Unfortunately, the progress in science and technology is accompanied with negative developments. For example, the noise pollution of environment. The reason of it is the increasing of traffic speed and density, and also the increasing of intensity of work of different industrial machines and so on.

It is well known that the noise with level higher than 65 dB can cause complete or partial hearing loss of a person [1]. At present, the offices and residential premises are affected by traffic noise, which exceeds the permissible noise level. The noise levels in hospitals, schools and pre-schools are also unacceptable. Absent-mindedness when performing intellectual work, increased fatigability, irritability, and headache are the most evident consequences of human exposure to noise impact. The consequences can be even more serious. According to present knowledge, municipal noise increases the incidence of neuroses, different autonomic disturbances, hypertension and other diseases, caused by the chronic-fatigue of brain. It should be noted that the noise reaction of a definite person is very subjective. It depends on the acoustic properties of the source, the place, the exposure time and the state of nervous system. For example, noise level 70–80 dB(A) is sometimes not perceived as irritatory agent when a person is in a subway car (the tired people can even sleep in subway). But an exterior noise of the same level penetrating in a living room is unbearable.

In the developed countries, a lot of attention is paid to the investigation of cities acoustic pollution and to the elaboration of its reduction measures. This is evident from the fact that there is a continuous stream of publications in top scientific and technical journals (see, for example, [2–5] and bibliography in them).

Noise reduction with the aid of acoustic barriers (acoustic baffles) holds a special place. Barriers are placed between the noise source and the area, which should be protected against noise. The examples of such areas are the residential buildings, the sidewalks near the traffic arteries, and the place of work on the factory floor. The barriers are popular because they are relatively cheap and simple-to-use. Figure 5.1 shows the photographs of real barriers.

Such barriers are made of non-transparent materials (concrete, wood, metal), and also from transparent plastics [6,7]. As we can see, they can have different design.

The described barriers can protect from noise due to the effect of acoustic shielding. It means that the acoustical shadow appears around a protected object. Within the scope
of ray acoustics (geometrical acoustics) theory [8], it is assumed that noise does not penetrate into the acoustical shadow zone. Acoustical shadow zone is separated from illuminated zone of space by semi-infinite right line, coming from the source through upper edge of sound barrier (Fig. 5.2). At first glance, it seems that it is enough to select the corresponding height of acoustic barrier to protect some object (all the object should be in geometric shadow) and the noise will be removed. But the results of ray theory application are satisfactory only when the sound wave length is much less than the barrier height and we can neglect a diffraction effect.

In most cases, the lion’s share of sound energy of traffic and industrial noise lies in the range of relatively law frequencies. As an example, Fig. 5.3 shows averaged experimental noise spectrums of traffic flows. And we can see, that the maximum of noise energy is in domain 50, ..., 200 Hz, which is equivalent of wave length 1, 5, ..., 6 m.
The height of acoustic barriers built near traffic arteries range from 3 to 8 m. When studying the sound scattering by bodies, comparable with the length of sound wave, it
is necessary to use the rigorous methods for statement and solution of corresponding
diffraction problem [10,11]. In the contrary case, the results of scattered field estimation
can be false.

The bulk of publications, dedicated to the estimation of sound fields scattered by
barriers, is performed on the base of using the approximate approaches (ray acoustics
methods, Keller method and other asymptotical methods [9,12]). The specified methods
allow us to obtain good fields estimations behind the barrier (in the zone of its acoustic
shadow) for the cases, when the barrier height exceeds the length of incident sound wave.

Diffraction problems, concerned with sound barriers models, are considered in this
chapter on the base of partial domains method. The solutions are obtained, which al-
low us to analyze the sound fields scattered by barriers in all the practically interesting
frequency range.

5.1 CLASSICAL SOUND BARRIER

5.1.1 Physical and Mathematical Models of Barrier

Classical barriers are the solid vertical walls made of rigid material; the height of such
barriers is much more than their thickness. The barriers are mounted on the sides of the
highways as a continuous wall. The length of this wall is much more than its height.
Assume the traffic flow is heavy (more than two thousand transport units per hour). In
this case, the traffic noise level almost does not depend on a separate transport unit noise
level.

In view of the foregoing, let us consider the following ideal physical model of clas-
sical barrier, situated along the highway (Fig. 5.4).

Assume that an infinite acoustically hard surface simulates the ground. Acoustically
hard thin barrier is at point \( O \). This barrier is infinite and perpendicular to the figure plane.
The height of barrier \( r_h \), and its inclination angle about the ground \( \theta_0 \). The effective
height of barrier \( h = r_h \sin \theta_0 \) (see Fig. 5.4). Linear harmonic sound source \( S \) with
frequency \( \omega \) is on the right hand, parallel to the barrier at distance \( b \) from the barrier and
at a height \( y_S \) from the ground. The source is an infinite thread. The observation point is
denoted with \( P \). All the semi-space is filled with ideal acoustic medium with density \( \rho \)
and sound velocity \( c \). These parameters correspond to the air.

![Physical model geometry](image-url)
The physical model, described mathematically, is equivalent to a plane problem. In this model the parameters of sound field are independent of one coordinate (in our case this coordinate is perpendicular to the figure plane). The normal component of sound field particle velocity on the ground and barrier is equal to zero.

This model corresponds to the practical situations and allows us to construct rigorous analytical solution [13].

5.1.2 Analytical Solution Construction

To construct the solution of the problem, let us introduce the polar coordinate system \((r, \theta)\) with point \(O\) as center (Fig. 5.4). Let us partition all the sound field existence domain into three domains: domain \(I\) is determined by formulas \(r \geq r_h, \ 0 \leq \theta \leq \pi\); domain \(II\) occupies the right sector of semicircle with radius \(r_h\): \(r \leq r_h, \ 0 \leq \theta \leq \theta_0\); domain \(III\) is the left sector of semicircle: \(r \leq r_h, \ \theta_0 \leq \theta \leq \pi\).

The described above partial domains were defined in such a way as to construct the problem solution. We can construct the general solution of Helmholtz equation for such domains. The solution of initial boundary problem is reduced to the fulfillment of matching conditions at the boundaries of partial domains. We do not write time factor \(\exp(-i\omega t)\).

Let us place center \(O_1\) of the second polar coordinate system \((r_1, \theta_1)\) in the location of source \(S\), Fig. 5.4. The pressure field of linear source with the amplitude equal to unity is determined by expression [14]:

\[
p_0 = H_0^{(1)}(kr_1),
\]

where angle functions \(\cos(n\theta)\) are chosen in such a way as to automatically fulfill boundary conditions on rigid surface \(\theta = 0\) and \(\theta = \pi\); the prime on the Hankel function means argument derivative. The totality of arbitrary elements \(A_n\) allows us to fulfill matching conditions at the boundaries with domains \(II\) and \(III\).

Let us represent the pressure field in domain \(II\) as the superposition of standing waves:

\[
p_{II} = \sum_{n=0}^{\infty} B_n J_{\alpha_n}(kr) \cos(\alpha_n \theta), \quad \alpha_n = \frac{n\pi}{\theta_0}.
\]

The angle functions here \(\cos(\alpha_n \theta)\) are chosen according to the boundary conditions on the rigid surfaces of the plane and screen (when \(\theta = 0\) and \(\theta = \theta_0\). If the condition \(r_S < r_h\) is fulfilled, then the expression for the field of source \(H_0^{(1)}(kr_1)\) should be transferred from Eq. (5.1) into the right-hand side of Eq. (5.2).

We write the pressure field for domain \(III\) as follows:
\[ p_{III} = \sum_{n=0}^{\infty} C_n J_{\beta_n}(kr) \cos(\beta_n(\theta - \theta_0)), \quad \beta_n = \frac{n\pi}{\pi - \theta_0}. \quad (5.3) \]

Standard notations for Bessel and Hankel functions are used in Eqs. (5.1)–(5.3).

The sound source can be in domain I, and in domain II. The linear source field \( H_{0}^{(1)}(kr_1) \) should be written in coordinate system \((r, \theta)\). For this purpose, we will use the Green function for an infinite wedge with acoustically rigid boundaries (the field of linear source differs from Green function only in constant factor [14]):

\[ G(r, \theta; r_S, \theta_S) = \frac{\pi i}{2\alpha} \sum_{n=0}^{\infty} \varepsilon_n J_{\nu_n}(kr_S) H_{\nu_n}^{(1)}(kr) \cos(\nu_n \theta_S) \cos(\nu_n \theta), \quad (5.4) \]

where \( \alpha \)—opening angle of wedge, \( r_< \)—is the least distance from \( r \) and \( r_S \), \( r_> \)—the largest distance from two indicated distances.

When the source is spaced in domain I, the opening angle of wedge corresponds to the plane and is equal \( \pi \). In such a situation the expression for pressure field \( p_{0I} \), created by linear source, is as follows

\[ p_{0I} = i \left\{ \begin{array}{l} \sum_{n=0}^{\infty} \varepsilon_n J_{\nu_n}(kr) H_{\nu_n}^{(1)}(kr_S) \cos(n\theta_S) \cos(n\theta), \quad r_0 < r < r_S, \\ \sum_{n=0}^{\infty} \varepsilon_n J_{\nu_n}(kr_S) H_{\nu_n}^{(1)}(kr) \cos(n\theta_S) \cos(n\theta), \quad r > r_S. \end{array} \right. \quad (5.5) \]

According to (5.1), the field in domain I

\[ p_I = p_{0I} + \sum_{n=0}^{\infty} A_n \frac{H_{n}^{(1)}(kr)}{H_{n}^{(1)}(kh)} \cos(n\theta). \quad (5.6) \]

If the source is situated in domain II, the opening angle of the wedge is equal \( \theta_0 \) and the expression for the pressure field of the source is as follows:

\[ p_{0II} = \frac{\pi i}{2\theta_0} \left\{ \begin{array}{l} \sum_{n=0}^{\infty} \varepsilon_n J_{\nu_n}(kr) H_{\nu_n}^{(1)}(kr_S) \cos(\nu_n \theta_S) \cos(\nu_n \theta), \quad r < r_S, \\ \sum_{n=0}^{\infty} \varepsilon_n J_{\nu_n}(kr_S) H_{\nu_n}^{(1)}(kr) \cos(\nu_n \theta_S) \cos(\nu_n \theta), \quad r_S < r < r_0, \end{array} \right. \quad (5.7) \]

where \( \nu_n = \alpha_n = n\pi/\theta_0 \). Taking into account expression (5.2), we have

\[ p_{II} = p_{0II} + \sum_{n=0}^{\infty} B_n \frac{J_{\alpha_n}(kr)}{J_{\alpha_n}(kh)} \cos(\alpha_n \theta). \quad (5.8) \]
Let us write the system of functional equations, which defines the conditions of sound field continuity at the boundaries of domains $I$ and $II$, $III$:

$$
\begin{align*}
 p_I &= \begin{cases}
   p_{II}, & r = r_h, \quad \theta = [0, \theta_0], \\
   p_{III}, & r = r_h, \quad \theta = [\theta_0, \pi],
\end{cases} \\
\frac{\partial p_I}{\partial r} &= \frac{\partial p_{II}}{\partial r}, \quad r = r_h, \quad \theta = [0, \theta_0], \\
\frac{\partial p_I}{\partial r} &= \frac{\partial p_{III}}{\partial r}, \quad r = r_h, \quad \theta = [\theta_0, \pi].
\end{align*}
$$

(5.9) (5.10) (5.11)

Taking into account the orthogonality of the corresponding sets of functions, we will algebraize the functional system of Eqs. (5.9)–(5.11). We obtain an infinite system of linear algebraic equations in unknown coefficients $A_n, B_n, C_n$. We do not cite the final form of this system because of its awkwardness.

We turn our attention now to the solution of system of algebraic equations and the estimation of accuracy of numerical results. The solution of infinite system of equations was considered in the previous chapters. Such algorithms are effective due to the calculation of the singularities in the vicinity of angular points (in our case it is an upper edge of a barrier). It allows us to obtain the quantitative estimations of sound fields characteristics in the domains arbitrarily close to angular points. If we want to find out the field properties in the points distant from the angles, the accurate results can be achieved using simple reduction method. We will proceed exactly this way.

Let us consider the accuracy of fulfillment of matching conditions at the boundaries of partial domains. To improve the accuracy of sound field estimation, we should increase the number of kept unknown complex coefficients when solving the algebraic system of equations by reduction method. It is especially important for the high frequency domain of noise spectrum. The numerical experiments were conducted, which allow us to estimate the degree of fulfillment of sound fields matching conditions for the wide range of values of geometric parameters of barrier and frequency. We can obtain the results, which can be used in practice, if the total number of kept unknown complex coefficients is $120 \div 270$. The relative accuracy of pressure matching and particle velocity matching will be no worse than 0.01, except for the points in the immediate neighborhood of barrier edge.

### 5.1.3 Numerical Results Analysis

Let us begin with the analysis of sound field distribution around the barrier. This is the most illustrative and informative property of sound field. Figures 5.5–5.8 shows such data, calculated for the different values of frequency and for the different distances from the source to the barrier $h = 5.5$ m high. The barrier is perpendicular to the ground ($\theta_0 = 90^\circ$, Figs. 5.5 and 5.6), or it is inclined (Figs. 5.7 and 5.8). The source $S$ is situated at the ground level ($y_S = 0$, Fig. 5.4).

Each figure shows the adjacent to the barrier fragment of space distribution of difference in levels of sound pressures in decibels (i.e., pressure relative value)
FIG. 5.5: Value $\Delta L$ distribution around the barrier; $h = 5.5$ m, $b = 2.5$ m, $\theta_0 = 90^\circ$, $y_S = 0$; (a)—$f = 34$ Hz; (b)—$f = 85$ Hz; (c)—$f = 850$ Hz

$$\Delta L = 20 \log \frac{p}{p_0},$$

(5.12)

here $p$—pressure in the observation point when there is a barrier, $p_0$—pressure in the observation point without a barrier. The coordinates of the considered domains on $Ox$ and $Oy$ axis are given in meters: 20 m in horizontal direction in front of the barrier and behind the barrier, 20 m up from the ground.

