leq r \leq r_0) \), and domain II \( (r \geq r_0) \). We use the plane wave expansion by spherical wave functions \([8,11]\). We represent the velocity potentials as follows:

\[
\Phi_I = \sum_{n=0}^{\infty} A_n j_n(kr)P_n(\cos \theta), \quad (4.2)
\]

\[
\Phi_{II} = \sum_{n=0}^{\infty} i^n (2n + 1) j_n(kr)P_n(\cos \theta) + \sum_{n=0}^{\infty} B_n h_n^{(1)}(kr)P_n(\cos \theta). \quad (4.3)
\]

It follows from condition (4.1) that each elementary reflector sector moves as a single unit under the influence of wave \( \Phi_0 \). Consequently, the fields matching conditions on the surface \( r = r_0 \) will be as follows

\[
\psi_{II} = \psi_I, \quad 0 \leq \theta \leq \pi, \quad (4.4)
\]

\[
p_{II} = \begin{cases} p_I - Z \psi_I, & 0 \leq \theta \leq \theta_0, \\ p_I, & \theta_0 \leq \theta \leq \pi. \end{cases} \quad (4.5)
\]

Here \( \psi = -\partial \Phi/\partial r, p = -i\omega \rho \Phi, \rho \)—surrounding medium density. Condition (4.4) allows us to relate the coefficients \( A_n \) and \( B_n \):

\(^*\)If \( \alpha > \pi/2 \), the reflector is called deep.
The algebraization of functional Eq. (4.5) is performed using the properties of completeness and orthogonality of Legendre polynomial $P_n(\cos \theta)$, taking into account (4.6). We obtain the infinite system of algebraic equations of the second kind:

$$A_n + (kr_0)^2(2n + 1)h_n^{(1)}(kr_0)\frac{Z}{2\rho c} \sum_{m=0}^{\infty} A_m j_m'(kr_0)N_{nm} = (2n + 1)i^n,$$  \hspace{1cm} (4.7)

where $N_{nm} = \int_{\cos \theta_0}^{1} f(\theta)P_n(\cos \theta)P_m(\cos \theta)d(\cos \theta)$, $c$—sound velocity in the medium.

System (4.7) is used as initial to obtain the quantitative characteristics of the field, scattered by reflector.

Now we turn to the analysis of the parabolic reflector acoustic properties. All the numerical results correspond to the following parameters of the reflector and surrounding medium: $D/F = 2.8$, $r_0 = 2.14F$, $\alpha = 70^\circ$, $f(\theta) = \text{const}$, $\rho = 10^3$ kg/m$^3$, $c = 1.5 \times 10^3$ m/s. The indicated value $r_0$ ensures (in the range of the considered frequencies) the absence of phase errors, which are more than $\lambda/12$ in the reflector opening. Such error is acceptable in practice [3] and it justifies the approximation of parabolic reflector by spherical reflector. The number of unknowns in system (4.7) is 36, when we solve this problem numerically.

Let us determine the influence of reflector transparency on its pressure gain coefficient $K_p$. Figure 4.2 shows the frequency dependencies of $K_p$ for different values of $M$ kg/m$^2$. Curve 4 [1] is given here for comparison, for an absolutely rigid (non-transparent) reflector. As we can see, there is a decrease of reflector gain coefficient with an increase of its transparency. For large $M$ (curve 3), when the reflector transparency is close to

![FIG. 4.2: Frequency dependency of reflector gain coefficient $D/\lambda = 2.6$; curves 1, 2, 3 correspond to $M = 106, 177, 1770$ kg/m$^2$; 4—absolutely rigid (non-transparent) reflector [1]; 5—$s = 2.1 \times 10^8$ kg/(ms$^2$)](image)
zero, the coefficient $K_p$ approaches the value of absolutely rigid (non-transparent) reflector (curve 4).

Now we turn to the analysis of the field in the area of reflector focus. Figure 4.3 shows the dependencies of normalized sound pressure (with respect to the pressure in focus) along the axis $z$. As we can see, for the large $M$, sound pressure distribution along the axis $z$ is little different from the pressure calculated in work [1]. The situation is different when the reflector transparency becomes noticeable. The amplitude of sound pressure oscillation along the axis $z$ increases with an increase of reflector transparency.

The influence of reflector transparency on the field in its focal region was considered above. The influence of reflector transparency on its directional properties is also of interest. We restate the initial problem to obtain the quantitative results. We assume that $Q$ of point sources are in the area of reflector focus along the axis $z$. The representation of fields (4.2) and (4.3) in domains $I$ and $II$ should be changed. We exclude the series determining the field of plane wave in expression (4.3). Expression (4.2) should be supplemented with the series, which describe the fields of point sources [11]. Taking into account this changes, the fields in domains $I$ and $II$ can be represented as follows:

$$
\Phi_1 = \sum_{n=0}^{\infty} A_n j_n(kr) P_n(\cos \theta)
+ ik \sum_{q=1}^{Q} \xi_q e^{i\psi_q} \sum_{n=0}^{\infty} (2n+1) j_n(kr^{(q)}) h_{n}^{(1)}(kr) P_n(\cos \theta), \quad (4.8)
$$

$$
\Phi_2 = \sum_{n=0}^{\infty} B_n h_{n}^{(1)} P_n(\cos \theta). \quad (4.9)
$$

Here $r^{(q)}$—is radial coordinate of the $q$th point sound source (the sources are on the axis $z$, angular coordinate for all the sources $\theta^{(q)} \equiv 0$); $\xi_q$ and $\psi_q$—amplitude and excitation phase of the $q$th point source.

FIG. 4.3: Sound pressure distribution along the axis $z$
At first we consider the situation when there is only one point source in focus. Figure 4.4 shows the curves, giving the general idea of reflector transparency influence on its directivity pattern $R(\theta)$. The analysis of calculated data allows us to find some peculiarities of the value of $R(\theta)$ behavior when the frequency and reflector transparency change. We partition all the angle range $\pi - \theta$ into domains, to simplify the analysis. We obtain the following domains: main lobe domain, first side lobe domain, side radiation domain and geometrical shadow domain.

Let us consider the main lobe domain. The calculations show that starting from $D/\lambda \geq 2.5$, the width $\theta_{0.7}$ of the main lobe depends almost not at all on the reflector transparency (the value of $\theta_{0.7}$ is determined at level 0.7 by pressure). When $D/\lambda < 2.5$, an increase of reflector transparency leads to the main lobe broadening. For example, when $D/\lambda = 1.75$ the ratio of values $\theta_{0.7}$, calculated for $M = 106$ and $M = 1770$ kg/m$^2$, is 1.5.

The level of the first side lobe $\sigma_1$ increases with a decrease of $M$ in the domain of the first side lobe, as it is seen from Fig. 4.5. Frequency dependency $\sigma_1$ has an oscillating character (it can be explained by the presence of interference effect between the

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**FIG. 4.4:** Directivity pattern of reflector with one point source, which is situated in focus, $D/\lambda = 2.6$: curves 1, 2, 3—calculation for $M = 106, 177, 1770$ kg/m$^2$; 4—experiment for $M = 177$ kg/m$^2$.

**FIG. 4.5:** Frequency dependency of the first side lobe level of directivity pattern; curves 1, 2, 3 correspond to $M = 106, 177, 1770$ kg/m$^2$; 4—$s = 2.1 \times 10^8$ kg/(ms$^2$).
incident and reflected waves). The oscillation amplitude decreases with a decrease of $D/\lambda$, because of the relative reduction of incident wave contribution. The middle level $\sigma_1$ approaches the value $\sim 0.105$, which coincides with the data in work [1].

The change of $R(\theta)$ is relatively small in the side radiation domain (angles $\pi-\theta$ about $90^\circ$). That is why we confine ourselves to its value determination when $\pi - \theta = 90^\circ$, which is denoted as $\sigma_2$. Figure 4.6 shows that value $\sigma_2$ reduces with an increase of frequency and a decrease of reflector transparency.

