PREFACE

From a formal point of view, this monograph may be regarded as the second edition of the author's book "An Introduction to Anisotropic Elasticity" (Nauka-Fizmatlit, Moscow, 1988). However this is, essentially, a new book updated and considerably extended. Since it has preserved its character, I think it to be worthwhile to retain the preface written by my teacher and friend Valentin Valentinovich Novozhilov (see preceding page).

It is impossible even to list all the numerous publications in which various problems in the analysis and the design of structures of anisotropic materials are discussed. In this connection, the fundamental work of N. G. Chentsov and S. G. Lekhnitsky must be mentioned above all. At the same time the number of publications on the anisotropy of elastic properties is strikingly small. Among them the following ones [1.1, 1.2, 2.4–2.6, 2.8, 3.4, 3.5, 4.1, 7.1–7.3, 11.6] should be noted.

The present volume was designed as a brief systematic account of the class of problems that are connected with the laws of elasticity of anisotropic materials. Emphasis is placed on either new problems or those that are seldomly treated in the literature—problems that are of great significance (generalized plane strain and plane state of stress, incompressible and reinforced materials, geometrical and physical nonlinearity, the nonlinear theory of shells, plane problem, brittle fracture, nonlinear Volterra's dislocations).

A few words on the contents of the book.

In Chapter I a brief but systematic account is given of the use of symmetry considerations in the mechanics of solids (deformable bodies). The concept of a symmetry group for a finite body is made clear. Restrictions on possible types of symmetry due to the existence of a space crystal lattice are revealed. The existing crystal classes and textures are listed. The Neumann principle is formulated.

In Chapter II the structure of Hooke's law for anisotropic materials is discussed. A nontraditional approach makes it possible to introduce symmetric Poisson coefficients of different orders. This enables solution of P. Bekhterev's problem [2.2] of finding narrowest (unimprovable) bounds on the variation of elastic constants [3.2]. The ideas of V. V. Novozhilov

concerning the principal axes of anisotropy and the reduction in the number of essentially different elastic constants are further developed. Attention is given to the problem of unification of elastic constant matrices within each crystal system.

In Chapter III nonlinearly elastic anisotropic materials are dealt with. The structures of strain-energy density functions corresponding to various anisotropic materials are investigated. For small deformations, the conditions for the appropriate law of elasticity to acquire the form of the corresponding Hooke's law are given.

In Chapter IV we deal with deformation anisotropy which occurs for large deformations of an elastic material. An incompressible isotropic material reinforced by families of cords (filaments) is considered.

In Chapter V the results obtained are used for shells subjected to large deformations. A brief outline of the simplest "working" nonlinear theory of elastic shells (without loss of generality) is given. The problem of the choice and design of strain-energy density functions is treated.

In Chapter VI we are concerned with shells reinforced by two families of inextensible or slightly extensible filaments. Two types of reinforcement are considered: in the middle surface and continuous through the shell thickness. Solutions to a number of applied problems are given.

In Chapter VII the linear plane problem for an orthotropic material is discussed. Extensive use is made of the complex variable method developed by the author and also of his algebraic method for solving boundary-value problems.

In Chapter VIII we deal with the nonlinear plane problem. An original approach makes it possible to obtain in finite form solutions to geometrically nonlinear problems.

In Chapter IX a brief outline of the results obtained by the author on the nonlinear theory of cracks in an isotropic material is given. The ways to extend the theory to anisotropic materials are discussed.

In Chapter X we deal with the essentials of the nonlinear theory of Volterra's dislocations. The wedge disclination is considered in detail.

In Chapter XI we are concerned with those aspects of theory that are closely connected with the subject matter of the preceding chapters. The theoretical material was published at different times by the author [11.2–11.6] and is, essentially, an extension of the fundamental work of V. V. Novozhilov and L. I. Sedov. The relevant published theoretical results bear witness to the undiminished attention paid to the problems of the structure and properties of the constitutive equations of the mechanics of deformable bodies. Instead of going into the contents of this chapter, we shall only mention that in §12 the general relations for large deformations are introduced, and in §13 the reasons are given for the substantial advantages of the

conventional stresses (the Biot stresses) over the true stresses (the Cauchy stresses) in geometrically nonlinear mechanics.

Appendices A and B are given for the convenience of the reader. They contain the fundamentals of curvilinear coordinates, curves, and surfaces, which are needed for reading the book.

At the end of the book a list of references and a subject index are given. The bibliography mostly contains references to Russian authors that are not widely known outside the former Soviet Union.

Many of the chapters in the book may be read independently of the other chapters, yet there are many cross references. Equations within the same chapter are referred to by the section number followed by the number of the equation, and equations in the other chapters are indicated using the triple-number notation, the first number being that of the chapter in which the equation is given. A similar triple-number notation is used for both figures and tables throughout the book. In the book the repetition of a *Greek* index implies summation with respect to that index.

When writing this book, the author sometimes felt as if he were performing a sort of a balancing act on a nebulous boundary between the fundamental theoretical results and their applications. While having preference for the former, the author, nevertheless, also wanted to show how the theory works (naturally, trying not to get bogged down in cumbersome details). The book contains both the standard material and some of the new results obtained by the author and his colleagues. Certain "nonhomogeneity" of the material may be justified by the attempt to stimulate the interest of the reader in doing research work in this difficult but promising area of the mechanics of solids.