The dynamic range of $\Delta L$ variation in decibels is shown in figures with the aid of a scale, which is situated to the right of each figure. The negative values of $\Delta L$ are plotted
FIG. 5.6: Value $\Delta L$ distribution around the barrier; $h = 5.5$ m, $b = 6$ m, $\theta_0 = 90^\circ$, $y_S = 0$; (a)—$f = 34$ Hz; (b)—$f = 85$ Hz; (c)—$f = 850$ Hz

down from 0, that corresponds to sound level reduction caused by the barrier, and the values plotted upwards correspond to the sound level increasing in the given point of space, caused by the presence of the barrier.

The analysis of data on sound field distribution around barrier allows us to determine the general regularities of the system source-barrier:

1. the whole sound shadow depth behind the barrier increases with an increase of frequency; it should be expected due to the increase of barrier wave height;
2. the total field nonuniformity behind the barrier increases with an increase of frequency; this is because of the linear source field interference with diffraction field on the barrier edge;

3. wave interference before the barrier becomes more noticeable when moving the source away from the barrier; this is because of the interaction of direct waves from the source and the waves reflected from the barrier in the opposite direction;
4. when analyzing the influence of the angle of slope of the barrier about the ground, it may be noted, that this parameter influences on the size of sound shadow domain (it increases when the barrier bends to the sound source, it decreases when the barrier bends in the opposite side from the source). The angle of slope influences on the depth of the sound shadow to a lesser extent. The middle level of $\Delta L$ value in the domain of geometrical shadow remains almost unchangeable for all three variants of barrier slope.
Pressure space distribution in the far region of the source is described by the directivity pattern. The directivity pattern is defined as the angular distribution of pressure amplitude normalized to maximum. The “source-barrier” system can be considered as a radiating system in this case. Expression (5.6) is computational for the field in domain $I$, taking into account the presence or absence of the sound source in domain $I$. The asymptotic representation of the Hankel function is used $H_n^{(1)}(kr)$ when $kr \gg 1$.

Figures 5.9 and 5.10 show directivity patterns at frequencies 34 and 85 Hz; the distance between the source and barrier 2.5 m [figures (a)], 6 m [figures (b)].

The height of barrier is the parameter of the curves. As one should expect, the noise-protective properties of the barrier weaken with a decrease of barrier height. The decrease of barrier height from 5.5 m to 4 m influences the field distribution in the far field to a lesser extent than the further decrease of the height to 2 m.

It is of interest to compare the results, obtained using the given algorithm, with the results in work [3]. The geometry of problem considered in work [3], is shown in Fig. 5.11(a). The approximate solution of wave diffraction of point source $S$ on the screen of finite height is obtained here. The screen is placed on a rigid surface. The point of observation $P$ is in the far field of radiating system “source-barrier.”

Figure 5.11(b) shows two frequency characteristics of the noise reduction value $\Delta L$, computed using two methods for fixed point $P$ of acoustic shadow zone. Curve 1

FIG. 5.9: Directivity patterns of “source-barrier” system; $f = 34$ Hz, $\theta_0 = 90^\circ$, $y_S = 0$:
(a)—for $b = 2.5$ m; (b)—for $b = 6$ m; 1—$h = 2$ m, 2—$h = 4$ m, 3—$h = 5.5$ m

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FIG. 5.10: The directivity patterns of system “source-barrier,” \( f = 85 \) Hz, \( \theta_0 = 90^\circ \), \( y_S = 0 \): (a)—for \( b = 2.5 \) m; (b)—for \( b = 6 \) m; 1—\( h = 2 \) m, 2—\( h = 4 \) m, 3—\( h = 5.5 \) m

FIG. 5.11: Problem geometry (a); (b)—frequency dependencies of value \( \Delta L \); 1—point source [3], 2—linear source
corresponds to the data in work [3] given the point sound source, and curve 2—to the algorithm for linear harmonic sound source described above. As we can see, there is a satisfactory fit between the both curves, while the sources are of various kinds. A little divergence of the curves can be explained by the fact the point sound source is used in one case, and linear source in the other. The linear source has a smaller decrease of pressure amplitude per distance unit. It is for this reason the sound levels of the linear source behind the barrier are larger than of the point source (3 dB bigger for the frequencies from 100 to 300 Hz), and interference minimums are deeper.

In conclusion of this paragraph, we provide an example of estimation of classical barrier acoustical properties. Let us assume that it is necessary to protect the pedestrians, walking on a sidewalk near a highway, from traffic noise. It was agreed that a concrete barrier of four-meter height should be built between the sidewalk and the highway. Let us estimate (at some characteristic frequencies) the efficiency of such barrier. Let us estimate how the barrier can decrease the level of traffic noise in the sidewalk area. We assume that the pedestrians are two meters apart from the barrier, when they are walking on the sidewalk, and the sound source is at the ground level.

Figure 5.12 shows the levels of sound pressure (for the point of observation without a barrier), calculated along $Oy$ axis at the distance 2 m from the barrier in the shadow zone. We assume that the height of a person does not exceed 2 m. It follows from the graphs, that barrier can reduce the traffic noise level in the sidewalk area in the amount for at least 12 dB in the frequency range from 34 to 850 Hz.

The given regularities allow us to draw a conclusion. The barrier efficiency of noise level reduction depends on the source noise spectrum, and on mutual arrangement of the source, the barrier, and the area protected from noise. Barriers efficiency depends also on the form and the physical properties of their surfaces (these questions are considered further). There are a lot of parameters of the problem to solve and the field character

![FIG. 5.12: Vertical distribution of sound pressure levels 2 m apart from the barrier in its shadow zone: $h = 4 \text{ m}, b = 2.5 \text{ m}, \theta_0 = 90^\circ, y_S = 0$](image)
behind the barrier is rather complex. Consequently, it is possible to predict the efficiency of the barriers basing on the strict solution of the problems of sound dissipation. We should solve the problems for specific purposes, with specific conditions and barrier location.

5.2 ACOUSTIC PROPERTIES OF √-BARRIER

5.2.1 Basic Idea

Sound barriers in the form of vertical rigid wall of specified height are widely used to protect residential areas from traffic noise due to their simple design and relative cheapness. The barrier height is the main parameter influencing the barrier efficiency. The higher barrier is the higher is its efficiency, all other parameters being equal. But this solution is irrational from the stand point of economy and aesthetics. Consequently, we are looking for the ways how to increase the barriers efficiency without increasing their height [4,15]. One of the possible constructions of the barrier with advanced efficiency is given bellow. And theoretical analysis of its acoustic properties estimation is given.

Let’s get back to the traditional barrier in the form of a vertical wall, and we recall its basic limitations. Figure 5.13(a) shows such a barrier and the noise source \( S \), from which it is necessary to protect the zone at the left, behind the barrier. Let us consider the given situation within the framework of geometrical theory of diffraction [12]. According to this theory, all the semi-space above the surface level of the road can be divided into illuminated area and shadow area behind the barrier. The line separating these areas lies on semi-infinite line, coming through the source and the edge of the barrier. The boundary [Fig. 5.13(a)] begins near the barrier edge. The sound field in the illuminated area is determined by five components:

1. the direct rays from the source,
2. the rays reflected from the barrier,
3. the rays reflected from the road surface,
4. the rays reflected from the road and from the barrier, and also
5. the ray’s fan, which appears when the direct ray falls on the barrier edge.

The sound field in geometric shadow area is defined by the rays scattered by the barrier edge. These rays are the main reason of sound field appearance in shadow area behind the barrier.

One of the ways to weaken the intensity of rays scattered on the edge of the barrier, is to create some additional field in the zone of the barrier edge. This field should be equal in intensity to the field created by diffraction rays fan on the edge of the barrier, but it should be different from this field in 180° phase. These fields may give total intensity close to zero, when being added in the barrier edge area. It weakens the field in the barrier.
shadow area. A similar idea was investigated experimentally in work [4], but there was not a rigorous analytical solution of a problem.

To implement this idea, we place a quarter-wave resonator in the form of a glass with the rigid walls and bottom near the barrier edge [Fig. 5.13(b)]. When the sound wave is incident on the inlet of resonator, two sound rays will propagate inside of it: direct, moving from the inlet to the bottom and the ray, reflected from the bottom and moving in opposite direction. A standing wave appears along the resonator axis because of the interference of these waves. If the resonator depth is equal to quarter wavelength, the maximum sound pressure area is near the resonator bottom, and minimum sound pressure area is near its inlet. Consequently, the level of noise, “flowing” behind the barrier, will be reduced.

Let us consider the structural scheme of the barrier, with a quarter-wave resonator in the area of its edge. We will show that the resonator efficiency can be higher than the efficiency of traditional barrier in the form of a simple wall of the same height.
5.2.2 Physical Barrier Model

The proposed construction of sound barrier with quarter-wave resonator near the edge is represented in Fig. 5.14. It contains two inclined to each other rigid panels 1. The bottom part of the space between them is filled with acoustically hard material 2 up to a definite level, and the top part is empty, and it is open to the surrounding space. Consequently, the upper part of the space between the walls is a peculiar kind of resonator. The depth of resonator can be changed due to the choice of material filling the bottom part of the space between the walls. Such barrier is called V-barrier.

We consider that the location of V-barrier and the traffic flow characteristics are the same as, for a classical barrier. Thus, as in the previous problem, we can confine ourselves to the consideration of two-dimensional problem in the plane perpendicular to the barrier. Then we can write the physical model of V-barrier, located along the highway (Fig. 5.14) as follows.

Let the earth surface be represented as an infinite acoustically hard surface. The V-barrier is located in the point $O$. The barrier consists of two rigid walls inclined to each other at an angle $\delta$. The height of the walls $r_h$ and inclination angle relative to the earth’s surface $\theta_0$ (Fig. 5.15). The filling level between the walls is equal $r_d$. The height of the walls $r_h$ is related to the height of V-barrier $h$ by expression $r_h = h/\sin \theta_0$. The linear

![FIG. 5.14: Structural scheme of V-barrier section: 1—barrier walls; 2—acoustically hard material, filling the space between the walls; 3—highway](image1)

![FIG. 5.15: Physical model geometry](image2)
harmonic sound source $S$ is to the right of the barrier at the height $y_S$ at the distance $b$ from barrier. The point of observation is denoted by $P$. This model allows us to construct rigorous analytical problem solution [16].

### 5.2.3 Analytical Solution Construction

Let us introduce the polar coordinate system $(r, \theta)$ with the center in the point $O$. Let us partition all the domain of sound field existence into four different domains, according to partial domains method. Domain $I$ is on the exterior of semicircle with radius $r_h$, it is bounded with the coordinates $r \geq r_h, 0 \leq \theta \leq \pi$. The other three domains are the adjoining sectors inside the circle with radius $r_h$. Thus, domain $II$, situated between the ground and the right wall of the barrier, is given by the following inequality $r \leq r_h$, $0 \leq \theta \leq \theta_0$. Domain $III$ is between the ground and the left wall of the barrier, that is $r \leq r_h, 0 + \delta \leq \theta \leq \pi$. And domain $IV$ is inside the resonator: $r_d \leq r \leq r_h$, $0 \leq \theta \leq \theta_0 + \delta$. All the model surfaces (Fig. 5.15) are ideally rigid.

The sound source can be in domain $I (r_S > r_h)$, or in domain $II (r_S < r_h)$. We should use the Green function for an infinite wedge with acoustically hard boundaries [Eq. (5.4)] to write the sound field of linear source.

Let $r_S > r_h$. The sound field in domain $I$ should then be written as Eq. (5.6)

$$p_I = p_{0I} + \sum_{n=0}^{\infty} A_n \frac{H_n^{(1)}(kr)}{H_n^{(1)}(kh)} \cos (n\theta), \quad (5.13)$$

The aggregate of arbitrary coefficients $A_n$ allows us to fulfill the matching conditions at the boundary with domains $II$, $III$, and $IV$. Let us represent the pressure field in domain $II$ as the superposition of standing waves:

$$p_{II} = \sum_{n=0}^{\infty} B_n \frac{J_{\alpha_n}(kr)}{J_{\alpha_n}(kh)} \cos (\alpha_n \theta), \quad \alpha_n = \frac{n\pi}{\theta_0}. \quad (5.14)$$

The angle functions here $\cos (\alpha_n \theta)$ are chosen according to the boundary conditions on the rigid surfaces of the ground and the barrier ($\theta = 0$ and $\theta = \theta_0$). The sequence of coefficients $B_n$ provides for the fulfillment of matching conditions at the boundary with domain $I$. Likewise, the pressure field in domain $III$ is as follows:

$$p_{III} = \sum_{n=0}^{\infty} C_n \frac{J_{\beta_n}(kr)}{J_{\beta_n}(kh)} \cos (\beta_n (\theta - \theta_0 - \delta)), \quad \beta_n = \frac{n\pi}{\pi - \theta_0 - \delta}. \quad (5.15)$$

Sound field in domain $IV$ should be written as follows:

$$p_{IV} = \sum_{n=0}^{\infty} \left( D_n \frac{J_{\gamma_n}(kr)}{J_{\gamma_n}(kh)} - C_n \frac{N_{\gamma_n}(kr)}{N_{\gamma_n}(kr_d)} \right) \cos (\gamma_n (\theta - \theta_0)), \quad \gamma_n = \frac{n\pi}{\delta}. \quad (5.16)$$

From the boundary condition on the resonator bottom $\partial p_{IV}/\partial r = 0, r = r_d, \theta_0 \leq \theta \leq \theta_0 + \delta$, we find the relation between coefficients $C_n$ and $D_n$: $C_n = -D_n (J_{\gamma_n}(kr_d)/J_{\gamma_n}(kr_h))$. Then Eq. (5.16) takes the form of
\[
\sum_{n=0}^{\infty} D_n \left( \frac{J_{\gamma_n}(kr)}{J_{\gamma_n}(kr_h)} - \frac{J'_{\gamma_n}(kr)}{J'_{\gamma_n}(kr_h)} \frac{N_{\gamma_n}(kr)}{N_{\gamma_n}(kr_d)} \right) \cos (\gamma_n (\theta - \theta_0)).
\] (5.17)

We write the system of functional equations determining the continuity conditions of the sound field at the boundaries of domains I and II, III, IV.

\[
p_I = \begin{cases} 
p_{II}, & r = r_h, \quad 0 \leq \theta \leq \theta_0, \\
p_{IV}, & r = r_h, \quad \theta_0 \leq \theta \leq \theta_0 + \delta, \\
p_{III}, & r = r_h, \quad \theta_0 + \delta \leq \theta \leq \pi,
\end{cases}
\] (5.18)

\[
\frac{\partial p_I}{\partial r} = \frac{\partial p_{II}}{\partial r}, \quad r = r_h, \quad 0 \leq \theta \leq \theta_0.
\] (5.19)

\[
\frac{\partial p_I}{\partial r} = \frac{\partial p_{IV}}{\partial r}, \quad r = r_h, \quad \theta_0 \leq \theta \leq \theta_0 + \delta.
\] (5.20)

\[
\frac{\partial p_I}{\partial r} = \frac{\partial p_{III}}{\partial r}, \quad r = r_h, \quad \theta_0 + \delta \leq \theta \leq \pi.
\] (5.21)

We algebraize Eqs. (5.18)–(5.21) using orthogonality properties of the corresponding sets of angle functions. Consequently, we obtain an infinite system of linear algebraic equations of the second kind in unknown coefficients \( A_n, B_n, C_n, D_n \). When solving the infinite system, we will follow recommendations given in Section 5.1.