The change of $R(\theta)$ is of complex character in the domain of reflector geometrical shadow. It is caused by the interference effect between the wave passing through the reflector and the wave diffracted by its edge. The field estimation is done by mean value of $R(\theta)$ in the domain of shadow:

$$\sigma_3 = \frac{1}{1 - \cos \alpha} \int_0^\alpha R(\theta) \sin \theta d\theta.$$

The data in Fig. 4.7 determine the dependency of $\sigma_3$ on reflector frequency and transparency. Note that curve 3 characterizes the field level, caused by diffraction on the reflector edge.

**FIG. 4.6:** Frequency dependency of the level of sidelight radiation: curves 1, 2, 3 correspond to $M = 106, 177, 1770 \text{ kg/m}^2$; 4—$s = 2.1 \times 10^8 \text{ kg/(ms}^2\text{)}$

**FIG. 4.7:** Frequency dependency of radiation level in the domain of shadow: curves 1, 2, 3 correspond to $M = 106, 177, 1770 \text{ kg/m}^2$; 4—$s = 2.1 \times 10^8 \text{ kg/(ms}^2\text{)}$; circles—experiment for $M = 177 \text{ kg/m}^2$
The experimental tests were performed using the model of reflector in order to confirm the applicability of obtained results for the evaluation of acoustic properties of real reflectors. The obtained results were compared with the calculation data. The dummy of reflector was made of metal with density $7.1 \times 10^3 \text{ kg/m}^3$ and it has the following parameters: $D/F = 2.8$, $\alpha = 70^\circ$, $f(\theta) = \text{const}$, $M = 177 \text{ kg/m}^2$. Piezoceramic sphere is the source of sound waves. It is situated in reflector focus. The sphere diameter does not exceed $0.07\lambda$ on the upper boundary of the considered frequency range. The reflector with the source of sound waves was in water. Figure 4.4 (curve 4) shows experimental directivity pattern, and Fig. 4.7—experimental values of $\sigma_3$ (indicated by circles). Comparing the curves 2 and 4 in Fig. 4.4, and the curve 2 with the curve marked with circles, in Fig. 4.7, we can see that the calculation data coincide with the experimental data.

We consider now the simplest grating, which is of practical interest. It is a linear grating, made of two point nondirective sound sources ($Q = 2$). We assume that the amplitudes and excitation phases of these sources are equal $\xi_1 = 1$, $\xi_2 = -1$, $\psi_1 = 0$, $\psi_2 = \beta kd$, respectively, where $d = (r^{(1)} - r^{(2)})$—the distance between the sources (assume that $r^{(1)} > r^{(2)}$), and $\beta$—real parameter, which varies within the limits from 0 to 1. When $\beta = 0$ we have dipole [12], which is characterized by directivity pattern in the form of “eight” when $d < \lambda/2$ (radiation maxima are directed along the axis $z$). If $\beta = 1$, the directivity pattern (when $d < \lambda/4$) takes the form of cardioids. The maximum of cardioids is in the positive $z$-axis direction and zero is in the opposite direction. The directivity pattern of two point sources has some intermediate form between “eight” and cardioids within the range $0 < \beta < 1$. Concentration factor is a maximum when $\beta = 0.33$. The directivity pattern can be considered as “optimal.”

The calculations of the far field of reflector were performed for the indicated above values $\beta = 0, 0.33, 1$. Figures 4.8 and 4.9 show the results of this calculation, when the sources are on the axis $z$, in the points with coordinates $r^{(1)} = 1.157F$ and $r^{(2)} = 1.144F$.

We turn now to Fig. 4.8, which shows the directivity patterns of reflector with two sources in focal area for different parameter $\beta$ values. Comparative analysis of the curves

![FIG. 4.8: Directivity patterns of reflector when $M = 177 \text{ kg/m}^2$; curves 1, 2, 3 correspond to $\beta = 0, 1, 0.33$ when $Q = 2$; 4—one source in focus; 5—$Q = 6$; circles—experiment when $Q = 2$, $\beta = 1$](image-url)
FIG. 4.9: Frequency dependencies of concentration coefficient: curves 1, 2, 3 for $Q = 2$, $\beta = 0, 1, 0.33; 4$—one sound source in focus

1, 2, 3, and 4 shows that when we use only two sources, which are correctly spaced and phased, it allows us to suppress sound radiation almost in the entire area of angles $\theta$ behind the main lobe of directivity pattern. We can also observe the other singularities of the directivity pattern of reflector for different relations of the phases of sources. When the sources are in anti-phase and they form the dipole ($\beta = 0$), the reflector directivity pattern has the low level in the area of angles $\theta$, close to $90^\circ$. When $\beta = 1$ we can observe the low radiation level just behind the main lobe of directivity pattern ($25^\circ \leq \theta \leq 45^\circ$). When the value is “optimal” $\beta = 0.33$ the area of angles with the low radiation level expands considerably and is within the limits $40^\circ \leq \theta \leq 90^\circ$.

So, we can draw a conclusion: when the two simple sources are on the axis of reflector in focal area and when their excitation is selected correctly, we can steer the directivity pattern of reflector. The general level of radiation can be reduced in the area of angles $\theta$ behind the main lobe, and the radiation can be suppressed in a particular direction.

When analyzing the concentration coefficient $K$ dependency on the reflector wave dimensions (Fig. 4.9), we can conclude that the nature of excitation of the sources also influences the concentration coefficient dependency on the reflector wave dimensions. The reflector with one sound source has less concentration coefficient than the reflector with two sound sources, especially when the parameter values are $\beta = 1$ and $\beta = 0.33$. Consequently, we can control the value of the concentration coefficient of reflector, when selecting the excitation character of the sources.

The simplest cases of the sources excitation are shown above. It is shown that we can influence the directional properties of reflector even if the number of sources is minimum ($Q = 2$). As the numerical experiments show, if we increase the number of sources and select the phases, we will be able to influence the directivity pattern of reflector more efficiently.

Figure 4.8 shows the directivity pattern (curve 5) for the case when there are six sources ($Q = 6$), situated on the axis of reflector in the points with coordinates $r^{(1)} =$

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1. Calculated and experimental data comparison in Fig. 4.8 validates this conclusion (the experiment was carried out for $\delta/\lambda = 0.04$ and $\delta/\lambda = 0.0154$, where $\delta$—sources diameter).
0.89 \, F, \, r^{(2)} = 0.99 \, F, \, r^{(3)} = 1.09 \, F, \, r^{(4)} = 1.19 \, F, \, r^{(5)} = 1.29 \, F, \, r^{(6)} = 1.39 \, F. \) These points are characterized by the following amplitudes and excitation phases \( \xi_1 = 1, \, \xi_2 = 1.17, \, \xi_3 = 1.31, \, \xi_4 = 1.35, \, \xi_5 = 1.04, \, \xi_6 = 0.83, \, \psi_1 = 0^\circ, \, \psi_2 = 61^\circ, \, \psi_3 = 126^\circ, \, \psi_4 = 204^\circ, \, \psi_5 = 255^\circ, \, \psi_6 = 325^\circ. \) The comparison of the curves shows that, when there are a lot of sources, the area of angles \( \theta \), with the low level of radiation, can be expanded. It is to be noted that, when the surface density \( M \) decreases the level of the values of directivity pattern in the area of angles \( 30^\circ \leq \theta \leq 180^\circ \) increases. It is a natural phenomenon because the transparency of reflector increases.

Finally, it is interesting to consider the case, when the impedance is of elastic character \( Z = isf(\theta)/\omega \). This is common for the thin layers of gas in liquid [8,9]. Here \( s \) \( [\text{kg/(ms}^2\text{)]—surface elasticity}. \) When investigating this case, conditions (4.4) and (4.5) on the surface \( r = r_0 \) should be replaced with the equalities

\[
p_I = p_{II}, \quad 0 \leq \theta \leq \pi, \quad v_{II} = \begin{cases} v_I - \frac{p_I}{s}, & 0 \leq \theta \leq \theta_0, \\ v_I, & \theta_0 \leq \theta \leq \pi. \end{cases}
\]

The calculations of the values \( K_p \) and \( R(\theta) \) are performed when \( s = 2.1 \times 10^8 \) \( \text{kg/(ms}^2\text{)} \) and \( f(\theta) = \text{const} \) (this value of \( s \) provides the same sound transmission coefficient as in the case when \( M = 106 \, \text{kg/m}^2 \)). Curves 1 and 5 in Fig. 4.2 correspond to the different properties of reflector surface. As we can see, the maximum oscillations of curve 1 correspond to the minimums of curve 5. The behavior of value \( \sigma_1 \), and the behavior of \( \sigma_2 \) to some extent, has the similar characteristic (see, Figs. 4.5 and 4.6). Comparing the data in Figs. 4.5–4.7, we can see that if the impedance has elastic character the general level of value \( \sigma_1 \) is a little higher. The values \( \sigma_3 \) are lower than in the case, when the impedance has the character of mass.