### 5.2.4 Numerical Results Analysis

Let us perform a comparative analysis of the space distribution of pressure sound fields for two types of barriers: classical barrier in the form of simple wall and \( V \)-barrier. Figures 5.16 and 5.17 shows the data, calculated for different frequencies, when the effective barriers height is \( h = 4 \) m. The resonator dimensions for each frequency are selected individually, according to the following rules:

1. the depth of the cavity between the walls is about quarter-wavelength
   \[ r_h - r_d \approx \lambda/4, \]

2. the distance between the upper edges of the walls is small in relation to the wavelength
   \[ 2r_h \sin (\delta/2) < \lambda/3. \]

As we can see, the barrier and the resonator have the noise level in the shadow zone lower than the noise level of a traditional barrier at all the frequencies under consideration. In particular, the field’s levels near the ground behind the \( V \)-barrier are lower, than behind the traditional barrier about 6 dB (for the frequencies 34 Hz and 85 Hz) and about 10 dB for the frequency 850 Hz. Thus, the calculation data shows, that \( V \)-barrier is more effective noise protection due to the resonator.

Let us consider in more detail, what is happening inside the resonator in the area adjacent to its inlet (throat), Fig. 5.17(a). The pressure near the resonator bottom is high
FIG. 5.16: The value of $\Delta L$ distribution around a classical barrier: $h = 4 \text{ m}$, $b = 6 \text{ m}$, $\theta_0 = 90^\circ$, $y_S = 0$; (a)—$f = 34 \text{ Hz}$; (b)—$f = 85 \text{ Hz}$; (c)—$f = 850 \text{ Hz}$

because of the field distribution here (about $+8 \text{ dB}$). The pressure near the inlet of resonator is low (about $-10 \text{ dB}$). The ideal situation for sound protection is when this zone of low pressure (like a “lid”) covers completely the inlet of resonator and is parallel to the ground. It would be like this, if the source of sound was on $Oy$ axis above the resonator. But if the sound source is alongside of the barrier (and the resonator), the low pressure zone near the inlet of resonator is above the right wall (on the line connecting the source with the right edge of resonator).
To analyze the barrier with resonator efficiency, we have to analyze the influence of its parameters, such as the opening angle of resonator $\delta$, the angle of slope $\theta_0$ and the resonator depth. As it follows from the numerical analysis, we have found some important regularities:

1. the $V$-barrier efficiency is higher than the efficiency of traditional barrier in the form of a wall in almost all the frequency range under consideration;
2. the most efficient barrier is when its symmetry axis coincides with axis $O_y$;

3. the optimum resonator aperture width is about $0.2\lambda$, $\lambda$—wave length;

4. optimum resonator depth is about $0.25\lambda$.

The following recommendations may be useful in practice:

1. the barrier under consideration is especially efficient with relatively narrow-range noise source, or the source with several dominant frequency components;

2. it makes sense to include the resonator into the barrier construction, if the problem frequencies are relatively low (barrier wave dimension $k\lambda \leq 10$, ..., 15). There is no difference in efficiency between traditional and $V$-barrier at higher frequencies.

5.3 INTEGRAL CHARACTERISTICS OF CLASSICAL AND $V$-BARRIER

5.3.1 Estimation Method of Integral Characteristics of the Acoustic Barriers

Sound pressure variation value $\Delta L$ is calculated to analyze the sound field’s properties in the vicinity of the barrier. It gives the comprehensive information about the local values of sound pressure levels in any point of space. It should be noted that the local values of sound pressure depend on the barrier configuration, frequency and noise source position relative to the barrier. Such detailed comparative evaluation of different barriers ability to protect from noise is rather labor-intensive task. We also need some generalized estimations of noise-protective properties of the barriers to design the barriers. Such estimations allow us to choose the barrier design without a detailed comparison of sound field’s local values.

It is worthwhile to use the energetic characteristics of sound field such as: full power radiated by source $W_0$ (with barrier) and sound field power $W_D$, penetrating into the zone of barrier geometric shadow due to diffraction [17]. Figure 5.18 shows the barrier with all the parameters, which are necessary to find the opening angle of geometric shadow $\phi$.

We take the value

$$G = \frac{W_D}{W_0},$$  \hspace{1cm} (5.22)

as an integral space criterion for the estimation noise-protective properties of the barrier. This value is an energy coefficient of sound transmission into barrier shadow zone. Plane problem of sound diffraction, medium loss free condition, is under consideration. The average power of the source per unit length in a period can be calculated by arc $L$ integration. This arc is of arbitrary radius $r$, and it encloses the barrier and source $S$:

$$W_0 = \int_0^\pi I(r, \theta) d\theta,$$  \hspace{1cm} (5.23)
the intensity $I$ in each point of the arc $L$ is expressed as sound pressure $p$ in this point and radial (normal to the arc) particle velocity component $v_r$: $I = 0.5 \text{Re} (pv_r^*)$, the asterisk means complex conjugation.

We can find the value $W_D$ by calculating the power flow, through the arc $L_W$ with radius $r_W$ (Fig. 5.18). This arc is situated in shadow zone. It is bounded by the ground and by the boundary line of geometric shadow $SP$, coming from the source through the barrier edge:

$$W_D = \int_{\pi - \phi}^{\pi} I(r_W, \theta) d\theta. \quad (5.24)$$

According to the problem geometry of $V$-barrier (Fig. 5.18), the opening angle of integration arc $\phi$ is expressed through the angle of geometric shadow $\alpha$, barrier height $h$ and arc radius $r_W$ as follows:

$$\phi = \alpha + \arcsin \left( \frac{h}{\sin \theta_0} \frac{\sin (\alpha + \theta_0)}{r_W} \right), \quad (5.25)$$

and for classical barrier ($\delta = 0, \theta_0 = 90^\circ$)

$$\phi = \alpha + \arcsin \left( h \frac{\cos \alpha}{r_W} \right). \quad (5.26)$$

The behavior of the value of $\phi$ results from the last two formulas when $r_W$ increases infinitely: $\lim_{r_W \to \infty} \phi = \alpha$. The power level of sound field, coming through the arc $L_W$, does not changes for rather large distances ($r_W \gg \lambda, r_W \gg h, r_S$) (in this case the integration arc is the part of the common wave front, formed by system “source-barrier-ground”).

Let us introduce the integrated space-frequency criterion to make a quantitative description of frequency dependency of barrier efficiency

$$\Psi = \int_{f_1}^{f_2} G(f) df, \quad (5.27)$$

**FIG. 5.18:** Problem geometry: $SP$ line, connecting the source with the barrier edge, separates the shadow zone from the illuminated zone
where \( G(f) \)—frequency dependency of space integrated criterion, \( f_1 \) and \( f_2 \)—the boundaries of frequency band in which the integration is carried out. This criterion allows us to describe the efficiency of sound barrier, taking into account the frequency dependencies of barrier. It makes easier the comparative analysis of efficiency of sound barriers of varying design.

### 5.3.2 Numerical Results Analysis

We turn our attention now to the calculation of space criterion of barrier efficiency \( G \). We begin with studying its dependence on the integration arc radius \( r_W \). It allows us to find the value of radius, starting with which, the value of \( G(r_W) \) can be considered as constant value. Figure 5.19 shows the corresponding calculations for classical barrier.

**FIG. 5.19:** Space criterion \( G \) dependency on normalized radius \( r_w/h \); (a)—classical barrier, (b)—\( V \)-barrier; 1—\( f = 34 \) Hz, 2—\( f = 85 \) Hz, 3—\( f = 125 \) Hz, 4—\( f = 250 \) Hz, 5—\( f = 500 \) Hz.
ACOUSTIC PROPERTIES OF SOUND BARRIERS

[Fig. 5.19(a)] and V-barrier [Fig. 5.19(b)]. Every figure shows the graphs for five different frequencies. The value of integration radius \( r_W \) is plotted along the abscissa axis. It is normalized to the efficient barrier height \( h \). The height of both barriers is \( h = 4 \) m. The depth and opening angle of \( V \)-barrier for each curve are tuned to the frequency under consideration [opening angle \( \delta \) is shown in Fig. 5.19(b)]. The source is 6 m apart from the barrier and it is situated on the ground.

The graphs behavior is the same in both figures. The data for the curves are variable when \( r_W/h \leq 5 \). The curves are almost parallel to abscissa axis for the large values of \( r_W/h \). We note that \( V \)-barrier is much more efficient than classical (6 times larger and more).

Figure 5.20 shows the dependencies of the value of \( G \) on frequency for the both types of the barriers under investigation. We turn our attention first to curves 1 and 2. Curve 1 corresponds to classical barrier. Curve 2 is constructed for \( V \)-barrier. The resonator depth here is tuned to each frequency in order to correspond to quarter sound wave length. Opening angle \( \delta = 10^\circ \) is unchanged. It is optimal for resonator frequency 96 Hz. In real life it is impossible to realize such tuning. But it is interesting to investigate this situation. From the graphs shown in Fig. 5.20 it is seen that the value of \( G \) depends on frequency for classical barrier, and for \( V \)-barrier. The curves oscillations in both cases are caused by interferential phenomena, arising because of the interaction of direct wave from the source with the wave scattered by the barrier.

We turn our attention now to the frequency domain from 25 to 34 Hz (Fig. 5.20). We can see that the curve corresponding to the ideal \( V \)-barrier, which can be tuned to the current frequency, is on the higher level than for classical barrier. This points to the

**FIG. 5.20:** Frequency dependency of space criterion \( G \): \( r_w = 20 \) m, \( h = 4 \) m, \( b = 6 \) m, \( y_s = 0 \), \( \delta = 10^\circ \), \( \theta_0 = 85^\circ \); 1—classical barrier, 2—\( V \)-barrier, the resonator is tuned to the current frequency, 3—resonator is tuned to frequency 34 Hz, 4—resonator is tuned to frequency 85 Hz, 5—resonator is tuned to frequency 125 Hz
fact that $V$-barrier is less efficient than classical barrier in this frequency domain. It is related to the fact that for the frequencies less than 34 Hz the opening angle of resonator $\delta = 10^\circ$ is too small and the sound wave “passes over” the resonator. When the wave distance between the top edges of resonator is so small (in this case it is about $0.07\lambda$, instead of the recommended $0.2\lambda$) all the construction behaves as the barrier with rather broad continuous top edge. The sound-protective properties of such barrier are worse than the properties of classical thin screen [2].

Curve 2 in Fig. 5.20 is constructed for $V$-barrier. The depth of its resonator varies with the frequency under consideration. To make a $V$-barrier with such frequency characteristic is almost impossible. Consequently, we will consider a more real $V$-barrier with fixed resonator dimensions. Curves 3, 4, 5 in Fig. 5.20 correspond to the situations, when the resonator of $V$-barrier is tuned to frequencies 34, 85, and 125 Hz. Such frequencies are chosen not coincidentally. Frequency 34 Hz is of interest, because it is one of the lowest frequencies, in the domain under consideration. 85 Hz and 125 Hz correspond to maximum levels in transport noise spectrum (see Fig. 5.3). Space-frequency criterion $\Psi$ was calculated for each of these curves. The values of this criterion are given in Table 5.1.

As we might expect, the efficiency of barrier with fixed resonator tuning is lower than the efficiency of barrier with the resonator, which can be tuned to the frequency of noise source. Nevertheless, its efficiency is higher, than the efficiency of classical barrier. The $V$-barrier is the most efficient at the frequencies to which it is tuned. The tonal noise or narrow band is a rare phenomena. The efficiency of the barrier for such noise is maximum. When we choose the geometrical dimensions of resonator for broadband noise, we should focus on its dominant frequency components. The $V$-barrier tuned to frequency 85 Hz is the most efficient for the spectrums of transport noise given in Fig. 5.3 (Fig. 5.22, curve 3). The $V$-barrier, tuned to frequency 125 Hz, is well suited for protection from noise with dominant frequency components higher than 100 Hz. The $V$-barrier, tuned to frequency 34 Hz is valid for sound protection for the frequencies lower than 70 Hz. The analysis of value of space-frequency criterion shows, that the relation between the values of $\Psi$ for the different barriers changes depending on the frequency domain. When integrating in domain from 100 to 400 Hz, we obtain the set of values of $\Psi$, which correspond to the graphs in Fig. 5.20. The efficiency of tunable $V$-barrier is five times larger than the efficiency of classical barrier. The efficiency of $V$-barrier, tuned to 34 Hz or to 85 Hz—is 3.5 times larger, and the efficiency of barrier tuned to 125 Hz—4 times larger than the efficiency of classical barrier.

**TABLE 5.1:** Integral criterion of barrier efficiency

<table>
<thead>
<tr>
<th>Frequency domain, Hz</th>
<th>Classical barrier</th>
<th>$V$-barrier, $\delta = 10^\circ$, $\theta_0 = 85^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Tuneable 34 Hz 85 Hz 125 Hz</td>
</tr>
<tr>
<td>25–400</td>
<td>7.96</td>
<td>2.13 2.85 2.90 3.13</td>
</tr>
<tr>
<td>50–400</td>
<td>7.00</td>
<td>1.44 2.29 2.28 2.56</td>
</tr>
<tr>
<td>100–400</td>
<td>5.18</td>
<td>1.15 1.50 1.48 1.39</td>
</tr>
</tbody>
</table>
Basing on the conducted analysis, we may say that the integral criterions, proposed for the estimation of the barrier acoustic properties, (space criterion $G$ and space-frequency criterion $\Psi$) are the clear and valid characteristic of their sound-protective properties. They can be used to analyze the different designs of the barriers to estimate their efficiency.

5.4 THE EFFECT OF SOUND BARRIER SURFACE PROPERTIES ON ITS EFFICIENCY

The design of classical barrier is simple. It is a thin vertical wall made of acoustically rigid material. We need to increase the height of the barrier in order to increase the shadow depth and its sizes behind the barrier. As it was mentioned above, such approach cannot be used in many circumstances. The ways to increase the efficiency of barriers, which are not higher than 4–6 meters, are looked for. The $V$-barrier was considered above as one of the possible problem solutions. The materials with special properties, used to improve the noise protective properties of the barriers, are considered in this paragraph [6,18].

As an example, Fig. 5.21 shows the photos of the barriers. These photos were made by one of the authors in the autumn of 2009 in Austria and Italy. It may be assumed that the surfaces of all three barriers possess some sound—absorbing properties. The barriers have different design. Figure 5.21(a) shows the barrier surface made of the panels of interlaced wooden ribbons. Such panels are often used for the treatment of walls and ceilings of concert halls. Figure 5.21(b) shows the barrier in the form of bookcases partially filled with sand. It is fair to assume that these bookcases are the quarter-wave resonators. Figure 5.21(c) shows the barrier in the form of the set of horizontal panels with pinpointing perforations. There is a rigid wall from backside of these panels at a distance.* It is so called resonant absorber [6].

There are a lot of different designs of barriers. It means that there is no optimal barrier design which meets several criteria, namely: acoustic efficiency, aesthetics and not very high price.

Let us add the following property to the model described in the Section 5.1.1 (see Fig. 5.4) in order to study the acoustic properties of barriers with the walls having sound absorption properties. The illuminated surface of the barrier is an impedance surface. It allows us to model the reflectance and absorption properties of real constructions. We assume that the sound source is on the ground and barrier is perpendicular to the ground.

The problem can be solved by partial domains method. Solution construction corresponds to Section 5.1.2. We write the pressure field in domain $\Omega$ as follows

$$p_{\Omega} = \sum_{n=0}^{\infty} B_n \frac{J_{2n}(kr)}{J_{2n}(kh)} \cos(2n\theta) + \sum_{n=0}^{\infty} D_n \cos\left(\frac{n\pi z}{h}\right) \exp(i\gamma_n x).$$  \hspace{1cm} (5.28)

*The barrier would be absolutely sound-transparent without a rigid wall.
The first sum here is the totality of particular solutions of Helmholtz equations in the domain in the form of quadrant with radius $h$. Consequently, with due choice of coefficients $B_n$, this sum allows us to fulfill the arbitrary matching conditions of the fields on the interface of domains $I$ and $II$. Angle functions $\cos(2n\theta)$ are chosen on the presumption that the ground and the illuminated side of barrier are acoustically rigid.

The illuminated side of barrier is characterized by some impedance $Z$. There is the second sum in Eq. (5.28) in order to fulfill the boundary condition on this surface. This sum is represented as the set of normal waves of plane-parallel waveguide with rigid boundary surfaces. The width of waveguide is $h$. Consequently, we can fulfill the impedance boundary condition on the illuminated side of the barrier by means of $D_n$ coefficients selection.
Let us write the system of functional equations. It shows the conditions of continuity of sound field on the interface of domains $I$, $II$, $III$ and boundary condition on the illuminated barrier surface:

$$\frac{\partial p_I}{\partial r} = \begin{cases} \frac{\partial p_{II}}{\partial r}, & r = h, \quad \theta = [0, \pi/2], \\ \frac{\partial p_{III}}{\partial r}, & r = h, \quad \theta = [\pi/2, \pi], \end{cases}$$

$$p_I = p_{II}, \quad r = h, \theta = [0, \pi/2],$$

$$\frac{1}{i\omega \rho} \frac{\partial p_{II}}{\partial r} = \begin{cases} 0, & x = 0, \quad y = [0, h_1], \\ \frac{p_{II}}{Z}, & x = 0, \quad y = [h_1, h], \end{cases}$$

$$p_I = p_{III}, \quad r = h, \theta = [\pi/2, \pi].$$

(5.29) (5.30) (5.31) (5.32)

Than we bring it to the infinite system of linear algebraic equations of the second kind in unknown coefficients $A_n, B_n, D_n, C_n$. We can obtain the satisfactory results for practical purposes if the total amount of unknown complex coefficient is about 130.

Let us set the following geometric parameters of barrier model (Fig. 5.4): barrier height $h = 4$ m, the distance from the barrier to the sound source $b = 6$ m, the radius of the arc where the energy flow is calculated $r_w = 20$ m. It is to be recalled that $y_s = 0$, $\theta_0 = 90^\circ$. Let us analyze the noise-protective properties of the barrier. We start with the situation when the illuminated side of the barrier has the properties of acoustically ideal surface (Fig. 5.22). Curve 1, in this figure, corresponds to acoustically rigid surface of the illuminated barrier side, curve 2—acoustically soft surface, curve 3—absorbing surface. For these three variants, the value of specific (per unit area) and normalized to the value of $\rho c$ impedance of the illuminated barrier surface is $Z = \infty$, $Z = 0$, $Z = 1$, respectively.

**FIG. 5.22:** Energy coefficient $G$ frequency dependency for the case of acoustically ideal surface of illuminated barrier side: 1—acoustically rigid, 2—acoustically soft, 3—totally absorbing
Curve 1 in Fig. 5.22 shows the potentials of classical barrier with the acoustically rigid surfaces. We can indicate three sections on this curve: the first—very-low frequencies domain to 30 Hz, the second—low frequencies from 30 Hz to 120 Hz, the third—middle and high frequencies (above 120 Hz). On the first section, the curve rises, with an increase of frequency. The sound-protective properties of barrier reduce here. Such barrier properties result from the small wave height of the barrier. On the second section, the value of coefficient $G$ decreases rapidly, when there are strong oscillations, with an increase of frequency. On the third section, curve 1 oscillates and approaches to some value of coefficient $G$ with an increase of frequency. The value of the coefficient is determined according to the geometrical theory of diffraction.

Curve 3 in Fig. 5.22 shows maximum possible values of energy coefficient $G$ for the barrier with an absorbing surface. As we can see, the barrier becomes almost sound non-transparent for the frequencies above 30 Hz. Curve 3 (as well as curve 1) rises steeply up on the section of very-low frequencies (lower than 30 Hz). And we can observe the improvement of sound-protective properties in this frequency range in comparison with the barrier with acoustically rigid surfaces.

The run of the curve 2, which corresponds to the acoustically soft surface of the illuminated barrier side, is different from curves 1 and 3. Curve 2 is between these curves. The value of $G$ coefficient is approximately constant in the domain of very-low frequencies. The barrier has such properties due to the fact that the phase of the wave reflected from acoustically soft wall is shifted $180^\circ$ with respect to the incident wave. The interaction between them leads to the fact that the pressure on the barrier surface becomes equal to zero, and the pressure in immediate proximity to the barrier decreases. The general efficiency improvement is the result of it. But it is impossible to create an acoustically soft surface in the air environment in the wide range of frequencies. That is why curve 2 is rather a guide, than the model of real barriers.

Let us turn attention to curve 3 in Fig. 5.22. It is of interest to find out, how the value of coefficient $G$ changes, if the ideally absorbing material is only on some section of the surface of illuminated barrier side. The rest of the surface is acoustically rigid.

The calculations given in Fig. 5.23 provide an answer for such question. Here curve 1 corresponds to the case when the barrier surface is covered completely, curves 2, 3, 4—when it is covered partially. The absorber is on the upper part of the barrier surface, starting from its top edge ($y = h$). Curve 2 corresponds to the case when 50% of the surface of illuminated barrier side is covered, curve 3—25% and curve 4—10%. As we can see, when 50% of the surface is covered (curve 2), the noise-protective properties are improved in almost all the considered frequency range (curve 1 in Fig. 5.22). When 25% of the surface is covered (curve 3) the noise-protective properties are improved in the domain of relatively high frequencies (above 120 Hz). As to the case when 10% of the surface is covered (curve 4), the effect is slight.

The curves represented in Figs. 5.22 and 5.23 allow us to conclude: the noise-protective barrier properties can be improved considerably, when an absorber covers not less than 25–50% of the surface of barriers illuminated side.

The sound-protective properties of the barriers with an absorber were considered above, regardless of sound frequency. Such study is useful for the modeling of absorbing
coatings close to the real constructions (see, Fig. 5.21). Two of such models are shown in Fig. 5.24. We turn now to the first of them [Fig. 5.24(a)], which is the grating of quarter-wave resonators [19,20]. It is fair to assume that this model corresponds to a real barrier, which is shown in Fig. 5.21(b). Specific and normalized to the value $\rho c$ impedance of such construction is as follows [11]:

$$Z = \frac{r}{\rho c} + i\frac{\omega}{c}\tan^{-1}\left(\frac{\omega b}{c}\right).$$

(5.33)

where the coefficient $r$ determines the sound energy loses inside this construction, $b$—resonator depth. Note that expression (5.33) makes sense in the domain of frequencies, for which the dimension $d$ [Fig. 5.24(a)] is less than half sound wave length.

FIG. 5.23: Frequency dependency of energy coefficient $G$ for the cases, when the ideally absorbing material is on some part of the surface of illuminated barrier side, starting from its top edge $(y = h)$: 1—100% area of surface, 2—50%, 3—25%, 4—10%

FIG. 5.24: Absorbing barrier cover constructions; (1—acoustically rigid barrier surface, 2—resonator walls): a—quarter-wave resonators grating, b—Helmholtz resonator grating
The value of $b$ is equal to quarter sound wave length at the frequency of the main resonance of construction [Fig. 5.24(a)]. The pressure in the plane of inlet section of resonators is almost equal to zero (when $r = 0$ it is equal to zero). This construction simulates the situation close to the wave incidence on the acoustically soft surface.

The frequency dependencies of energy coefficient $G$ are shown in Fig. 5.25 for the cases, when absorbing cover is made in the form of quarter-wave resonators [Fig. 5.24(a)]. Curve 1 in the figure corresponds to the acoustically rigid screen surface, curves 2 and 3 correspond to the construction in the form of the resonator grating, which covers all the illuminated side of the barrier. Resonator depth is $b = 0.25$ m for curve 2, $b = 0.50$ m for curve 3. The resonance frequencies corresponding to such dimensions are approximately equal 330 Hz and 165 Hz. The normalized coefficient is set equal $r/(\rho c) = 0.1$. As we can see in Fig. 5.25, each of the quarter-wave resonators constructions reduces the value of coefficient $G$ in rather wide frequency range. The domain of these frequencies depends on resonator depth $b$.

We consider now the second model of absorbing cover in the form of the grating of Helmholtz resonators [Fig. 5.24(b)]. The grating covers all the surface of the illuminated barrier side. It is fair to assume that this model corresponds to a real barrier. The photo of this barrier is shown in Fig. 5.21(c). The parameters of a particular Helmholtz resonator depend on its geometrical dimensions: depth $b$ and width $a$ of resonator chamber, with diameter $2r_0$ and length $l$ of Helmholtz resonator throat (we assume that, the resonator throat cross section is a circle, the area of which is $\sigma = \pi r_0^2$, and the cross section of chamber—a square with an area $S = a^2$). The natural frequency of Helmholtz resonator in the air [21,22]

$$f_0 \approx \frac{93r_0}{a\sqrt{b(l + \Delta l)}}$$

(5.34)

**FIG. 5.25:** Frequency dependency of energy coefficient $G$ for the cases, when all the surface of illuminated barrier side is made in the form of a grating of quarter-wave resonators (loss coefficient $r/(\rho c) = 0, 1$): 1—barrier surface is acoustically rigid, 2—$b = 0.25$ m, 3—$b = 0.5$ m
The impedance of Helmholtz resonator (in fractions $\rho c$ and calculated per unit area of resonator chamber base)

$$Z = \frac{n \tilde{r}}{\rho c} + i \left( \frac{\omega b}{c} \right) \left( \frac{\omega n}{c} (l + \Delta l) \right),$$

(5.35)

here $\tilde{r}$—friction coefficient in the constructional elements of Helmholtz resonator (it is usually small net or fabric, near the resonator throat, $[22]$), $n = S/\sigma$. The value $\Delta l = \pi r_0/2$ is determined by the added mass at the ends of resonator throat.

We obtain the barrier models with different frequency characteristics of energy coefficient $G$ by changing the geometric and physical parameters of resonator construction. Figure 5.26 shows three calculation variants (curves 2, 3, and 4). Resonator throat diameter $2r_0$, chamber width $a$ and friction coefficient $\tilde{r}$ are constant. Variable parameters are the resonator chamber depth $b$ and throat length $l$. The chamber depth $b$ increases, and the throat length $l$ decreases with an increase of the curves 2, 3, 4 number. It leads to a decrease of equivalent elasticity and Helmholtz resonator mass. And the values of natural resonator frequencies change little, but its $Q$ factor changes substantially.

As we can see in Fig. 5.26, for curves 2, 3, 4, there is a substantial coefficient $G$ reduction for the range of frequencies in the vicinity of Helmholtz resonator natural frequency. Curve 2 corresponds to the narrowest band construction, and curve 4—construction with the widest range of the effective noise protection. According to the calculations, the extending of range of effective noise protection may cause the essential nonuniformity in coefficient $G$ frequency characteristic. Curve 4 is an example. If we know the frequency band of intense noise, the grating of resonators should be used with the resonance frequency in the middle of this frequency band.