**4.2 SOUND DIFFRACTION BY THE WEDGE OF FINITE DIMENSIONS**

The problems of waves scattering by bodies of varying configuration are of significant theoretical and practical importance. The solution results may be the basic for the construction of location systems of the objects detection and their classification. Waves scattering by wedge bodies is of interest, because the form of such bodies is similar to the forms of some parts of the aviation or space objects. The problem of the waves scattering by wedge bodies of finite sizes is considered in this paragraph.

Let us consider the plane problem of scattering by wedge object (Fig. 4.10). We assume that the object has infinite extension along the axis, perpendicular to the figure plane. The object is a finite wedge with an angle \( 2\theta_1 \). The sides of the wedge are closed by arc \( AB \) with radius \( a \). The surfaces of the object are acoustically rigid. The wedge object is in an ideal medium with density \( \rho \) and sound velocity \( c \). Let us introduce the polar coordinate system \( r \theta \) with the center in the angle point of the wedge.

We partition all the space of sound field existence into two domains: \( I \)—outside the circle of radius \( a \), i.e., \( r \geq a, \, 0 \leq \theta \leq 2\pi; \) \( II \)—sector of a circle of radius \( a \), namely: \( 0 \leq r \leq a, \, \theta_1 \leq \theta \leq 2\pi - \theta_1 \).
Let a plane harmonic wave with a unit pressure amplitude be incident on the wedge object in domain $I$

$$p_0 = \exp(-ikr \cos(\theta - \theta_0)). \quad (4.10)$$

The sign “minus” in the exponential index points out that the direction of the plane wave propagation is opposite to the direction of growth of radial coordinate $r$. Angles $\theta_0$ and $\theta$ determine the angular direction of the wave incidence and the direction of the observation point.

The field of incident wave (4.10) can be written as follows [10]

$$p_0 = \sum_{n=0}^{\infty} (-i)^n \varepsilon_n J_n(kr) \cos(n(\theta - \theta_0)), \quad (4.11)$$

where $\varepsilon_0 = 1$, $\varepsilon_n = 2$, if $n > 0$, $J_n(kr)$—Bessel function of the first kind.

Scattered field appears when the plane wave interacts with the wedge object. We represent it in the form of superposition of cylindrical running waves $H_n^{(1)}(kr) \cos(n\theta)$, $n = 0, 1, 2, ...$ and $H_n^{(1)}(kr) \sin(n\theta)$, $n = 1, 2, 3, ...$. The angle functions $\cos(n\theta)$ and $\sin(n\theta)$ are necessary, because the angle of incidence $\theta_0$ is arbitrary. Pressure field in domain $I$ is as follows:

$$p_I = \sum_{n=0}^{\infty} (-i)^n \varepsilon_n J_n(kr) \cos(n(\theta - \theta_0))$$

$$+ \sum_{n=0}^{\infty} A_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} \cos(n\theta) + \sum_{n=1}^{\infty} B_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} \sin(n\theta). \quad (4.12)$$

The field in domain $II$ we represent as the superposition of standing waves:

$$p_{II} = \sum_{n=0}^{\infty} C_n \frac{J_{\alpha_n}(kr)}{J_{\alpha_n}(ka)} \cos(\alpha_n(\theta - \theta_1)), \quad (4.13)$$
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when the values of $\alpha_n$ are determined from the boundary conditions on the rigid wedge borders $\partial p_{II}/\partial \theta = 0$ when $\theta = \theta_1$ and $\theta = 2\pi - \theta_1$. Consequently,

$$\alpha_n = \frac{n\pi}{2(\pi - \theta_1)}, \quad n = 0, 1, 2, \ldots$$  \hspace{1cm} (4.14)

Let us write matching conditions on the interface of partial domains $I$ and $II$ (Fig. 4.10):

$$p_I = p_{II}, \quad r = a, \quad \theta = [\theta_1, 2\pi - \theta_1], \hspace{1cm} (4.15)$$

$$\frac{\partial p_I}{\partial r} = \begin{cases} \frac{\partial p_{II}}{\partial r}, & r = a, \quad \theta = [\theta_1, 2\pi - \theta_1], \\ 0, & r = a, \quad |\theta| \leq \theta_1. \end{cases}$$  \hspace{1cm} (4.16)

We algebraize functional Eq. (4.15) due to the orthogonality of the system of functions $\cos(\alpha_n (\theta - \theta_1))$, $n = 0, 1, 2, \ldots$ on the interval $\theta = [\theta_1, 2\pi - \theta_1]$. We algebraize functional Eq. (4.16) two times: the first time—using the system of functions $\cos(n\theta)$, $n = 0, 1, 2, \ldots$, the second time—using the system $\sin(n\theta)$, $n = 1, 2, \ldots$. Both systems are orthogonal on the interval $\theta = [0, 2\pi]$. We obtain an infinite system of linear algebraic equations of the second kind in unknown coefficients $A_n, B_n, C_n$. An infinite system of equations was solved by reduction method. To estimate the accuracy of the fulfillment of conditions (4.15) and (4.16). We find the discrepancy as the ratio of difference modulus of the field characteristics (pressure or particle velocity) to the left and to the right from the boundary between the partial domains to the plane wave amplitude (4.10). For example, pressure discrepancy $\delta_p = |p_I - p_{II}| / |p_0|$. For example, Fig. 4.11 shows the pressure discrepancy graphs $\delta_p$ (solid line) and particle velocity discrepancy $\delta_v$ (dashed line) along the circle with radius $a$: $ka = 15$, $\theta_1 = 45^\circ$, $\theta_0 = 90^\circ$. When calculating, 40 coefficients $A_n, B_n$, and $C_n$ are kept. Angular range $\theta = [-\theta_1, \theta_1]$ corresponds to the rigid object surface (Fig. 4.10).

![FIG. 4.11: Pressure discrepancy curves $\delta_p$ (solid curve) particle velocity discrepancy $\delta_v$ (dashed) along the circle of radius $a$ (Fig. 4.10): $ka = 15$, $\theta_1 = 45^\circ$, $\theta_0 = 90^\circ$]
The discrepancy increases $\delta$ in the vicinity of edges of wedge object $\theta = \pm \theta_1$, $r = a$. It is a logical result, because there is a singularity of particle velocity. When $ka = 15$, $\theta_1 = 45^\circ$ the rigid boundaries of the object have the considerable wave dimensions. Consequently, pressure amplitude on this surface is approximately equal to the double amplitude of incident wave. If the discrepancy is $\delta < 0.1$, the fulfillment of boundary conditions on the interface between partial domains is satisfactory.

It should be noted that the curves in Fig. 4.11 are of “irregular” character and it means that the calculations are reliable. The limitation on amount of equations in the infinite system introduces an error when fulfilling the conditions for high harmonics.