![FIG. 5.26: Frequency dependency of energy coefficient $G$ for the cases, when the whole surface of illuminated barrier side is covered with the grating of Helmholtz resonators](image-url)

$G$ vs. frequency $f$ Hz

$a = 0.05$ m, $2r_0 = 4 \times 10^{-3}$ m, $\tilde{r} = 2$; 1—barrier surface is acoustically rigid, 2—$b = 0.08$ m, $l = 10 \times 10^{-3}$ m, $f_0 \approx 115$ Hz, 3—$b = 0.32$ m, $l = 2.5 \times 10^{-3}$ m, $f_0 \approx 88$ Hz, 4—$b = 0.40$ m, $l = 0.5 \times 10^{-3}$ m, $f_0 \approx 97$ Hz
5.5 NOISE-PROTECTIVE PROPERTIES OF THE BARRIERS, SITUATED ALONG THE BOTH SIDES OF A HIGHWAY

The sources of literature analysis (see, for example, [6,23] and bibliography in them) show, that, as a rule, the cases are considered, when there is a barrier only on the one wayside. There are a lot of situations in practice, when it is necessary to protect the area adjacent to the both sides of highway from transport noise and the barriers are along the both waysides. Figure 5.27 shows the photo, made by one of the authors in Italy. As we can see, sound-protective barriers are along the both waysides of highway. It is fair to assume that if the distance between barriers is divisible by some part of wave length, the resonance phenomena may appear. These phenomena can decrease the efficiency. This presumption is to a certain degree proved by work [24]. The fact of sharp acoustic pressure fluctuations in the zone between the barriers is proved experimentally (using the model) in this work.

5.5.1 Physical and Mathematical Models

Let us consider the following physical model of two barriers system [25], Fig. 5.28. Assume that two infinite (along the direction perpendicular to the figure plane) barriers (2 and 3) with height $h$ are on the infinite acoustically rigid surface 1, modeling the earth surface, in points $x = 0$ and $x = a$. The surface between the barriers $0 < x < a, y = 0$ simulates the highway. The sound source $S$, in the form of infinite pulsating band, is on this surface and it is parallel to the barriers. The position of the sound source is specified by coordinates $x = a_1, x = a_2$. The observation point is denoted by $M$. The semi-space $y > 0$ is filled with air with density $\rho$ and sound velocity $c$. The barriers surface exposed to the source $S$ (illuminated barriers surface), is characterized by complex conductivity $Y$. The barrier surface in the shade is always acoustically rigid. From a mathematical standpoint, the described physical model is equivalent to a plane problem.

FIG. 5.27: The highway section with sound-protective barriers along its both sides
5.5.2 Analytical Solution Construction

Let us introduce Cartesian coordinate system \((x, y)\) with point \(O\) as center and polar coordinate system \((r, \theta)\) with point \(O_1\) as center (Fig. 5.28). As it follows from Fig. 5.28, the relation between the coordinates of point \(M\) in the indicated coordinate systems is as follows:

\[
x = r \cos \theta + b, \quad y = r \sin \theta, \quad b = OO_1,
\]

\[
r = \sqrt{(x - b)^2 + y^2}, \quad \cos \theta = \frac{x - b}{r}.
\]

We partition all the domain of sound field existence into five domains according to the partial domains method:

1. domain \(I\) is the space between the barriers \(0 \leq x \leq a, 0 \leq y \leq h\);

2. domain \(II\) is bounded by the surfaces \((0 \leq x \leq a, 0 \leq y \leq h)\) and \((r = d, \theta_0 \leq \theta \leq \pi - \theta_0)\);

3. domain \(III\) is bounded by the surfaces \((a \leq x \leq d + b, y = 0), (x = a, 0 \leq y \leq h)\), and \((r = d, 0 \leq \theta \leq \theta_0)\);

4. domain \(IV\) is bounded by the surfaces \((-d - b) \leq x \leq 0, y = 0), (x = 0, 0 \leq y \leq h)\), and \((r = d, \pi - \theta_0 \leq \theta \leq \pi)\);

5. domain \(V\) is the exterior of semicircle with radius \(d = \sqrt{b^2 + h^2}\), i.e., \(r \geq d, 0 \leq \theta \leq \pi\).

The field in domain \(I\) should be represented in such form, as to fulfill the conditions: 1) boundary conditions on the surface of the road \((0 \leq x \leq a, y = 0)\), 2) impedance
conditions on the barriers surfaces, 3) matching conditions of the sound fields on the surface \((0 \leq x \leq a, y = h)\).

The conditions indicated above can be fulfilled, if the field in domain \(I\) is represented as superposition of normal waves of two parallel-plate waveguides with rigid surfaces of width \(a\) and \(h\), respectively. Thus,

\[
p_I = \sum_{n=0}^{\infty} A_n \cos \left( \beta_n^{(1)} x \right) \exp \left( i \gamma_n^{(1)} y \right) + \sum_{n=0}^{\infty} A_n^{(1)} \cos \left( \beta_n^{(1)} x \right) \exp \left( -i \gamma_n^{(1)} (y - h) \right) + \sum_{n=0}^{\infty} A_n^{(2)} \cos \left( \beta_n^{(2)} y \right) \exp \left( i \gamma_n^{(2)} x \right) + \sum_{n=0}^{\infty} A_n^{(3)} \cos \left( \beta_n^{(2)} y \right) \exp \left( -i \gamma_n^{(2)} (x - a) \right),
\]

(5.38)

where

\[
\beta_n^{(1)} = \frac{n \pi}{a}, \quad \gamma_n^{(1)} = \sqrt{k^2 - \left( \beta_n^{(1)} \right)^2}, \quad \beta_n^{(2)} = \frac{n \pi}{h},
\]

(5.39)

\[
\gamma_n^{(2)} = \sqrt{k^2 - \left( \beta_n^{(2)} \right)^2}, \quad k = \frac{\omega}{c}.
\]

Boundary condition on the road surface \((0 \leq x \leq a, y = 0)\) is as follows:

\[
\left. \frac{1}{i \omega \rho} \frac{\partial p_{II}}{\partial y} \right|_{y=0} = V(x),
\]

(5.40)

where

\[
V(x) = \begin{cases} 
\nu_0, & a_1 \leq x \leq a_2, \\
0, & 0 \leq x < a_1 \cup a_2 < x \leq a
\end{cases}
\]

the function, specifying the distribution of sound field particle velocity amplitude along the surface of highway. We substitute (5.38) into boundary condition (5.40), and we find the relation between the coefficients \(A_n\) and \(A_n^{(1)}\):

\[
A_n = A_n^{(1)} \exp \left( i \gamma_n^{(1)} h \right) + \frac{\rho c V_n}{a \delta_n \gamma_n^{(1)}},
\]

(5.41)

where \(\delta_0 = 1, \delta_n = 0, 5\) when \(n > 0\), and coefficients

\[
V_n = \int_0^a V(x) \cos \left( \beta_n^{(1)} x \right) dx.
\]

(5.42)

With allowance for (5.41), expression (5.38) for the field in domain \(I\) is as follows:
\[ p_I = \sum_{n=0}^{\infty} A_n^{(1)} \cos (\beta_n^{(1)} x) \left[ \exp (i \gamma_n^{(1)} (y + h)) + \exp (-i \gamma_n^{(1)} (y - h)) \right] \\
+ \sum_{n=0}^{\infty} \rho c V_n \cos (\beta_n^{(1)} x) \exp (-i \gamma_n^{(1)} y) \\
+ \sum_{n=0}^{\infty} A_n^{(2)} \cos (\beta_n^{(2)} y) \exp (i \gamma_n^{(2)} x) \\
+ \sum_{n=0}^{\infty} A_n^{(3)} \cos (\beta_n^{(2)} y) \exp (-i \gamma_n^{(2)} (x - a)). \] 

(5.43)

Consequently, we can fulfill the impedance conditions on the barrier surfaces due to the choice of \( A_n^{(2)} \) and \( A_n^{(3)} \) coefficients. The set of coefficients \( A_n^{(1)} \) allows us to fulfill the matching conditions of sound fields on the boundaries of domains \( I \) and \( II \).

Pressure field in domain \( II \) we write as follows

\[ p_{II} = \sum_{n=0}^{\infty} A_n^{(4)} \cos (\beta_n^{(1)} x) \exp (i \gamma_n^{(1)} (y - h)) \\
+ \sum_{n=0}^{\infty} \frac{A_n^{(5)} J_{\beta_n^{(1)}}(kr)}{J'_{\beta_n^{(1)}}(kd)} \cos (\beta_n^{(3)} (\theta - \theta_0)), \] 

(5.44)

where

\[ \beta_n^{(3)} = \frac{n\pi}{\pi - 2\theta_0}. \] 

(5.45)

The first sum here is the superposition of normal waves of parallel-plate waveguide of width \( h \). It provides the sound fields matching on the boundary of domains \( I \) and \( II \). The second sum is the set of particular solutions of Helmholtz equation in the domain in the form of the sector of a circle of radius \( d \). This sum allows us to fulfill the arbitrary conditions of fields matching on the interface of domains \( II \) and \( V \).

The expressions for the fields in domains \( III \) and \( IV \) also have the structure similar to Eq. (5.44):

\[ p_{III} = \sum_{n=0}^{\infty} A_n^{(6)} \cos (\beta_n^{(2)} y) \exp (i \gamma_n^{(2)} (x - a)) \\
+ \sum_{n=0}^{\infty} \frac{A_n^{(7)} J_{\beta_n^{(2)}}(kr)}{J'_{\beta_n^{(2)}}(kd)} \cos (\beta_n^{(4)} \theta), \] 

(5.46)

\[ p_{IV} = \sum_{n=0}^{\infty} A_n^{(8)} \cos (\beta_n^{(2)} y) \exp (-i \gamma_n^{(2)} x) \\
+ \sum_{n=0}^{\infty} \frac{A_n^{(9)} J_{\beta_n^{(2)}}(kr)}{J'_{\beta_n^{(2)}}(kd)} \cos (\beta_n^{(4)} (\theta - (\pi - \theta_0))), \] 

(5.47)

where
\[ \beta^{(4)}_n = \frac{n\pi}{\theta_0}. \] (5.48)

And the field in domain \( V \) is as follows:
\[ p_V = \sum_{n=0}^{\infty} A_n^{(10)} \frac{H^{(1)}_n(kr)}{H^{(1)}_n(kd)} \cos(n\theta). \] (5.49)

Let us write the system of functional equations. It determines the continuity conditions of sound field on the interface of domains \( I, II, III, IV, V \) and boundary conditions of barriers surfaces:
\[ p_I = p_{II}, \quad x = [0, a], \quad y = h, \] (5.50)
\[ \frac{\partial p_I}{\partial y} = \frac{\partial p_{II}}{\partial y}, \quad x = [0, a], \quad y = h, \] (5.51)
\[ \frac{1}{i\omega \rho} \frac{\partial p_I}{\partial x} = -Y p_I, \quad x = 0, \quad y = [0, h], \] (5.52)
\[ \frac{1}{i\omega \rho} \frac{\partial p_I}{\partial x} = Y p_I, \quad x = a, \quad y = [0, h], \] (5.53)
\[ \frac{\partial p_{III}}{\partial x} = 0, \quad x = a, \quad y = [0, h], \] (5.54)
\[ \frac{\partial p_{IV}}{\partial x} = 0, \quad x = 0, \quad y = [0, h], \] (5.55)
\[ p_{II} = p_V, \quad r = d, \quad \theta = [\theta_0, \pi - \theta_0], \] (5.56)
\[ p_{III} = p_V, \quad r = d, \quad \theta = [0, \theta_0], \] (5.57)
\[ p_{IV} = p_V, \quad r = d, \quad \theta = [\pi - \theta_0, \pi], \] (5.58)
\[ \frac{\partial p_V}{\partial r} = \begin{cases} \frac{\partial p_{III}}{\partial r}, & r = d, \quad \theta = [0, \theta_0], \\ \frac{\partial p_{II}}{\partial r}, & r = d, \quad \theta = [\theta_0, \pi - \theta_0], \\ \frac{\partial p_{IV}}{\partial r}, & r = d, \quad \theta = [\pi - \theta_0, \pi]. \end{cases} \] (5.59)

The standard procedure of functional system algebraization (5.50)–(5.59) leads to the infinite system of linear algebraic equations of the second kind in unknown coefficients \( A_n^{(1)}, \ldots, A_n^{(10)} \). As in the problems considered above, the field characteristics away from barriers edges are of interest. And simple reduction method can be used for solution of infinite system of equations \([10,11]\). As a rule, the number of unknowns in algebraic equations system is determined experimentally. We omit this standard analysis \([10,11]\). Note that we can obtain the results, which can be used in practice, if the total number of unknown complex coefficients is about 300.
Along with the amplitude of sound field distribution in the vicinity of barriers, the integral estimations of noise-protective barriers properties are of special interest. We use value \( G = W_D/W_0 \) as an integral criterion for noise-protective barrier properties estimation. It shows the part of sound energy, radiated by the source, which is in the zone of barriers geometric shadow. The expression for full power of the source per unit length is as follows:

\[
W_0 = \frac{1}{2} \int_0^a \Re [p(x, y = 0)] V(x) \, dx. \tag{5.60}
\]

The value of \( W_D \) is defined as sound field power flow, which goes through the arcs \( l_1 \) and \( l_2 \) of radius \( r_w \), situated in the zone of the barriers geometric shadow (see Fig. 5.29). The arcs \( l_1 \) and \( l_2 \) are bounded by ground from one side, and by rays 1 and 2 from the other side. These rays go through the barrier edges. The position of rays 1 and 2 is determined by angles \( \psi_1 \) and \( \psi_2 \). The values of angles \( \psi_1, \psi_2 \) and radius \( r_w \) allows us to find the range of variation of angle \( \theta \) of polar coordinate system along the arcs \( l_1 \) and \( l_2 \) (see Fig. 5.29). Consequently, we can find the sought-for value \( W_D \) as follows:

\[
W_D = \int_{l_1} I(l_1) \, dl + \int_{l_2} I(l_2) \, dl. \tag{5.61}
\]

Note that when the value of radius \( r_w (r_w \gg \lambda, r_w \gg h) \) is rather large the level of power of sound field, going through the arcs \( l_1 \) and \( l_2 \), almost does not change. In this case, the integration arc is the part of wave front, formed by the system “source-barriers-ground.”

**5.5.3 Calculation Data Analysis**

We set the following geometric parameters of the considered mathematical model (Fig. 5.28): barrier height \( h = 4 \) m, the width of highway \( a = 12 \) m, the band width, simulating the noise source \( S \) is defined by the coordinates \( a_1 = 1 \) m and \( a_2 = 3 \) m. The

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**FIG. 5.29:** Geometric shadow of the source behind the barriers
radius of arc, on which the power flow is calculated \( r_w = 2d \approx 7.2 \) m. The amplitude of sound source velocity \( v_0 = 1 \) m/s.

Figure 5.30 shows frequency dependencies of energy coefficient \( G \) for different acoustic properties of the illuminated barrier sides: curve 1 corresponds to acoustically rigid surface, curve 2—absorbing surface, curve 3—the grating of Helmholtz resonators covers the surface of illuminated barriers sides (the grating of resonators description was given in Section 5.4). We suppose that the acoustic properties of two barriers are the same and constant over the whole surface.

The parameters of a specific Helmholtz resonator depend on its geometric dimensions: the depth \( b_1 \) and width \( b_2 \) of resonator chamber, diameter \( 2r_0 \) and length \( l \) of Helmholtz resonator throat (we assume that the cross section of resonator throat is a circle with area \( \sigma_1 = \pi r_0^2 \), and the cross section of chamber is a square \( \sigma_2 = b_1 b_2 \) in area). If we know these values, we can calculate the natural Helmholtz resonator frequency and impedance (in fractions of \( \rho c \) per unit area of resonator chamber base \( \sigma_2 = b_1 b_2 \), \( n = \sigma_2 / \sigma_1 \)) using the Eqs. (5.34) and (5.35). Curve 3 in Fig. 5.30 was obtained for the following parameters of Helmholtz resonator: \( b_1 = 0.40 \) m, \( b_2 = 0.05 \) m, \( 2r_0 = 4 \times 10^{-3} \) m, \( l = 0.5 \times 10^{-3} \) m, \( \tilde{r} = 2 \). The natural frequency of Helmholtz resonator \( f_0 \approx 97 \) Hz for the given parameters. For curves 1 and 2, the value of specific (per unit area) and normalized to the value of \( \rho c \) impedance of illuminated barrier surface is \( Z = \infty \) and \( Z = 1 \), respectively.

Curve 1 in Fig. 5.30 shows the potentials of classical barrier with acoustically rigid surfaces. Four frequency domains can be marked on the given curve: the first—very-low frequencies domain to 30 Hz, the second—low frequencies from 30 Hz to 120 Hz, the third—medium frequencies from 120 Hz to 300 Hz, the fourth—high frequencies (above 300 Hz). In the first domain, the curve goes up with a decrease of frequency. The noise-protective barrier properties worsen. It is due to the small wave height of the barrier. In the second domain, we can see a series of abrupt and relatively large spikes of \( G \) coefficient. But its middle level decreases rapidly with an increase of frequency. There are also the

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**FIG. 5.30:** Frequency dependency of energy coefficient \( G \) for three variants of surface of illuminated barrier side: 1—acoustically rigid, 2—completely absorbing, 3—grating of Helmholtz resonators (resonance sound absorbers)
spikes in the third domain, but the general level is lower than in the second domain. In the fourth domain, curve 1 asymptotically approaches zero with further increase in frequency.

Curve 2 in Fig. 5.30 lies lower than curve 1 and it has not the spikes in the entire considered frequency domain. It shows the minimum possible levels of energy coefficient $G$, which can be reached, using the barriers with completely absorbing surfaces. In the domain of very-low frequencies (below 30 Hz), curve 2 comes up, but not so sharply, as curve 1 and it causes a significant improvement of noise-protective properties, in comparison to the barrier with acoustically rigid surfaces.

The run of curve 3, which corresponds to the case when the surface of the illuminated barrier side is covered with the grating of Helmholtz resonators, differs from the curves 1 and 2. As we can see, curve 3 is close to curve 1 in the domain of low frequencies, up to 55 Hz. Thus, the barrier with the grating of Helmholtz resonators has much in common with the barrier with acoustically rigid surfaces in the domain where the frequencies are below the resonance frequency of Helmholtz resonators. But in the domain of resonators resonance and up to 250 Hz we can see a significant improvement of noise-protective barrier properties. This is particularly significant, because there is an irregularity of coefficient $G$ for the barriers with acoustically rigid walls in this frequency domain. The resonators efficiency decreases for the frequencies higher than 250 Hz, i.e., for the frequencies which are higher than the frequency of resonators resonance. The curves 1 and 3 are almost the same. Thus, if we know the frequency band of intense noise, the gratings of resonators can be used, the resonance frequency of which is in the middle of this band.

Figure 5.30 shows the irregularity of curve 1 (especially at frequencies from 50 Hz to 120 Hz). We can assume that there are the resonance phenomena in partial domain $I$ (Fig. 5.28). They may be caused by standing waves between barriers 2 and 3. We will show that it is possible.

Taking into account that noise source $S$ is loaded on partial domain $I$, we should estimate its radiation specific impedance. Taking into account the normalization to $\rho c$, its calculation formula is as follows:

$$Z' = R' + iX' = \frac{1}{\rho c (a_2 - a_1) \nu_0} \int_{a_1}^{a_2} p(x, y = 0) dx. \quad (5.62)$$

Figure 5.31 shows the frequency dependency of real $R'$ (curve 1) and imaginary $X'$ (curve 2) parts of radiation impedance of source $S$. As we can see, 1 tends to unity with an increase of frequency, and curve 2 tends to zero. Such tendency is typical for any sound source. The positions of splashes along the frequency axis for curves 1 in Figs. 5.30 and 5.31 almost coincide. The sharp fluctuations of imaginary part $X'$ (curve 2) of radiation impedance correspond to the real part $R'$ splashes. The negative values $X'$ here change to positive values and vice versa. Such behavior of the value of $X'$ points out that we deal with a complex (multiresonant) sound source in the form of a system “radiating band—space between the barriers”. The sequence of resonances and anti-resonances is typical [26]. It is to be recalled that for the chosen time dependency $\exp (-i\omega t)$, the values of $X' < 0$ correspond to mass impedance, and the values of $X' > 0$—correspond to elastic
impedance. Such behavior of $R'$ and $X'$ points at the fact that there are the resonance phenomena in partial domain I and the connected with it splashes of radiated acoustic power of source $S$ and coefficient $G$.

Space distribution of sound field at resonance frequencies is of interest. Figure 5.32 (for the resonance frequencies) shows the pressure amplitude distribution normalized to maximum values between the barriers and in the vicinity of the highway with acoustically rigid barriers. For Figs. 5.3(a)–5.3(e) these frequencies are as follows: 31, 57, 71, 84, and 249 Hz. These are the frequencies at which curve 1 (Fig. 5.30) have sharp spikes. As we can see, there is clearly defined standing wave along $Ox$ axis for the indicated frequencies of sound source in the space between the barriers. Such distribution is typical for the pressure amplitude field of appropriate modes of parallel-plate waveguide with rigid boundaries. Function $\cos \left(\frac{n\pi x}{a}\right)$ determines pressure distribution in the cross section of $n$-th mode of parallel-plate waveguide of width $a$ with the rigid boundaries. Consequently, the standing waves shown in Figs. 5.32(a)–5.32(e) are typical for the modes enumerated $n = 2, 4, 5, 6, 9$, respectively. Figure 5.32(f) shows the situation, when there is no space resonance. The frequency of 64 Hz is between the resonance frequencies here.

Thus, the resonance phenomena appear for the model under investigation (Fig. 5.28) at certain frequencies of sound source, in the space between the barriers. We can observe the space resonance, when the source energy is in one of the waveguides modes. Consequently, there is no interference effect between the modes and, as a consequence, there is a sharp increase of energy flow in the waveguide.

One of the effective ways to remove the space resonance phenomena is to use the barriers with the absorbing walls. The space distributions of pressure amplitude, represented in Figs. 5.33 and 5.34, illustrates it. Here Fig. 5.33 corresponds to the situation with the ideally absorbing illuminated barrier sides, and Fig. 5.34—corresponds to the case when the barriers surface is covered with the grating of Helmholtz resonators.

It is easy to note the difference between the space structures of sound field, when comparing these graphs with Figs. 5.32(b), 5.32(c), and 5.32(e). The periodicity in
FIG. 5.32: Pressure amplitude distribution between the barriers and in the vicinity of highway; the barriers surfaces are acoustically rigid \((Y = 0)\): (a)—\(f = 31\) Hz, (b)—\(f = 57\) Hz, (c)—\(f = 71\) Hz, (d)—\(f = 84\) Hz, (e)—\(f = 249\) Hz, (f)—\(f = 64\) Hz

Pressure amplitude variation in Fig. 5.34 is not very noticeable. The resonance field structure is absent in Fig. 5.33, which corresponds to the completely absorbing barrier surfaces.

The resonance phenomena reduction, which appear in the space between the barriers, can be gained by covering the barrier walls with sound-absorbing covers. As may be supposed, the slope of the barriers (may be small) away from the highway is a simpler way.
FIG. 5.33: Pressure amplitude distribution between the barriers and in the vicinity of highway; ideally absorbing illuminated barriers surfaces \((Y = 1)\): (a)—\(f = 71\) Hz, (b)—\(f = 249\) Hz

FIG. 5.34: Pressure amplitude distribution between the barriers and in the vicinity of highway; illuminated barriers surfaces covered with the grating of Helmholtz resonators \((Y = 1/\bar{Z})\): (a)—\(f = 57\) Hz, (b)—\(f = 71\) Hz

If we look closely at photo in Fig. 5.27, we can note, that the barriers are gently sloped. In theory, the slope should weaken the resonance phenomena in the space between the barriers. To make the quantitative estimation of the slope influence on the coefficient \(G\) and sound field distribution between the barriers, it is necessary to develop the corresponding physical and mathematical models and to perform all the necessary calculations. Let us show how it can be done.

5.5.4 Sloped Barriers

If the barriers are sloped, the mathematical model geometry (Fig. 5.28) changes and takes the form as in Fig. 5.35. Let us introduce one more polar coordinate system \((R, \psi)\) with point \(O_2\) as centre, to analyze this problem. Barriers slope is defined by angle \(\psi_0\). The
slope angle $\psi_0$ is rather small in real constructions. To simplify the mathematical model it is necessary to represent the sloped barrier as the construction with triangular profile. In Fig. 5.35 such barriers 1 and 2 are shown as blacken triangles.

The new problem geometry leads to the change of partial domains form and to the occurrence of one more partial domain. This domain is denoted as $I'$ in Fig. 5.35. Thus, we partition the whole space of sound field existence into six domains:

1. domain $I'$ is bounded by the surfaces $(0 \leq x \leq a, y = 0)$ and $(R = d_1, \pi/2 - \psi_0 \leq \psi \leq \pi/2 + \psi_0)$, where $d_1 = O_2O = b/\sin \psi_0, b = OO_1$;

2. domain $I$ is bounded by the barriers and the surfaces $(R = d_1, \pi/2 - \psi_0 \leq \psi \leq \pi/2 + \psi_0)$ and $(R = d_2, \pi/2 - \psi_0 \leq \psi \leq \pi/2 + \psi_0)$, where $d_2 = O_2A = h + d_1$;

3. domain $II$ is bounded by the surfaces $(R = d_2, \pi/2 - \psi_0 \leq \psi \leq \pi/2 + \psi_0)$ and $(r = d, \theta_0 \leq \theta \leq \pi - \theta_0)$, where $d = \sqrt{(h \cos \psi_0)^2 + (b + h \sin \psi_0)^2}$;

4. domain $III$ is bounded by the surfaces $(a + h \sin \psi_0 \leq x \leq d + b, y = 0), (x = a + h \sin \psi_0, 0 \leq y \leq h \cos \psi_0)$, and $(r = d, 0 \leq \theta \leq \theta_0)$;

5. domain $IV$ is bounded by the surfaces $(- (d - b) \leq x \leq -h \sin \psi_0, y = 0), (x = -h \sin \psi_0, 0 \leq y \leq h \cos \psi_0)$, and $(r = d, \pi - \theta_0 \leq \theta \leq \pi)$;

6. domain $V$ is the exterior of semicircle of radius $d$, i.e., $r \geq d, 0 \leq \theta \leq \pi$. 

**FIG. 5.35:** Geometry of barriers with slope system: 1—ground, 2 and 3—barriers
Let us write the expressions for the sound fields in the indicated partial domains.

\[ p_I = \sum_{n=0}^{\infty} B_n^{(1)} \cos (\alpha_n x) \exp (i \eta_n y) + \sum_{n=0}^{\infty} B_n^{(2)} \frac{J_{\beta_n^{(1)}} (k R)}{J_{\beta_n^{(1)}} (k d_1)} \cos \left( \beta_n^{(1)} (\psi - \psi_{00}) \right), \]

where \( \alpha_n = n \pi/a, \eta_n = \sqrt{k^2 - (\alpha_n)^2}, \psi_{00} = \pi/2 - \psi_0, \beta_n^{(1)} = n \pi/2 \psi_0; \)

\[ p_I = \sum_{n=0}^{\infty} A_n^{(1)} \frac{H^{(1)}_{\beta_n^{(1)}} (k R)}{H^{(1)}_{\beta_n^{(1)}} (k d_1)} \cos \left( \beta_n^{(1)} (\psi - \psi_{00}) \right) + \sum_{n=0}^{\infty} A_n^{(2)} \frac{J_{\beta_n^{(1)}} (k R)}{J_{\beta_n^{(1)}} (k d_2)} \cos \left( \beta_n^{(1)} (\psi - \psi_{00}) \right), \]

\[ p_{II} = \sum_{n=0}^{\infty} A_n^{(3)} \frac{H^{(1)}_{\beta_n^{(1)}} (k R)}{H^{(1)}_{\beta_n^{(1)}} (k d_2)} \cos \left( \beta_n^{(1)} (\psi - \psi_{00}) \right) + \sum_{n=0}^{\infty} A_n^{(4)} \frac{J_{\beta_n^{(1)}} (k r)}{J_{\beta_n^{(1)}} (k d)} \cos \left( \beta_n^{(3)} (\theta - \theta_0) \right), \]

where \( \beta_n^{(3)} = n \pi/(\pi - 2 \theta_0); \)

\[ p_{III} = \sum_{n=0}^{\infty} A_n^{(5)} \cos \left( \beta_n^{(2)} y \right) \exp \left( i \gamma_n^{(2)} (x - (a + h \sin \psi_0)) \right) + \sum_{n=0}^{\infty} A_n^{(6)} \frac{J_{\beta_n^{(4)}} (kr)}{J_{\beta_n^{(4)}} (kd)} \cos \left( \beta_n^{(4)} \theta \right), \]

\[ p_{IV} = \sum_{n=0}^{\infty} A_n^{(7)} \cos \left( \beta_n^{(2)} y \right) \exp \left( -i \gamma_n^{(2)} (x + h \sin \psi_0) \right) + \sum_{n=0}^{\infty} A_n^{(8)} \frac{J_{\beta_n^{(4)}} (kr)}{J_{\beta_n^{(4)}} (kd)} \cos \left( \beta_n^{(4)} (\theta - (\pi - \theta_0)) \right), \]

where \( \beta_n^{(2)} = n \pi/h \cos \psi_0, \gamma_n^{(2)} = \sqrt{k^2 - (\beta_n^{(2)})^2}, \beta_n^{(4)} = n \pi/\theta_0; \)

\[ p_V = \sum_{n=0}^{\infty} A_n^{(9)} \frac{H^{(1)}_{\beta_n^{(4)}} (kr)}{H^{(1)}_{\beta_n^{(4)}} (kd)} \cos \left( n \theta \right). \]

We omit the following analytic transformations: the sound fields matching conditions at the boundaries of partial domains and analytical solution construction. It is not different from the data given above. We turn our attention now to the numerical results analysis. We need to find out how the barrier slope helps to improve their noise–protective properties?
Figure 5.36 shows the calculated $G$ coefficient values depending on the frequency for two cases: without barriers slope and with their slope equal to ten degrees. Curve 1 (barriers without a slope) shows an irregularity in the frequency band from 40 to 120 Hz and there are the big sharp splashes. These irregularities are weakened to a certain extend when the barrier is sloped (curve 2). The value of the splashes near the frequencies 57 Hz and 71 Hz decreases, and the splashes almost disappear in the vicinity of frequencies 84 Hz and 98 Hz. The estimates suggest that, curve 2 irregularity decreases considerably at the higher frequencies comparing to curve 1. Thus, the slope of the barriers improves the noise–protective properties of the barriers.