When slowing the problem of sound scattering by an obstacle, the far sound field is usually explored. Scattered field here is a spherical expanding wave. It corresponds to the cylindrical expanding wave in the plane problem under consideration. According to (4.12), complex amplitude of scattered wave is as following

$$p_s(r, \theta, \theta_0) = A_0 \frac{H_0^{(1)}(kr)}{H_0^{(1)}(ka)} + \sum_{n=1}^{\infty} \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} [A_n \cos (n\theta) + B_n \sin (n\theta)].$$  \hspace{1cm} (4.17) 

Taking into account the Hankel functions asymptotic behavior when $kr \to \infty$ formula (4.17) can be written as follows:

$$p_s(r, \theta, \theta_0) = \sqrt{\frac{2}{\pi kr}} \exp \left( ikr - i \frac{\pi}{4} \right) \times \left[ \frac{A_0}{H_0^{(1)}(ka)} + \sum_{n=1}^{\infty} \frac{\exp \left( -in\pi/2 \right)}{H_n^{(1)}(ka)} (A_n \cos (n\theta) + B_n \sin (n\theta)) \right].$$  \hspace{1cm} (4.18) 

The intensity in the far field of scattered wave is determined by formula $I_{rs} = |p_s|^2/(2\rho c)$. Then, according to (4.18) we have

$$I_{rs}(r, \theta, \theta_0) = \frac{2I_0}{\pi kr} L(\theta, \theta_0) L^*(\theta, \theta_0),$$  \hspace{1cm} (4.19) 

where $I_0 = 1/(2\rho c)$—intensity of the plane incident wave of unit amplitude (4.10), asterisk—complex conjugation

$$L(\theta, \theta_0) = \frac{A_0}{H_0^{(1)}(ka)} + \sum_{n=1}^{\infty} \frac{\exp \left( -in\pi/2 \right)}{H_n^{(1)}(ka)} (A_n \cos (n\theta) + B_n \sin (n\theta)).$$  \hspace{1cm} (4.20) 

The power of scattered wave, when integrating (4.20) over the circle of radius $r$ is determined by relation

$$P_s(\theta_0) = \int_0^{2\pi} I_{rs}(r, \theta, \theta_0) r d\theta = \frac{2I_0}{\pi kr} \int_0^{2\pi} L(\theta, \theta_0) L^*(\theta, \theta_0) d\theta.$$  \hspace{1cm} (4.21)
The full cross-section of scattering is determined by the ratio of scattered wave power to the intensity of incident plane wave

\[
\sigma_s (\theta_0) = \frac{P_s (\theta_0)}{I_0} = \frac{2}{\pi k} \int_0^{2\pi} L (\theta, \theta_0) L^* (\theta, \theta_0) d\theta.
\]  

(4.22)

Substituting (4.20) into (4.22), we obtain the following formula for the non-dimensional value of the full cross-section of dissipation:

\[
\frac{\sigma_s (\theta_0)}{a} = \frac{2}{ka} \left[ 2 \left| \frac{A_0}{H_0^{(1)} (ka)} \right|^2 + \sum_{n=1}^{\infty} \frac{|A_n|^2 + |B_n|^2}{|H_n^{(1)} (ka)|^2} \right].
\]  

(4.23)

It is of interest to find the positional cross-section of dissipation \( \sigma (\theta, \theta_0) \). This value is equal to the ratio of power of dummy of non-directional cylindrical radiator with intensity equal to the intensity of scattered wave in the given direction \( \theta \), to the intensity of incident plane wave. According to the definition \( \sigma (\theta, \theta_0) \), taking into account Eq. (4.19), we obtain the relation for the non-dimensional value of positional cross-section:

\[
\frac{\sigma (\theta, \theta_0)}{a} = \frac{1}{a} \frac{2 \pi r I_s (r, \theta, \theta_0)}{I_0} = \frac{4}{ka} L (\theta, \theta_0) L^* (\theta, \theta_0).
\]  

(4.24)

If we take angle \( \theta = \theta_0 \) in Eq. (4.24) we obtain the cross-section of back-scattering \( \sigma_L (\theta_0) = \sigma (\theta = \theta_0, \theta_0) \).

We turn now to the analysis of numerical results. Figure 4.12(a) shows the values of the cross-section of dissipation \( \sigma_s, \sigma, \sigma_L \) as the functions of the angle of incidence of the plane wave \( \theta_0 \) on the wedge object; \( ka = 15 \) (or \( a \approx 2.4\lambda \), \( \theta_1 = 45^\circ \). The object has the considerable wave dimensions at such parameters. The arc \( AB \) and the wedge boundaries \( OA, OB \) are approximately equal in wave dimensions. As we can see, the full cross-section of dissipation \( \sigma_s \) (curve 1) almost does not depend on the angle of wave incidence \( \theta_0 \). It is caused by the given dimensions of the object. The positional cross-section of scattering \( \sigma (\theta = 0, \theta_0) \) (curve 2) characterizes the energy of scattered wave in the direction \( \theta = 0 \) as the function of the angle of incidence of plane wave \( \theta_0 \). There is a “shade” lobe (the angle of incidence \( \theta_0 = 180^\circ \)). The cross-section of back scattering \( \sigma_L \) (curve 3) is of specific interest.

When the angles of incidence are \( \theta_0 \approx 0, ..., 70^\circ \), curve 3 is a flat curve. It corresponds to the wave reflection from surface \( AB \) (Fig. 4.10) with large wave dimension (the arc length \( AB \) is about 3.75\( \lambda \)). The range of angles \( \theta_0 \approx 70, ..., 180^\circ \) may be defined as the reflection of the wave from the flat area \( OA \) of the wedge object surface. We can see here the considerable “irregularity” of curve 3. When the angle of incidence is \( \theta_0 \approx 135^\circ \) we have a so called “fleck.” It means that there is a sharp increase of the reflected wave amplitude in the opposite direction. It is caused by the normal incidence of the plane wave on the boundary \( OA \) of large wave dimension.

We reduce the wave dimension of object, \( ka = 0.7; \theta_1 = 45^\circ \), Fig. 4.12(b). As we can see the values of the cross-section of scattering \( \sigma_s, \sigma, \sigma_L \) decrease and three curves become flat.
Figure 4.12 shows the graphs for the object with large value $ka = 15$, but with small angle $\theta_1 = 3^\circ$. The geometry of object changes. The wave length of arc $AB$ here is approximately ten times smaller than the value of $ka$ and is approximately $0.24\lambda$. As we can see, curve 1 for the full cross-section of scattering slopes steeply down in the vicinity of angles $\theta_0 = 0$ and $\theta_0 = 180^\circ$. Positional cross section of scattering (curve 2) is less than in Fig. 4.12(a). The cross-section value of back scattering (curve 3) has pronounced peak in the vicinity of the angle of incidence $\theta_0 = 90^\circ$.

Thus, the cross-section of scattering depends on the object geometry, wave dimensions of the surfaces, which form it, and the direction of wave incidence.

4.3 SOUND DIFFRACTION BY CORNER

The corner is widely used in physics and engineering. If the symmetry axis of corner coincides with the direction of the flat incident wave, it reflects this wave effectively in the opposite direction. This property is widely used in architectural acoustics for the sound
field distribution correction in theater auditoriums and concert halls. The corner reflectors are widely used in radiotechnics and optics for precise measurements for topographical mapping, in marine navigation and so on. American astronauts mounted the mirror corner reflectors on the moon in 1969. If we radiate these reflectors by laser from the earth surface and receive the reflected ray we can find the distance from the earth to the moon. The corner reflectors are also used in corner antennas [3,4,7]. The source of cylindrical waves (or the series of sources) is placed inside the corner opening. The corner reflector transforms the cylindrical waves of the sources into the plane waves, which are radiated by corner antenna.

The indicated above corner properties are considered in this paragraph.

4.3.1 Problem Solution Construction

Let us consider the plane problem of the wave incidence on the corner of finite dimensions with the opening angle $2\theta_1$, Fig. 4.13. We assume that the surfaces of corner are acoustically rigid. The corner is immersed in the ideal medium with density $\rho$ and sound velocity $c$. To construct the problem solution, we introduce the polar coordinate system $rO\theta$. According to the partial domains method all the space of sound field existence can be partitioned into three domains: $I$—exterior of the circle of radius $a$, i.e., $r \geq a$, $0 \leq \theta \leq 2\pi$; $II$—sector $0 \leq r \leq a$, $0 \leq \theta \leq 2\theta_1$; $III$—sector $0 \leq r \leq a$, $2\theta_1 \leq \theta \leq 2\pi$.

Let us consider two variants of the wave incidence on the corner: the first—the incidence of plane wave $p_0$ at an angle $\theta_0$, the second—the incidence of cylindrical wave, created by the source $S$. The source is situates at an angle $\theta_1$ at a distance $R$ from the origin of coordinates $O$ (Fig. 4.13).