5.6 SOUND-PROTECTIVE BARRIERS PROPERTIES, SITUATED ALONG THE STREETS

Intra-urban traffic is the most widely-spread and intense noise source in the cities [24,27–30]. The traffic flow contribution into acoustic contamination of the streets is from 60% to 80% depending on the traffic area covering (asphalt or block stone), the rate of traffic, the concentration of traffic [30]. The people living near the traffic areas are affected by high noise level. In the towns, the noise protection of sidewalks and windows in the buildings, situated along the streets, is different from the noise protection outside the town. Residential accommodations outside the towns can be situated rather far from highways. Consequently, the problem of sound protection in a town requires detailed consideration [24,28–30]. The articles [24,28] deserve special attention. These articles describe the model experiments, which were conducted to estimate the possibility to reduce the transport noise level using classical rigid barriers, situated along the sidewalks. Unfortunately, there is no absolute answer in these articles for a question whether it is advisable to mount the barriers along the streets in the towns. It is shown in these articles, that frequency dependencies of sound pressure of traffic noise have oscillating character with the differential in pressure, up to 60 dB.

The analysis of the barriers effectiveness in an urban setting is conducted in the given paragraph, basing on the constructed physical and mathematical models [31].

FIG. 5.36: Energy coefficient $G$ dependency on frequency: 1—barriers without a slope (dependency curve in Fig. 5.30), 2—sloped barriers (angle $\psi_0 = 10^\circ$)
5.6.1 Physical and Mathematical Models

Let us consider the physical model of typical urban street with close development. The sound barriers are situated along the both sides of the street Fig. 5.37. In this figure, acoustically rigid surface \( y = 0, \; x = [0, a + b + c] \) simulates the surfaces of sidewalk and traffic area. The street is bounded with buildings. The faces of these buildings are described as acoustically rigid surfaces \( x = 0, \; y = [0, H] \cup x = [-\infty, 0], \; y = H \) and \( x = a + b + c, \; y = [0, H] \cup x = [a + b + c, \infty], \; y = H \). The models geometry does not depend on the direction, which is perpendicular to the figure plane. Two barriers of height \( h \) are in the planes \( x = b \) and \( x = a + b \). The surfaces between the buildings and the barriers \( x = [0, b], \; y = 0 \) and \( x = [a + b, a + b + c], \; y = 0 \) simulates the sidewalk. The surface between the barriers \( x = [b, a + b], \; y = 0 \) is the traffic area. There is a sound source \( S \) as an infinite pulsing band of width \( a_2 - a_1 \) on this surface. The source simulates the noise of traffic flow. The semi-space \( y > 0 \) is filled with area with density \( \rho \) and sound velocity \( c \). The barriers surfaces, faced to the source \( S \) (illuminated barriers surface), are characterized by complex conductivity \( Y \). The shade side of the barriers is always acoustically rigid. It should be reminded that the conductivity of ideal acoustically rigid surface is equal to zero \((Y = 0)\). And the conductivity of ideally absorbing surface (normalized to the value \( \rho c) \) \( Y = 1 \).

FIG. 5.37: Model geometry: 1—sidewalk surface and traffic area surface, 2, 3—barriers, 4, 5—buildings, 6—windows
5.6.2 Analytical Solution Construction

Let us introduce the Cartesian coordinate system \((x, y)\) with point \(O\) as center and polar coordinate system \((r, \theta)\) with point \(O_1\) as center (Fig. 5.37). As it follows from the figure, the relation between the point \(M\) coordinates in the given coordinate systems is as follows:

\[
x = r \cos \theta + d, \quad y = r \sin \theta + H, \tag{5.63}
\]

\[
r = \sqrt{(x - d)^2 + (y - H)^2}, \quad \cos \theta = (x - d)/r, \tag{5.64}
\]

where \(d = b + a/2\).

We partition all the field existence domain into six domains (Fig. 5.37):

1. domain \(I\) is the space between barriers 2 and 3 \(b \leq x \leq a + b, 0 \leq y \leq h\);
2. domain \(II\) is the space between building 4 and barrier 2 to the left of the traffic area \(0 \leq x \leq b, 0 \leq y \leq h\);
3. domain \(III\) is the space between building 5 and barrier 3 to the right of the traffic area \(a + b \leq x \leq a + b + c, 0 \leq y \leq h\);
4. domain \(IV\) is the space between the buildings \(0 \leq x \leq a + b + c, h \leq y \leq H\);
5. domain \(V\) is bounded by surfaces \(0 \leq x \leq a + b + c, y = H\) and \(r = d, 0 \leq \theta \leq \pi\);
6. domain \(VI\) \(r \geq d, 0 \leq \theta \leq \pi\).

The field in domain \(I\), is represented in the form that fulfills the following conditions:

boundary conditions on the traffic area surface \(b \leq x \leq a + b, y = 0\), impedance conditions on the barriers surfaces, sound fields matching conditions on the surface \(b \leq x \leq a + b, y = H\). These conditions can be fulfilled, if the field in domain \(I\) is represented as superposition of normal waves of two parallel-plate waveguides with the rigid surfaces of width \(a\) and \(h\), respectively. Consequently,

\[
p_I = \sum_{n=0}^{\infty} A_n \cos \left(\beta_n^{(1)} (x - b)\right) \exp \left(i \gamma_n^{(1)} y\right)
+ \sum_{n=0}^{\infty} A_n^{(1)} \cos \left(\beta_n^{(1)} (x - b)\right) \exp \left(-i \gamma_n^{(1)} (y - h)\right)
+ \sum_{n=0}^{\infty} A_n^{(2)} \cos \left(\beta_n^{(2)} y\right) \exp \left(i \gamma_n^{(2)} (x - b)\right)
+ \sum_{n=0}^{\infty} A_n^{(3)} \cos \left(\beta_n^{(2)} y\right) \exp \left(-i \gamma_n^{(2)} (x - a - b)\right), \tag{5.65}
\]

where
Boundary condition on the traffic area surface \((b \leq x \leq a + b, y = 0)\) is as follows:

\[
\left| \frac{1}{i \omega \rho} \frac{\partial p_I}{\partial y} \right|_{y=0} = V(x),
\]

where

\[
V(x) = \begin{cases} 
\nu_0, & a_1 \leq x \leq a_2, \\
0, & b \leq x < a_1 \cup a_2 < x \leq b + a
\end{cases}
\]

the function, setting the distribution of particle velocity amplitude on the source \(S\) surface and traffic area surface. Substituting (5.65) into boundary condition (5.67), we find the relation between coefficients \(A_n\) and \(A_n^{(1)}\):

\[
A_n = A_n^{(1)} \exp \left( i \gamma_n^{(1)} y \right) + \frac{\rho c V_n}{a \delta_n \gamma_n^{(1)}},
\]

where \(\delta_0 = 1, \delta_n = 0, 5\) when \(n > 0\), and coefficients

\[
V_n = \int_0^a V(x) \cos \left( \beta_n^{(1)} x \right) dx.
\]

Taking into account Eq. (5.68), expression (5.65) for the field in domain \(I\) changes to

\[
p_I = \sum_{n=0}^{\infty} A_n^{(1)} \cos \left( \beta_n^{(1)} (x - b) \right) \left[ \exp \left( i \gamma_n^{(1)} (y + h) \right) + \exp \left( -i \gamma_n^{(1)} (y - h) \right) \right]
\]

\[
+ \sum_{n=0}^{\infty} \frac{\rho c V_n}{a \delta_n \gamma_n^{(1)}} \cos \left( \beta_n^{(1)} (x - b) \right) \exp \left( i \gamma_n^{(1)} y \right)
\]

\[
+ \sum_{n=0}^{\infty} A_n^{(2)} \cos \left( \beta_n^{(2)} y \right) \exp \left( i \gamma_n^{(2)} (x - b) \right)
\]

\[
+ \sum_{n=0}^{\infty} A_n^{(3)} \cos \left( \beta_n^{(2)} y \right) \exp \left( -i \gamma_n^{(2)} (x - a - b) \right).
\]

We can fulfill the impedance conditions on the barriers surfaces due to the choice of coefficients \(A_n^{(2)}\) and \(A_n^{(3)}\). The set of coefficients \(A_n^{(1)}\) allows us to satisfy the matching conditions of sound fields on the boundaries of domains \(I\) and \(II\). We write the pressure field in domain \(II\) as follows:
\[ p_{II} = \sum_{n=0}^{\infty} A_n^{(4)} \cos (\beta_n^{(3)} x) \exp \left( i\gamma_n^{(3)} y \right) \]
\[ + \sum_{n=0}^{\infty} \tilde{A}_n^{(4)} \cos (\beta_n^{(3)} x) \exp \left( -i\gamma_n^{(3)} (y - h) \right), \tag{5.70} \]

where \( \beta_n^{(3)} = n\pi/b, \gamma_n^{(3)} = \sqrt{k^2 - \left( \beta_n^{(3)} \right)^2} \). Boundary condition:
\[ \frac{\partial p_{II}}{\partial y} = 0, \quad x = [0, b], \quad y = 0, \tag{5.71} \]

we find the relation \( \tilde{A}_n^{(4)} = A_n^{(4)} \exp \left( -i\gamma_n^{(3)} h \right) \). Taking into account this equality, we write expression (5.70) as follows:
\[ p_{II} = \sum_{n=0}^{\infty} A_n^{(4)} \cos (\beta_n^{(3)} x) \frac{\exp \left( i\gamma_n^{(3)} (y + h) \right) + \exp \left( -i\gamma_n^{(3)} (y - h) \right)}{1 + \exp \left( 2i\gamma_n^{(3)} h \right)}. \tag{5.72} \]

Likewise the field in domain \( III \) can be written as follows:
\[ p_{III} = \sum_{n=0}^{\infty} A_n^{(5)} \cos \left( \beta_n^{(3)} (x - a - b) \right) \frac{\exp \left( i\gamma_n^{(3)} (y + h) \right) + \exp \left( -i\gamma_n^{(3)} (y - h) \right)}{1 + \exp \left( 2i\gamma_n^{(3)} h \right)}. \tag{5.73} \]

The field in domain \( IV \), is similar to expression (5.70), and is as follows:
\[ p_{IV} = \sum_{n=0}^{\infty} A_n^{(6)} \cos \left( \beta_n^{(4)} x \right) \exp \left( i\gamma_n^{(4)} (y - h) \right) \]
\[ + \sum_{n=0}^{\infty} \tilde{A}_n^{(7)} \cos \left( \beta_n^{(4)} x \right) \exp \left( -i\gamma_n^{(4)} (y - H) \right), \tag{5.74} \]

where \( \beta_n^{(4)} = n\pi/(a + b + c), \gamma_n^{(4)} = \sqrt{k^2 - \left( \beta_n^{(4)} \right)^2} \).

We write the field in domain \( V \) as follows:
\[ p_V = \sum_{n=0}^{\infty} A_n^{(8)} \cos \left( \beta_n^{(4)} y \right) \exp \left( i\gamma_n^{(4)} (x - H) \right) + \sum_{n=0}^{\infty} A_n^{(9)} \frac{J_n (kr)}{J'_n (kd)} \cos (n\theta). \tag{5.75} \]

The first sum here is the superposition of the normal waves of parallel-plate waveguide of width \((a + b + c)\), it provides sound fields matching on the boundaries of domains \( IV \) and \( V \). The second sum is the totality of particular solutions of Helmholtz equation in the semi-circular domain of radius \( d \). This sum allows us to fulfill the fields matching conditions at the interface of domains \( V \) and \( VI \). The field in domain \( VI \) is as follows:
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\[ p_{VI} = \sum_{n=0}^{\infty} A_n^{(10)} \frac{H_n^{(1)}(kr)}{H_n^{(1)}(kd)} \cos(n\theta). \]  

(5.76)

We write the system of functional equations. It determines the sound field continuity conditions at the interface of domains I, II, III, IV, V, VI and boundary conditions on the barriers surfaces:

\[ p_I = p_{IV}, \quad x = [b, a + b], \quad y = h, \]  

(5.77)

\[ p_{II} = p_{IV}, \quad x = [0, b], \quad y = h, \]  

(5.78)

\[ p_{III} = p_{IV}, \quad x = [a + b, a + b + c], \quad y = h, \]  

(5.79)

\[ \frac{\partial p_{IV}}{\partial y} = \begin{cases} \frac{\partial p_{II}}{\partial y}, & x = [b, a + b], \quad y = h, \\ \frac{\partial p_I}{\partial y}, & x = [0, b], \quad y = h, \\ \frac{\partial p_{III}}{\partial y}, & x = [a + b, a + b + c], \quad y = h, \end{cases} \]  

(5.80)

\[ p_{IV} = p_{V}, \quad x = [0, a + b + c], \quad y = H, \]  

(5.81)

\[ \frac{\partial p_{IV}}{\partial y} = \frac{\partial p_{V}}{\partial y}, \quad x = [0, a + b + c], \quad y = H, \]  

(5.82)

\[ p_{V} = p_{VI}, \quad r = d, \quad \theta = [0, \pi], \]  

(5.83)

\[ \frac{\partial p_{V}}{\partial r} = \frac{\partial p_{VI}}{\partial r}, \quad r = d, \quad \theta = [0, \pi], \]  

(5.84)

\[ \frac{1}{i\omega \rho} \frac{\partial p_I}{\partial x} = -Y p_I, \quad x = b, \quad y = [0, h], \]  

(5.85)

\[ \frac{1}{i\omega \rho} \frac{\partial p_I}{\partial x} = Y p_I, \quad x = a + b, \quad y = [0, h], \]  

(5.86)

Then we obtain an infinite system of linear algebraic equations of the second kind in unknown coefficients \( A_n^{(1)}, \ldots, A_n^{(10)} \). We solve this system by the simple reduction method. The field characteristics away from the barriers edges are of main interest in our problem. Field properties at a distance from barrier edges are of interest in our problem. The number of unknowns in the system of algebraic equations are determined by experiment. The results appropriate for practical application can be obtained in this problem if the total amount of unknown complex coefficients is 350–600 for the frequency range from 30 Hz to 1000 Hz.

5.6.3 Calculated Data Analysis

Let us set the following geometric parameters of the considered model (Fig. 5.37): barrier height \( h = 4 \) m, traffic area width \( a = 12 \) m, sidewalks width at the left and at the right of the sidewalk is \( b = c = 4 \) m, the band, simulating the sound source \( S \), is specified by
the coordinates \( a_1 = 5 \text{ m} \) and \( a_2 = 7 \text{ m} \). Velocity amplitude of sound source \( v_0 = 1 \text{ m/s} \).

The height of five-storied building is \( H = 19 \text{ m} \). The height of windows is taken equal to 2; 5.5; 9; 12.5; 16 m, respectively. Air properties: density \( \rho = 1.27 \text{ kg/m}^3 \), sound velocity \( c = 333.3 \text{ m/s} \).

Let us see how the levels of sound pressures change in some characteristic points of space between the buildings, depending on whether the barriers are absent or present (barriers surface is acoustically rigid). As a criterion, we use the ratio from work [24], namely

\[
\Delta L = 20 \log \left( \frac{|p_1(x_j, y_j)|}{|p_2(x_j, y_j)|} \right) \text{ dB},
\]

where \( |p_1| \) — pressure amplitude in the point with coordinates \( x_j, y_j \) without the barriers, \( |p_2| \) — pressure in the same point with barriers.

We should simplify the original problem, and exclude domains \( I, II, \) and \( III \) (see Fig. 5.37), when performing the calculations for the case without barriers. But it can be omitted. It is enough to assume that the barriers height \( h \to 0 \) to satisfy the conditions \( h \neq 0 \) and \( h/\lambda \ll 1 \), where \( \lambda \) — the wave length. We compared the data of calculations performed for such two models. The comparison showed, that when \( h/\lambda \leq 0.1 \) the results agree up to fourth decimal.

Figure 5.38 shows the frequency dependencies of value \( \Delta L \) for two points on the face of the left building (see Fig. 5.37). When we compare these dependencies with similar dependencies from work [24], it may be noted that there is a good qualitative and quantitative correspondence. The main feature of these dependencies is their oscillating character with the pressure drops, reaching 50–60 dB. If we average these dependencies (even visually) we can see that the barriers help to reduce the pressure on some intervals of frequency domains (about 5–10 dB) in the points under consideration.

It is not possible to estimate the integrated efficiency of barriers, using expression (5.87), because it is of local character. We assume that the sound protection of the windows in buildings is the most important, because the noise penetrates into the offices and apartments through the windows. We introduce some parameter to estimate the noise effect on the windows surface. We introduce this parameter as the mean sound pressure on the windows surface, divide by volume velocity of the source

\[
J = \frac{1}{S_0'v_0(a_2-a_1)} \int_{S'} |p| dS',
\]

the integration here is performed on windows surface \( S' \) (computation is performed per unit windows width for plane problem), \( S_0' \) — windows area, \( v_0(a_2-a_1) \) — volume particle velocity of the source. The dependency of \( J \) parameter on frequency for the windows on different floors is of primary interest.

At first, we consider the case when there are no barriers. Figures 5.39(a)–5.39(e) shows frequency dependencies \( J \) for the windows from the ground-floor [Fig. 5.39(a)] to the fifth floor [Fig. 5.39(e)] of the building to the left of the barriers, and in
Figs. 5.39(f)–5.39(k), respectively for the building to the right of the barriers. As we can see, frequency dependencies of $J$ parameter have a lot of sharp relatively narrow-band spikes of large amplitude against relatively low (on the average) parameter $J$ value. The character of frequency dependency of $J$ vary only slightly from one floor to another. It ought to be noted that there is some reduction of spike values of function $J$ for the fourth and the fifth floors. The value of spikes, which are beyond the diagram boundaries and their frequencies are indicated near the spikes in fractional form. The upper number indicates the frequency, and the bottom number indicates the maximum value of spike amplitude.

What is the reason of appearance of such sharp spikes in $J$ parameter frequency dependency? To answer this question, we will we perform the computations of pressure field in the space between the buildings at the frequencies, corresponding to some of these spikes. Figures 5.40(a)–5.40(d) shows these data. It follows from their analysis:

1. the spikes in frequency dependencies of value of $J$ are caused by resonance phenomena; these phenomena appear in the area, bounded by acoustically rigid faces of buildings and by the surface of traffic area and of sidewalks; visually the field distribution is as the pulled bands vertically alternating;
FIG. 5.39: Frequency dependency of $J$ parameter without barriers: the left column—the building to the left of the traffic area, right column—the building to the right of the traffic area; the floors are numbered from the top down
2. the resonances may appear in the cases when the ratio of distance between the buildings to the wave length is \( n/2 \), where \( n = 1, 2, 3, \ldots \) (the scale of values \((b + a + c)/\lambda\) is given in Figs. 5.39(a) and 5.39(f), at the top of the graphs, here \( \lambda \)—wave length in the air); the surfaces of buildings faces are acoustically rigid, and for this reason, the pressure loops appear on them, i.e., the zones of maximum pressure value [21];

3. the area between the buildings is an open resonator; consequently, the resonance field structure “gets fuzzy” near the upper (open) part of the resonator; the pressure drops, resulting in the weakening of the spike values of function \( f \) for the fourth and the fifth floors (it was mentioned above).

It should be mentioned that not all the potentially-enable resonances can be realized to the full extent. Their efficiency depends largely on the source \( S \) wave dimensions and its position on the traffic area. In other words, the source is in good agreement with the area volume between the faces of the buildings only at some resonance frequencies. Consequently, on the curves in Fig. 5.39 we can see not only the large spikes (with relatively good agreement), but also a lot of small spikes, with poor agreement.
Is it of interest for you to observe the field distribution on the non-resonance frequencies? For this purpose we turn to Fig. 5.40(d), where the pressure field distribution is shown between the buildings faces at frequency 70 Hz. There are no resonance spikes here. As we can see, the field distribution is of rather “fuzzy” character. Mean value of the pressure amplitude decreases gradually, when moving from the source to the upper floors.

Let us consider now the situation, when there are the barriers with acoustically rigid surfaces along the street (Fig. 5.37). Figure 5.41 shows frequency dependencies of $J$ parameter for the same calculations variants, as in Fig. 5.39.

When comparing the graphs in Figs. 5.39 and 5.41, it may be noted that:

1. the mean level of parameter $J$ for the lower floors, and especially for the first floor, decreases considerably (about 4–6 dB); but its frequency dependency has a lot of spikes;

2. the levels of the most spike values of parameter $J$ decrease, at least, starting from the second floor, and above;

3. resonance spike at frequency 83 Hz almost does not change its high level.

Thus, the application of acoustically rigid barriers leads to some decrease of mean level of sound pressure on the windows of the lower floors and to decrease spikes values of the pressure on the upper floors. Nevertheless, there is only little decrease of the value of parameter $J$. The question arises, why the parameter $J$ almost does not change? We assume that the application of the barriers creates the conditions for the additional series of resonance frequencies appearance. The additional spikes appear in the frequency dependency of parameter $J$. The barriers are situated parallel to each other and parallel to the buildings faces. It leads to the formation of other three open resonators—one between the barriers and two between the barriers and the faces surfaces. To be sure in this fact, we turn to Fig. 5.42. We start with Fig. 5.42(a), which shows the pressure field distribution at frequency 45 Hz. We can see here the resonance phenomena between the barriers and the building faces. The condition $b/\lambda = c/\lambda = 1/2$, is fulfilled at this frequency, and we can see a pronounced spike, Fig. 5.41(a), in frequency dependency. Now we turn attention to the fact that the resonance is more pronounced between the left barrier and the left face of the building, than between the right barrier and the right face. It is due to the fact that the source is closer to the left barrier.

The resonances can appear at higher frequencies, when the following condition is fulfilled $b/\lambda = c/\lambda = m/2$, where $m = 1, 2, 3, ...$. Figures 5.41(a) and 5.41(e) shows the additional scales of values $a/\lambda, b/\lambda, (a + b + c)/\lambda$ in the upper part of the graphs. These scales allow us to estimate the frequencies of potentially-enable resonances between the barriers and the faces of the buildings.

Figure 5.42(b) illustrates the resonance between the barriers at frequency 58 Hz. It corresponds to the wave dimension $a/\lambda = 2$, (general condition of the resonance appearance between the barriers is as follows $a/\lambda = m/2$, where $m = 1, 2, 3, ...$).

Figure 5.42(c) is very interesting. Pressure field distribution is shown here, when the resonance phenomena appear simultaneously at one frequency (in this case at frequency...
FIG. 5.41: Frequency dependency of parameter $J$ with acoustically rigid barriers: left column—the building to the left of the traffic area, right column—the building to the right of the traffic area; 1–5—floors numbering (from the top down)
FIG. 5.42: Pressure amplitude field in the space between the buildings. When the frequency of the sound source is (barriers are acoustically rigid): (a)—45 Hz, (b)—58 Hz, (c)—83 Hz, (d)—249 Hz

83 Hz), in all the domains (between the buildings, between the barriers and between the barriers and buildings). It corresponds to the case, when \((b + a + c)/\lambda = 5\), \(a/\lambda = 3\) and \(b/\lambda = c/\lambda = 1\). It seems that the sound field “ignores” the barrier [compare Figs. 5.40(b) and 5.42(c)]. Such “global” resonance occupies all the area between the buildings. That is why this resonance dominates on all the floors (see Fig. 5.41) of both buildings. The situation is similar at the frequency 249 Hz [see Fig. 5.42(d)], where there is also a “global” resonance. But it is not so efficient here, as at frequency 83 Hz. This is due to the fact that at frequency 249 Hz the wave size of sound source \((a_2 - a_1)/\lambda = 1.54\), i.e., it is greater than wave length. The source becomes directional and it is unable to insonify uniformly all the space between the buildings. Figure 5.42(e) shows that the relatively narrow cone of acoustic energy comes from the source. It expands and “gets fuzzy.” That is why the spike is clearly seen only in the area of the first floor, at frequency 249 Hz. The spike level above the first floor is rather small.

In summary we can conclude: the application of barriers with acoustically rigid surfaces on the urban streets to protect the buildings and sidewalks from traffic noise may be ineffective because of the resonances between the buildings, between the barriers and
between the barriers and the faces of the buildings. It is fair to assume that the barriers with sound absorbing walls, can protect the sidewalks and windows in the buildings more efficiently.

Figure 5.43 shows frequency dependencies of function $J$ for the barriers with ideally absorbing surfaces ($Y = 1$) from the traffic way side.

Comparing the graphs in Fig. 5.43 with the graphs in Figs. 5.39 and 5.41 it is easy to verify that the level of all the resonance spikes decreases and the mean level of parameter $J$ decreases in the entire frequency band under consideration. There are the isolated resonance spikes, but their level is no more than 0.5–0.3. If we turn to Fig. 5.44, which shows the structure of pressure amplitude field for the barriers with ideally absorbing walls, we can see, that the “global” resonances disappeared. There are only the resonances between the barriers and the faces of buildings of relatively low levels. The walls of the barriers, faced to the buildings, should also be absorbing to remove the resonances.

Figure 5.45 shows frequency dependencies of parameter $J$ for the barriers, covered with the grating of Helmholtz resonators. We select the following parameters of Helmholtz resonator: $b_1 = 0.40$ m, $b_2 = 0.05$ m, $2r_0 = 4 \times 10^{-3}$ m, $l = 0.5 \times 10^{-3}$ m, $\tilde{r} = 2$. The natural frequency of Helmholtz resonator $f_0 \approx 97$ Hz.

As we can see, the frequency dependencies of parameter $J$ for the barriers with the walls in the form of the gratings of Helmholtz resonators are little different from the barriers with the ideally absorbing walls. Compare the graphs in Figs. 5.43 and 5.45. It is fair to say that, the barriers with the absorbing walls can be implemented in practice. They can ensure effective noise protection on the modern urban streets. We note finally,
FIG. 5.44: Pressure amplitude field in the space between the buildings when sound source frequency is: (a)—45 Hz, (b)—84 Hz (barriers surfaces, faced to the traffic way, are ideally absorbing, and the surfaces of the barriers, faced to the buildings—are acoustically rigid)

FIG. 5.45: Frequency dependency of parameter $J$ when the surfaces of the barriers, faced to the traffic area, are covered with the grating of resonant absorbers (the barriers surfaces faced to the buildings are acoustically rigid): the left column—the buildings to the left of the traffic area, the right column—the buildings to the right of the traffic area, the floors are numbered from top down

that the considered problem can easily be used in more complicated case, when all the walls of the barriers absorb sound energy.

The results given in Chapter 5, are based on works [13,16–18,25,31], which were published.
REFERENCES