![FIG. 4.13: Problem geometry](image)
For the case of plane wave \( p_0 \) incidence (see Eq. (4.11) in the previous paragraph), we write the field in domain I as follows:

\[
p_I = p_0 + \sum_{n=0}^{\infty} A_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} \cos(n\theta) + \sum_{n=1}^{\infty} B_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(ka)} \sin(n\theta),
\]

where \( p_0 \) is the incident wave, \( A_n \) and \( B_n \) are coefficients determined by the boundary conditions on the rigid boundaries of the wedge. Consequently

\[
A_n = \frac{\pi i}{4\theta_1} \sum_{n=0}^{\infty} \varepsilon_n J_{\alpha_n}(kr) J_{\alpha_n}^{(1)}(kR) \cos(\alpha_n \theta_0) \cos(\alpha_n \theta),
\]

\[
B_n = \frac{\pi i}{4\theta_1} \sum_{n=1}^{\infty} \varepsilon_n J_{\alpha_n}(kr) J_{\alpha_n}^{(1)}(kR) \cos(\alpha_n \theta_0) \cos(\alpha_n \theta),
\]

where \( \alpha_n \) and \( \beta_n \) are determined from the boundary conditions on the rigid boundaries of the wedge. Consequently

\[
\alpha_n = \frac{\pi n}{2\theta_1}, \quad \beta_n = \frac{\pi n}{2(\pi - \theta_1)}, \quad n = 0, 1, 2, ...
\]

We represent the field in domains II and III as the superposition of standing waves:

\[
p_{II} = \sum_{n=0}^{\infty} C_n J_{\alpha_n}(kr) J_{\alpha_n}^{(1)}(kR) \cos(\alpha_n \theta)
\]

\[
p_{III} = \sum_{n=0}^{\infty} D_n J_{\alpha_n}(kr) J_{\alpha_n}^{(1)}(kR) \cos(\beta_n (\theta - 2\theta_1)),
\]

where \( \alpha_n \) and \( \beta_n \) are determined from the boundary conditions on the rigid boundaries of the wedge. Consequently

\[
\alpha_n = \frac{\pi n}{2\theta_1}, \quad \beta_n = \frac{\pi n}{2(\pi - \theta_1)}, \quad n = 0, 1, 2, ...
\]

If we consider the incidence of cylindrical wave (the source of cylindrical wave \( S \) is situated in domain II), we should use the known representation of the source field in wedge domain [17]:

\[
p_0 = \frac{\pi i}{4\theta_1} \left\{ \sum_{n=0}^{\infty} \varepsilon_n J_{\alpha_n}(kr) H_{\alpha_n}^{(1)}(kR) \cos(\alpha_n \theta_0) \cos(\alpha_n \theta), \quad r < R, \right. \]
\[
\left. \sum_{n=0}^{\infty} \varepsilon_n J_{\alpha_n}(kr) H_{\alpha_n}^{(1)}(kR) \cos(\alpha_n \theta_0) \cos(\alpha_n \theta), \quad R < r < a \right\}
\]

In this case, the source field (4.28) should be added to the right part of Eq. (4.26), and value \( p_0 \) should be excluded in Eq. (4.25).

We write the matching conditions on the boundary between the partial domains I and II, III (Fig. 4.13):

\[
\frac{\partial p_I}{\partial r} = \begin{cases} \frac{\partial p_{II}}{\partial r}, & r = a, \quad \theta = [0, 2\theta_1], \\ \frac{\partial p_{III}}{\partial r}, & r = a, \quad \theta = [2\theta_1, 2\pi], \end{cases}
\]

\[
p_I = p_{II}, \quad r = a, \quad \theta = [0, 2\theta_1],
\]

\[
p_I = p_{III}, \quad r = a, \quad \theta = [2\theta_1, 2\pi].
\]

We perform the algebraization of functional Eq. (4.29) two times: the first time—we use the system of functions \( \cos(n\theta) \), \( n = 0, 1, 2, ... \); the second time—we use the system \( \sin(n\theta) \), \( n = 1, 2, ... \); both systems of functions are orthogonal on the interval \( \theta = \)
[0, 2\pi]. We perform the algebraization of functional Eq. (4.30) due to the orthogonality of the system of functions \( \cos(\alpha_n \theta) \), \( n = 0, 1, 2, ... \) on interval \( \theta = [0, 2\theta_1] \). We use the system of functions \( \cos(\beta_n (\theta - 2\theta_1)) \), \( n = 0, 1, 2, ... \) on the interval \( \theta = [2\theta_1, 2\pi] \) for Eq. (4.31). We obtain the infinite system of linear algebraic equations of the second kind in unknown coefficients \( A_n, B_n, C_n, D_n \). The infinite system of equations is solved by reduction method.

4.3.2 The Singularities of Plane Wave Reflection from the Corner

It was considered above that when angle is \( 2\theta_1 = 90^\circ \), the corner has the property to reflect the incident wave in the opposite direction in relatively wide range of incident angle \( \theta_0 \), (Fig. 4.13). Figure 4.14 illustrates it.

The difference in reflectance properties of the corner and the band is well seen here. The band efficiently reflects the waves when the directive beam of the incident wave is perpendicular to the surface of the band. The angles \( \theta_0 \) area of effective reflection narrows rapidly with an increase of frequency. The band behaves like a mirror. The behavior of the corner is quite different in contrast to the band. The corner reflects the waves in the direction of their source in relatively wide range of angles \( \theta_0 \). This property remains almost constant with an increase of frequency.

It is interesting to compare the structure of the field in the vicinity of the band and corner. Figure 4.15 shows that the field in the vicinity of the corner is more concentrated

**FIG. 4.14:** Dependency of the value of back scattering on incident angle of plane wave: 1—corner with \( 2\theta_1 = 90^\circ \), 2—band with \( 2\theta_1 = 180^\circ \); (a)—\( 2a/\lambda = 2.4 \), (b)—\( 2a/\lambda = 12 \)
in the direction of incident wave, unlike the field in the vicinity of the band. This character of the corner field structure remains in the relatively wide range of angles \( \theta_0 \).

### 4.3.3 The Characteristics of Cylindrical Wave Reflection from the Corner. Corner Antenna

We consider the case, when the source of cylindrical waves \( S \) is in the opening of the corner at distance \( R \) from point \( O \), Fig. 4.13. Such system can be considered as corner antenna [3,4,7]. Let us examine its acoustical properties.

We consider the pressure variation in the far field depending on the source position relative to the origin of coordinates \( O \), Fig. 4.16. It follows form Fig. 4.16 that the sound source should be placed near the point \( O \) for higher energetic efficiency of antenna. The directivity pattern almost does not change in the frequency band about octave. The width of directivity pattern \( \theta_{0.7} \) (at the pressure level 0.7) is within the limits \( 60^\circ \leq \theta_{0.7} \leq 70^\circ \), and the level of back radiation is \( \sigma < 0 < 0.19 \).

Let us consider the change of directivity pattern depending on opening angle \( 2\theta_1 \). Figure 4.17 shows the directivity patterns for the different opening angles. As it follows from these data the optimum opening angles are in the region \( 80^\circ \leq 2\theta_1 \leq 100^\circ \).
FIG. 4.17: The dependency of the change of shape of directivity pattern on opening angle $2\theta_1$ for $a/\lambda = 0.5$ and $R/a = 0.5$: 1–8 correspond to $2\theta_1 = 40^\circ$, $60^\circ$, $80^\circ$, $100^\circ$, $120^\circ$, $140^\circ$, $160^\circ$, $180^\circ$

In addition, Fig. 4.18 shows the generalized data. These data was obtained on the basis of analysis of the bulk of numerical calculations. As it follows from these data (for the indicated source position), the optimum angles of corner opening are in the region $80^\circ \leq 2\theta_1 \leq 100^\circ$. The pressure in the far field is maximum, the level of back radiation is minimum, the width of the main lobe of directivity pattern is moderate. We should choose the angle of opening $2\theta_1 \approx 140^\circ$ for high concentration of sound energy. The pressure on the directivity pattern axis decreases at about 25%, and the back level of radiation increases almost in 2 times. Consequently, there is no tangible benefit in energetic efficiency of antenna.
FIG. 4.18: The dependency of acoustic parameters of corner antenna on opening angle $2\theta_1$, $a/\lambda = 0.5$, $R/a = 0.5$: 1—relative pressure in the far field $p/p_{\text{max}}$, 2—the width of the main lobe of directivity pattern $\theta_{0.7}$, 3—the level of back radiation $\sigma$

4.4 SOUND WAVES DIFFRACTION BY WEDGE

4.4.1 The Waves Diffraction by Classical (Sharp) Wedge

The problem of wave diffraction by wedge is a fundamental problem. A lot of physical theories of the interaction between waves and bodies are based on the results of this problem solving. The literature on this problem is quite extensive. The physical phenomena, connected with the influence of the wedge point on the incident waves is under investigation. The waves are of different nature – sound, elastic, electro-magnetic, and light waves. There are two reasons of interest in this problem. The first reason is theoretical (mathematical). Within the context of Euclidean geometry, it is impossible to determine the normal derivative on the point of wedge (its edge) (in acoustics – normal component of particle velocity). It was discussed in the first chapter. It leads to the ambiguity of required solution of diffraction problem. Besides, particle velocity tends to infinity when approaching the edge. Works [13,20] and the first chapter of this monograph give detailed information about the behavior of the velocities field near the edge of the wedge and about the mathematical methods of solving this problem.

The second reason of interest in diffraction of incident wave by wedge is the practical importance of this process. It is know that when the wave is incident on the wedge, the cylindrical wave appears on its edge. This wave interferes with the incident wave, and it also causes the waves appearance in the geometrical shadow zone. But the physical reason of the cylindrical wave appearance on the edge of the wedge was unknown for a long time. Huygens was possibly the first who gave the correct explanation of the reason of cylindrical wave appearance on the edge of the wedge. The idea of his theory is as follows. Assume that there is the sharp wedge with the acoustically rigid edges $\sigma_1$ and $\sigma_2$, the plane wave front $p$ runs along the boundary $\sigma_1$, Fig. 4.19.

The edges are acoustically rigid, and the normal components of particle velocities on their surfaces are equal to zero. Consequently, the acoustic impedance of the edges
THE DISTINCTIVE FEATURES OF SOUND DIFFRACTION BY SOME COMPLEX BODIES

surfaces is equal to infinity. Thus, while the wave front is propagating along the boundary \( \sigma_1 \), the sound pressure on the surface of this face remains constant. As soon as the wave front passes the point \( O \), that is the edge of the wedge, the acoustical impedance decreases sharply. It leads to the stepwise fall of sound pressure and, as a result, to the appearance of cylindrical sound wave [25], which comes from the edge. This wave is often called the edge wave. Particle velocity is coordinate derivative of pressure. Consequently, there is a stepwise increasing of particle velocity in the vicinity of edge. Formally this velocity tends to infinity, when approaching the point \( O \). Mathematically it means that function, which describes particle velocity, in point \( O \) has a peculiarity. The speed of particle velocity increase depends on the wedge sharpness. The sharper it is, the higher is the speed. The speed is maximum in the case, when the wedge becomes the semi-plane. We note that the described process of the wave formation does not break the law of conservation of energy. The energy of the edge wave appears due to the some part of energy of incident wave. An increase of particle velocity when infinitely approaching the edge does not mean that sound energy increases infinitely. The peculiarity of the function of particle velocity in point \( O \) is integratable for all the parameters of the wedge (this question is considered in [20]).

Now we mention briefly a historical background [17,21]. The problem of waves diffraction by wedge with ideal edges was solved for the first time by variable separation method of Poincare in 1892–1897 [22,23]. A little later, A. Sommerfeld [24] solved the problem of waves diffraction by semi-infinite plane screen by the method of so called ramified solutions. The researches paid a lot of attention to the question of the diffraction of waves of different nature by wedge. The works [21,26–29] are of special interest. In the last of these works the problem of sound diffraction by wedge with impedance edges was consid. The variable separation method could not be used.

In view of the above mentioned, let us start the presentation of solution of problem of cylindrical wave diffraction by wedge with ideal properties of edges. Methodically we will construct the solution in the same way, as it was done in [17,28,30].

The geometry of two-dimensional problem of cylindrical wave scattering on the wedge is shown in Fig. 4.20. The wave is created by harmonic point source \( S \) in the form of infinite long line, which is perpendicular to the figure plane and parallel to the edge of the wedge. The center \( O \) of polar coordinate system \((r, \theta)\) is on the wedge point with an exterior angle equal \( \alpha \). On the edges of the wedge then \( \sigma_1 \) and \( \sigma_2 \) have the
coordinates $\theta = 0$, $0 \leq r < \infty$ and $\theta = \alpha$, $0 \leq r < \infty$. The source $S$ is in the point with coordinates $r_i = (r_i, \theta_i)$, and the point of observation $M$ is determined by vector $r = (r, \theta)$.

Let us write the function of the source (Green function) for the free space in two dimensional case [17,30,31]

$$G (r, r_0) = \frac{i}{4} H_0^{(1)} (k |r_0 - r|) . \quad (4.32)$$

To write the function of the source with wedge (Fig. 4.20) it is necessary to solve the corresponding diffraction problem (the details of its solution are given in [17,26,31]). We omit the problem solution and give the final formulas:

1. if the surfaces $\sigma_1$ and $\sigma_2$ are acoustically rigid, then

   $$G(r, r_i) = \frac{\pi i}{2\alpha} \sum_{m=0}^{\infty} \epsilon_m J_{\nu_m}(kr_<)H_{\nu_m}^{(1)}(kr_>) \cos(\nu_m \theta) \cos(\nu_m \theta_i), \quad (4.33)$$

   $$\nu_m = \frac{m\pi}{\alpha},$$

2. if the surfaces $\sigma_1$ and $\sigma_2$ are acoustically soft, then

   $$G(r, r_0) = \frac{\pi i}{\alpha} \sum_{m=1}^{\infty} J_{\nu_m}(kr_<)H_{\nu_m}^{(1)}(kr_>) \sin(\nu_m \theta) \sin(\nu_m \theta_i), \quad (4.34)$$

   $$\nu_m = \frac{m\pi}{\alpha},$$

3. if the surface $\sigma_1$ is acoustically rigid, and surface $\sigma_2$ is acoustically soft, then

   $$G(r, r_0) = \frac{\pi i}{\alpha} \sum_{m=0}^{\infty} J_{\nu_m}(kr_<)H_{\nu_m}^{(1)}(kr_>) \cos(\nu_m \theta) \cos(\nu_m \theta_i), \quad (4.35)$$

   $$\nu_m = \frac{(2m + 1)\pi}{2\alpha},$$
The distinctive features of sound diffraction by some complex bodies

$r_\prec$ is the smallest of two distances $r$ and $r_i$, and $r_\succ$ is the largest of two distances $r$ and $r_i$ [31], $\varepsilon_0 = 1$ and $\varepsilon_m = 2$ when $m > 0$.

If we know the Green function, it is easy to determine the sound pressure, created by the source of cylindrical waves [17]. It is enough to multiply the Green function by value $-ik\rho\nu_0r^*\alpha/2\pi$, where $r^*$—radius of the source of cylindrical waves, and $\nu_0$—particle velocity of its surface.

### 4.4.2 Plane Sound Wave Diffraction by Wedge

Let us consider the problem of plane wave diffraction by wedge. The source $S$ should be placed far enough from the wedge [17]. Let tend the value $kr_\succ$ in the formulas given above to infinity and use the asymptotic representation of Hankel function for $kr_i = kr_\succ \gg 1$:

$$H^{(1)}_{\nu_m}(kr_i) \approx \sqrt{\frac{2}{\pi kr_i}} \exp\left(ikr_i - \frac{i\nu_m\pi}{2} - \frac{i\pi}{4}\right).$$  \hspace{1cm} (4.36)

Besides, we normalize the sound pressure to the pressure on edge, which is created by point (linear) source in the free space. We use the Green function for it (4.34), where $r = 0, kr_i \gg 1$. Then

$$G(0, r_i) \approx \frac{i}{4} \sqrt{\frac{2}{\pi kr_i}} \exp\left(ikr_i - \frac{i\pi}{4}\right).$$  \hspace{1cm} (4.37)

We divide the expressions (4.33)–(4.35) by the right hand side (4.37) and we use asymptotics (4.36). Then we write the full pressure sound field, which arises in the consequence of scattering of plane sound wave by wedge of unit amplitude:

1. acoustically rigid wedge

\[ p = \frac{2\pi}{\alpha} \sum_{m=0}^{\infty} \varepsilon_m \exp\left(-\frac{i\nu_m\pi}{2}\right) J_{\nu_m}(kr) \cos(\nu_m\theta) \cos(\nu_m\theta_i), \]  \hspace{1cm} (4.38)

\[ \nu_m = \frac{m\pi}{\alpha}, \]

2. acoustically soft wedge

\[ p = \frac{4\pi}{\alpha} \sum_{m=1}^{\infty} \exp\left(-\frac{i\nu_m\pi}{2}\right) J_{\nu_m}(kr) \sin(\nu_m\theta) \sin(\nu_m\theta_i), \]  \hspace{1cm} (4.39)

\[ \nu_m = \frac{m\pi}{\alpha}, \]

3. acoustically rigid face $\sigma_1$ and acoustically soft face $\sigma_2$

\[ p = \frac{4\pi}{\alpha} \sum_{m=0}^{\infty} \exp\left(-\frac{i\nu_m\pi}{2}\right) J_{\nu_m}(kr) \cos(\nu_m\theta) \cos(\nu_m\theta_i), \]  \hspace{1cm} (4.40)

\[ \nu_m = \frac{(2m + 1)\pi}{2\alpha}. \]
Thus, using Eqs. (4.38)–(4.40) we can investigate the plane wave diffraction \( p_i = \exp (-ikr \cos (\theta - \theta_i)) \). The plane wave front is parallel to the edge, and the direction of incidence is determined by angle \( \theta_i \).

### 4.4.3 The Analysis of Calculation Results for the Sharp Wedge

As it was mentioned above, the wave processes in the vicinity of the edge of the wedge are of the main interest. Consequently, we fix attention on these processes.

When calculating, it is assumed that the source \( S \) is situated on the edge \( \sigma_1 \) (i.e., \( \theta_i = 0 \)) and the wave distance from the source to the edge is \( r_i/\lambda = 1.3 \) (here \( \lambda \)—the wave length in the acoustic medium surrounding the wedge). The additional Cartesian coordinate system \( xO^*y \) is introduced (see Fig. 4.20). The center \( O^* \) of this system is brought in coincidence with the point of the source \( S \) position. The number of the kept terms in the series (4.33)–(4.35), when calculating, is 100.

Figure 4.21 shows distributions of pressure and radial component of particle velocity along the \( O^*x \) axis for the wedge with acoustically rigid edges. The part of distance, equal \( O^*O \), coincides with the surface of edge \( \sigma_1 \), and the part after the point \( O \) is beyond the wedge in acoustical medium.‡

Let us consider some peculiarities of the behavior of the values \( \tilde{p} \) and \( \tilde{\upsilon} \) in Fig. 4.21. As we can see, Huygens was right—there is a pressure-jump on the edge. The jump becomes more abrupt with an increase of the wedge sharpness. This jump causes edge cylindrical wave on the edge of the wedge. Edge wave in the illuminated zone interferes with the incident wave (the pressure oscillations appear on the surface of the face \( \sigma_1 \)) and penetrates into the zone of geometrical shadow.

![Normalized pressure distribution and particle velocity](image)

**FIG. 4.21:** Normalized pressure distribution \( \tilde{p} \) and particle velocity \( \tilde{\upsilon} \) (both edges are acoustically rigid) along the axis \( O^*x \) when \( r_i/\lambda = 1.3 \): (a,b)—\( \alpha = 270^\circ \), (c,d)—\( \alpha = 360^\circ \)

‡Note that the interval of the axis \( O^*x \) beyond the edge \( \sigma_1 \) is (geometrically) a dividing line between the illuminated domain and the domain of shadow.
Pressure jump causes the appearance of peculiarity of particle velocity on the edge. The sharper is edge the stronger is peculiarity (compare the levels of particle velocity on the edge in Figs. 4.21(b) and 4.21(d)). The comparison of these levels gives only the relative idea about the character of peculiarity on the edge. The calculations were performed for the finite number of the terms of series (4.33). If we want to estimate rigorously the character of these peculiarities, it is necessary to use the method of Maue [20], as it was done in the first chapter. In this case, when $\alpha = 270^\circ$, the character of peculiarity of particle velocity $\nu$ when approaching the edge is as follows $\nu \approx A/r^{1/3}$, and when $\alpha = 360^\circ$ it is $\nu \approx A/r^{1/2}$ respectively. The peculiarity is stronger for the semi-plane than for the direct angle; $A$ is a constant here.

Let us consider now the behavior of pressure distributions of radial component of particle velocity along the axis $O^*x$ in the case when the face of the wedge $\sigma_1$ is acoustically rigid, and the face $\sigma_2$ is acoustically soft. Figure 4.22 shows such data. As we can see, the pressure jumps near the edge are more sharp, than in the case, when the both faces are acoustically rigid. Consequently, the peculiarities of particle velocity on the edge are more pronounced. In the case, when $\alpha = 270^\circ$, the character of particle velocity peculiarity is as follows $\nu \approx A/r^{2/3}$ when approaching the edge, and when $\alpha = 360^\circ$ it is $\nu \approx A/r^{3/4}$, respectively.

The space distribution of acoustic pressure in the vicinity of the wedge is also of interest. Figure 4.23 shows such data for the right-angled wedge. The strong interference effect between the incident and edge waves is observed in the illuminated area. The interference is shown as the alternation of zones of high and low pressure. It is clearly seen how the edge wave flows into the shadow zone. When the faces have different acoustic properties, the pressure level on shadow zone is about 10 dB lower, than in the case, when both faces are acoustically rigid.

FIG. 4.22: The distribution of normalized pressure and particle velocity (one edge is acoustically rigid, the other—soft) along the axis $O^*x$ when $r_1/\lambda = 1.3$: (a,b)—for $\alpha = 270^\circ$, (c,d)—for $\alpha = 360^\circ$

\*The peculiarity, that is an infinite velocity increase, is the consequence of the neglect of viscosity of acoustic medium and nonlinearity.
It is interesting to consider the situation when the wedge becomes a plane. Angle $\alpha$ becomes equal $180^\circ$. Fig. 4.24 shows pressure space distribution for this case. When the faces of the wedge are acoustically rigid, there are no special wave processes. Cylindrical wave, excited by source $S$, propagating in semi-plane and the level of its pressure drops gradually according to the law $1/\sqrt{r}$. But if the surface $\sigma_1$ is acoustically rigid, and $\sigma_2$ acoustically soft, the sound pressure on the rigid part of the surface is different from zero, and on the soft part—is equal to zero. That is why there is a pressure jump in the point $O$, and it is well seen in Fig. 4.24(b), and the peculiarity of particle velocity appears. The result of it is that the edge wave appears. This wave interferes with the incident wave, and creates the sequence of zones of high and low pressure over the surface $\sigma_1$. We have the zone of smooth and rapid drop of the pressure level down to zero over the surface $\sigma_2$, when approaching the surface $\sigma_2$. 

**FIG. 4.23:** Space distribution of pressure field when $\alpha = 270^\circ$ and $r_1/\lambda = 1.3$: (a)—both faces of the wedge are acoustically rigid, (b)—face $\sigma_1$ is acoustically rigid, and face $\sigma_2$ acoustically soft

**FIG. 4.24:** Space distribution of the field pressure when $\alpha = 180^\circ$ and $r_1/\lambda = 1.3$: (a)—surface $\sigma_1$ and $\sigma_2$ are acoustically rigid, (b)—surface $\sigma_1$ is acoustically rigid, and $\sigma_2$ acoustically soft
4.4.4 Waves Diffraction by Rounded Wedge

The diffraction of waves by shaped (classical) wedge was considered, and we found out that there is a pressure jump on its edge. The edge wave appears in the result of it. Let us consider now what happens if the sharp edge of the wedge is rounded as cylindrical surface. Figure 4.25 shows the geometry of such wedge and all the notations.

Let us consider the plane problem of cylindrical wave scattering by rounded wedge. All the surfaces of the wedge are acoustically rigid. Let us introduce two polar coordinate systems: \( rO\theta \) and \( r_1O_1\theta_1 \) to construct the problem solution (the coordinates of some point \( M \) are indicated on these coordinate systems in Fig. 4.25). The radius of the wedge rounding is \( a = O_1A = O_1B \). The angle of rounded wedge \( \alpha \) is determined as the angle of the wedge without rounding. The sound source \( S \) have the coordinates \( (r_i, \theta_i = 0) \), it is situated on wedge surface \( \sigma_1 \) and coincides with the origin of additional coordinate system \( xO^*y \).

Let us designate the following domains to use the partial domains method. Exterior domain \( I \) is set in the coordinate system \( rO\theta \): \( b \leq r < \infty, 0 \leq \theta \leq \alpha \). It is the exterior of the circle with radius \( b \), bounded by the faces of rounded wedge. Domain \( II \) is the interior of the circle with radius \( b \) excluding the zone of intersection with the circle of radius \( a \) with the point \( O_1 \) as a center.

The pressure in domain \( I \) is written as follows:

\[
p_I = \frac{\pi i}{2\alpha} \sum_{n=0}^{\infty} \varepsilon_n H_{\xi_n}^{(1)}(kr_i) J_{\xi_n}(kr) \cos(\xi_n\theta) + \sum_{n=0}^{\infty} A_n \frac{H_{\xi_n}^{(1)}(kr)}{H_{\xi_n}^{(1)'}(kb)} \cos(\xi_n\theta), \quad (4.41)
\]

where \( \xi_n = n\pi/\alpha \). The first sum here describes the field of the source of the usual (sharp) wedge. The second sum determines the field of the scattered wave in the form of the set of cylindrical waves coming away from the wedge. The prime means derivative of function with respect to argument.

The pressure in domain \( II \) is represented as follows

\[
p_{II} = p_{II}^{(1)} + p_{II}^{(2)}, \quad (4.42)
\]

FIG. 4.25: Rounded wedge geometry
Expression (4.43) is the general solution for the interior of the circle of radius $b$ with point $O$ as the center. The functions $\cos (n\theta)$ and $\sin (n\theta)$ have the orthogonality property on the full circle ($0 \leq \theta \leq 2\pi$). But partial domain $II$ is not a full circle. The zone of intersection with the circle of radius $a$ and point $O_1$ as the center is excluded. Consequently, when writing the conditions of the fields matching on the boundaries of partial domains $I$ and $II$ ($r = b$, $0 \leq \theta \leq \alpha$), we should add the boundary condition for solution $p_{II}^{(1)}$ on the nonphysical area ($r = b$, $\alpha \leq \theta \leq 2\pi$), (see below). The possibility of using such procedure was discussed in Chapter 1.

Solution (4.44) is general for the interior of dihedral domain $BO_1A$ (the functions $\cos (\eta_n \theta_1)$ are orthogonal when the angle coordinate changes $0 \leq \theta_1 \leq 2\gamma$). That is why the solution $p_{II}^{(2)}$ provides the fulfillment of the boundary condition on the rigid surface of the wedge rounding (arc $ADB$).

The sound fields matching conditions on the boundary of domains $I$ and $II$ are as follows:

$$ p_{II} = p_I, \quad r = b, \quad \theta = [0, \alpha], \quad (4.45) $$

$$ \begin{align*}
\frac{\partial p_{II}}{\partial r} &= \frac{\partial p_I}{\partial r}, \quad r = b, \quad \theta = [0, \alpha], \\
\frac{\partial p_{II}^{(1)}}{\partial r} &= F (\theta), \quad r = b, \quad \theta = [\alpha, 2\pi].
\end{align*} \quad (4.46) $$

It is to be recalled that it makes no difference whether we set pressure or radial velocity on the nonphysical part of the boundary ($r = b$, $\alpha \leq \theta \leq 2\pi$). The velocity was chosen in condition (4.46). Function $F (\theta)$ can be arbitrary. But it is desirable to decrease the field jumps on the ends of the region (points $A$ and $B$). We choose the function $F (\theta)$ in the following form

$$ F (\theta) = \frac{\partial G (b, \alpha; r_i, \theta_i)}{\partial r} + \left[ \frac{\partial G (b, 0; r_i, \theta_i)}{\partial r} - \frac{\partial G (b, \alpha; r_i, \theta_i)}{\partial r} \right] \frac{\theta - \alpha}{2\pi - \alpha}, \quad (4.47) $$

$G$—the source function for the usual wedge, $\alpha \leq \theta \leq 2\pi$.

The boundary condition on the surface of wedge rounding is as follows:

$$ \frac{\partial p_{II}}{\partial r_1} = 0, \quad r_1 = a, \quad \theta_1 = [0, 2\gamma]. \quad (4.48) $$

The algebraization of functional equations (4.45), (4.46) and boundary condition (4.48) is performed by standard method. The properties of orthogonality of the set of functions $\cos (\xi_n \theta)$ on the interval $0 \leq \theta \leq \alpha$ are used for condition (4.45) and the properties of orthogonality of functions $\cos (n\theta)$ and $\sin (n\theta)$ on the interval $0 \leq \theta \leq 2\pi$
are used for condition (4.46). The orthogonality of function $\cos(\eta_n \theta_1)$ on the interval $0 \leq \theta_1 \leq 2\gamma$ is used for condition (4.48). We obtain the infinite system of linear algebraic equations of the second kind in unknown coefficients $A_n, B_n, \tilde{B}_n, C_n$.

The number of the kept unknowns in the systems of linear algebraic equations is 100 when we perform the calculations. The matching conditions (4.46) were fulfilled with graphical accuracy, and boundary condition (4.45)—with relative accuracy not less than 1%.

### 4.4.5 Calculation Results Analysis for Rounded Wedge

The obtained above solution allows us to carry out the numerical analysis of sound field diffraction by wedge in the case when the edge is rounded. Figure 4.26 shows the normalized distributions of pressure and of radial component of particle velocity along the axis $O^*x$ for rounded wedge with acoustically rigid boundaries and acoustically rigid surface of rounding. Figure 4.27 shows the space distributions of pressure in the vicinity of rounded edge. Let us compare the dependencies of distribution of pressure and radial component of particle velocity in Figs. 4.21(a), 4.21(b), 4.26(a), 4.26(b) and also the space distribution of pressure in Figs. 4.23(a) and 4.27(a). As we can see, when the radiiuses of edge rounding are small, there are no special changes in the field of diffraction. Despite the rounding of the edge, the pressure jump is almost the same as in the case of sharp edge. Consequently, the peak particle velocity appears. The edge wave appears as a result of it. This wave interferes with incident wave and creates the succession of zones of high and low pressure over the surface $\sigma_1$ of the wedge [these zones are well seen in Fig. 4.27(a)]. It causes the ripple of the graphs of pressure and particle velocity on the intervals $O^*A$ of $O^*x$ axis [Figs. 4.26(a) and 4.26(b)]. The peculiarity of particle velocity cannot appear on the rounded edge of wedge from the mathematical point of

**FIG. 4.26:** Normalized distribution of pressure and particle velocity along $O^*x$ axis when the faces and the surfaces of rounding for $\alpha = 270^\circ$ are acoustically rigid: (a,b) $a/\lambda = 0.05$; (c,d) $a/\lambda = 0.5$
FIG. 4.27: Space distribution of pressure field with acoustically rigid faces and surface of rounding for $\alpha = 270^\circ$: (a) $a/\lambda = 0.05$; (b) $a/\lambda = 0.5$

view. But, as we can see, all the characteristic features of the field of diffraction remain almost the same.

The situation is quite different, when the radius of the edge rounding is comparable with the wave length. Let us consider the graphs of pressure and particle velocity in Figs. 4.26(c), 4.26(d) and space distribution of the pressure field in Fig. 4.27(c). There is no pressure jump in this case, and consequently, the peak of particle velocity does not occur. The edge wave is almost absent. This is proved by the absence of interference zones of high and low pressure over the surface $\sigma_1$ of wedge [Fig. 4.27(b)] and by the absence of ripple on the graphs of pressure and particle velocity on the intervals $O^*A$ of axis $O^*x$ [Figs. 4.26(c) and 4.26(d)].

It should be noted in conclusion that the results given in this chapter are based on the investigations performed by the authors in monograph [13] and articles [14,15,16].

REFERENCES

The distinctive features of sound diffraction by some complex bodies